A braneworld universe from colliding bubbles

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Abstract

Much work has been devoted to the phenomenology and cosmology of the so-called braneworld universe, where the \((3+1)\)-dimensional universe familiar to us lies on a brane surrounded by a \((4+1)\)-dimensional bulk spacetime that is essentially empty except for a negative cosmological constant and the various modes associated with gravity. For such a braneworld cosmology, the difficulty of justifying a set of preferred initial conditions inevitably arises. The various proposals for inflation restricted to the brane only partially explain the homogeneity and isotropy of the resulting braneworld universe because the three-dimensional homogeneity and isotropy of the bulk must be assumed a priori. In this Letter we propose a mechanism by which a brane surrounded by AdS space arises naturally in such a way that the homogeneity and isotropy of both the brane and the bulk are guaranteed. We postulate an initial false vacuum phase of \((4+1)\)-dimensional de Sitter, or possibly Minkowski, space subsequently decaying to a true vacuum of anti-de Sitter space, assumed discretely degenerate. This decay takes place through bubble nucleation. When two bubbles of the true AdS vacuum eventually collide, because of the degeneracy of the true AdS vacuum, a brane (or domain wall) inevitably forms separating the two AdS phases. It is on this brane that we live. The \(SO(3,1)\) symmetry of the collision geometry ensures the three-dimensional spatial homogeneity and isotropy of the universe on the brane as well as of the bulk. In the semi-classical \((\bar{h} \to 0)\) limit, this \(SO(3,1)\) symmetry is exact. We sketch how the leading quantum corrections translate into cosmological perturbations. \(\copyright\) 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

We propose a cosmogony based on collisions of true anti-de Sitter (AdS) vacuum bubbles in \((4+1)\) dimensions expanding at nearly the speed of light within a surrounding \((4+1)\)-dimensional de Sitter (dS) or Minkowski (M) space false vacuum. The bubble collisions produce a braneworld universe very similar to the cosmogony with a \((3+1)\)-dimensional, positive-tension brane surrounded by \((4+1)\)-dimensional AdS space proposed by Randall and Sundrum [1,2].

Initially a \((4+1)\)-dimensional spacetime consisting of either de Sitter space or Minkowski space is assumed. In the former case an initial epoch of \((4+1)\)-dimensional ‘old’ inflation [3] ensures a very nearly \(SO(5,1)\) symmetric state prior to bubble nucleation, regardless of whatever departures from de Sitter space may have initially existed. The homogeneity and isotropy of the resulting \((3+1)\)-dimensional braneworld universe is thus assured, as we shall explain in more detail. In the latter case, a metastable \((4+1)\)-dimensional Minkowski state vacuum must be
postulated at the outset; however, it is not at all implausible that some as yet unknown theory of the initial conditions of the universe prefers empty Minkowski space.

The false de Sitter or Minkowski vacuum decays through the nucleation by quantum tunnelling of bubbles filled with the lower energy true AdS vacuum [4–6]. The bubble wall separating the two phases may take the form of either a brane or an accelerating domain wall. We postulate that the AdS vacuum is discretely degenerate, so that the energy from the collision of two bubbles is not entirely transformed into energy dispersed into the $4+1$ dimensions. In the case of a degenerate AdS vacuum, when the two colliding bubbles contain differing AdS phases, after the collision at least part of the energy is transferred to a brane (or domain wall) that must mediate between the two phases. This is energy that remains localized in the fifth dimension. In this Letter we shall call this brane (or domain wall) our local brane because this is where the $(3+1)$-dimensional universe familiar to us resides.

To the extent that our universe has a violent beginning resulting from the collision of branes, the scenario presented here has much in common with the brane inflation proposed by Dvali and Tye [7] and the ekpyrotic universe recently proposed by Khoury et al. [8]; however, the physics by which preferred initial conditions are determined is quite different. The scenario proposed here also bears some similarities to the work of Gorsky and Selivanov [9], Perkins [10] considered a braneworld scenario in which our universe is situated on a bubble wall. However, in his scenario bubble collisions are regarded as catastrophic. The dynamics of bubble collisions have been studied by Guth and Weinberg [11], Hawking, Moss and Stewart [12], and Wu [13].

Before embarking on a detailed description of the colliding bubble scenario, we first highlight some of the problems arising from the assumption of a bulk with a negative cosmological constant. These difficulties, which render many braneworld cosmogonies problematic, are avoided in the scenario proposed here because of the presence of a prior epoch of de Sitter or Minkowski space. Most braneworld models, including those with inflation on the brane, are plagued by the same horizon and smoothness problems present in non-inflationary cosmogonies but in $(4+1)$ rather than $(3+1)$ dimensions. The persistence of very near spatial homogeneity and isotropy on the brane requires that the bulk at the outset be very nearly three-dimensionally homogeneous and isotropic [14]. Otherwise, through gravity an inhomogeneous bulk inevitably induces inhomogeneities on the brane. A successful braneworld cosmology must therefore explain why the bulk was very nearly homogeneous and isotropic at the beginning. A mechanism that merely smooths an initially inhomogeneous brane embedded in pristine AdS space, such as brane inflation, does not suffice because the necessary bulk homogeneity and isotropy must be put in by hand.

Anti-de Sitter space, or more broadly any spacetime with the stress-energy of a negative cosmological constant, lacks the ability to erase small initial perturbations from homogeneity and isotropy. For the case of a positive cosmological constant departures from homogeneity and isotropy must be put in by hand.

The differing evolution of $dS$ and AdS space is readily illustrated by considering the family of time-like geodesics emanating from an arbitrary point $P$ in the spacetime, as indicated in Fig. 1. One might, for example, interpret these geodesics as the worldlines of the shrapnel from an exploding bomb! In both cases the trajectories initially diverge in proportion to their relative velocities, just as in a Milne universe (which is but another coordinatization of flat Minkowski space). However, after a proper time comparable to the curvature radius, the trajectories in AdS start to converge, eventually refocusing to a point (where the bomb momentarily re-assembles itself)! This sequence of divergence and reconvergence repeats itself $ad$ infinitum. In de Sitter space, however, the non-vanishing spacetime curvature has precisely the opposite effect. Rather than re-converging, the initial linear divergence of the trajectories accelerates, so that eventually the pieces of
Fig. 1. Differing evolution of timelike geodesics in anti-de Sitter and de Sitter space. The left panel (a) shows a conformal diagram for anti-de Sitter space, which has the form of an infinite vertical strip of finite thickness. The horizontal and vertical directions indicate space and time, respectively, and null geodesics travel obliquely at 45 degrees. The right panel (b) shows the conformal diagram of de Sitter space which has the form of a cylinder of finite height. The dashed vertical boundaries are identified. In both panels the forward timelike geodesics of a spacetime point $P$ are indicated, as well as the asymptotic light cones forming the boundary of the causal future of $P$. In anti-de Sitter space the timelike geodesics periodically refocus ad infinitum. By contrast, in de Sitter space the geodesics diverge, eventually losing causal contact with each other.

Fig. 2. Causal structure of the single-brane Randall–Sundrum braneworld spacetime. The surfaces of constant time in the Randall–Sundrum coordinates are generated by the family of all space-like geodesics emanating from a certain fixed point on the conformal boundary. The worldlines of constant transverse coordinate (i.e., the “fifth dimension” of the Randall–Sundrum scenario) represent uniformly accelerating observers, all with the same uniform acceleration away from this point. The Cauchy horizons $H_-$ and $H_+$ coincide with the past and future boundaries of the region covered by these coordinates.

Thus, anti-de Sitter space is plagued with a bizarre causal structure. As indicated in Fig. 1(a), maximally extended AdS space is bounded by timelike boundaries at infinity from which and to which information flows. It does not make sense to postulate eternal AdS without some theory of appropriate boundary conditions on these edges or on the Cauchy horizons that result when one attempts to limit consideration to a subspace of the maximally extended spacetime. In the original Randall–Sundrum proposal (whose causal structure is indicated in Fig. 2), the usual Randall–Sundrum coordinates

$$ds^2 = dy^2 + \exp[2y] \left[ -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right]$$

cover only a minute portion of maximally extended AdS space. The coordinate patch covered by (1) forms a globally hyperbolic subspace of maximally extended anti-de Sitter space—that is, initial data on a slice of constant cosmic time in the Randall–Sundrum coordinates is not constrained by any consistency conditions and completely suffices to determine the fields in the triangular region covered by these coordinates. But the lower light-like boundary constitutes a Cauchy horizon, and one may legitimately inquire, what principle determines the initial conditions on this boundary? And if they are trivial, as is often assumed, why is this so? Although Fig. 2 illustrates the special case of a static Randall–Sundrum universe, the lower Cauchy horizon persists in Randall–Sundrum cosmological models of an expanding universe.

In the present proposal AdS bubbles arise through the decay of a false de Sitter space or Minkowski space vacuum. The AdS space that emanates inside the bubble is produced in a precise and predictable way, with quantum fluctuations that are predictable and calculable. The problems described above are avoided. In the next section, we describe the geometry and dynamics of the production and collisions of AdS bubbles, explaining why in the semi-classical ($\hbar \to 0$) limit the resulting brane universe is homogeneous and
2. AdS from colliding bubbles

The possibility has been previously advanced that the true vacuum might not coincide with what we commonly perceive as the true vacuum. That is, rather than being either empty Minkowski space or de Sitter space with a remarkably small positive cosmological constant, the true vacuum might take the form of some lower energy state with a negative cosmological constant. If this were true, we would live in a metastable false vacuum state susceptible to decay to the true vacuum through bubble nucleation. Phenomenologically, given the observed persistence of our universe, an approximate upper bound on the rate $\Gamma$ at which bubbles of true AdS vacuum spontaneously nucleate can be established, but it is not possible to reject this possibility altogether.

A manifestly covariant description of the dynamics of false vacuum was given by Sidney Coleman, first ignoring gravity [4] and then extended to include the gravitational corrections in work with F. de Luccia [5]. Important prior work is contained in [6]. This process is illustrated in Fig. 3. We summarize below the principal results of these papers to the extent that they are needed here and refer the reader to the original papers for a more detailed and rigorous discussion.

False vacuum decay takes place at zero temperature, or said another way, from an initial state no preferred time direction. The consequences of the lack of a preferred time direction are profound. They render false vacuum decay qualitatively different from the more familiar thermal tunnelling, which enjoys considerably less symmetry due to the fact that a thermal state singles out a preferred time direction. For false vacuum decay in $(d + 1)$ dimensions, the resulting classical expanding bubble solution possesses an $SO(d, 1)$ symmetry. The symmetry group in the absence of any bubbles (which in the case of de Sitter space would be $SO(d + 1, 1)$) is broken by the presence of a single bubble to $SO(d, 1)$, the subgroup of transformations that leaves invariant a spacetime point known as the nucleation center.

For a spacetime with two bubbles, the resulting symmetry is further reduced, but considerable residual symmetry remains. Suppose that two bubbles nucleate at spacelike separated nucleation centers $N_L$ and $N_R$, where $L$ and $R$ denote left and right. This separation must be spacelike, for otherwise one bubble would nucleate within the other. For two bubbles the solution remains symmetric under the subgroup of transformations that leave invariant the line (or spacelike geodesic) passing through $N_L$ and $N_R$. For a pair of colliding bubbles nucleating in $(4 + 1)$-dimensional dS space, the $SO(5, 1)$ symmetry breaks
to $SO(3, 1)$. This residual symmetry has the following consequences. First, one may always choose a coordinate system in which the two bubbles nucleate at the same time. Hence, unlike for thermal tunnelling, here it is not meaningful to ask which of the two bubbles is the bigger one. Moreover, once a coordinate choice is made in which the bubbles nucleate simultaneously, substantial residual symmetry remains. While in a particular coordinate system the bubbles first collide at a given spacetime point $P$, for any other point $P'$ of the locus of points where the bubbles collide, a coordinate transformation exists such that the bubbles first collide at $P'$. It is this symmetry mapping $P$ into $P'$ that is responsible for the three-dimensional spatial homogeneity and isotropy of the universe on the local brane.

We now turn to a more detailed consideration of what happens during the bubble collision. For vacuum decay with a single scalar field where the AdS vacuum is non-degenerate, the energy of the colliding bubble walls, absent some good reason to the contrary, dissipates in the fifth dimension (the direction parallel to the line connecting the two nucleation centers) but in an $SO(3, 1)$ symmetric way, much as in the initial stages of thermalization first envisaged for ‘old’ [3] or ‘extended’ [15] inflation. However, if the AdS vacuum is finitely degenerate (in the simplest case with two such AdS vacua related by a $Z_2$ symmetry), topology demands that a domain wall form after the bubble collision to separate the two distinct AdS domains when the colliding bubbles contain differing AdS phases. While energy that disperses in the fifth dimension could as well be produced in the collision, topology requires that a domain wall form to mediate between the two AdS states. This wall, which we call our local brane (on which we live) is at rest in the center of mass frame of the colliding bubble. Of the kinetic energy left over after this domain wall has been formed, a part is expected to stick to the brane (and to be confined to it, as is typically assumed in the Randall–Sundrum scenario) and another part is expected to disperse into the bulk. The energy dispersed into the bulk, however, is $SO(3, 1)$ symmetric, and therefore does not induce any irregularities on the brane. Moreover, this energy does not fall back onto the brane, because when the gravitation of the brane is taken into account, the brane accelerates away from this symmetric, dispersive debris.

Fig. 4 shows schematically first a $(2 + 1)$-dimensional representation of the colliding bubble geometry in the left panel and then in the right panel a cut-away of the surface of equal proper distance from the two nucleation centers. The point $M$ is the midpoint of $N_L$ and $N_R$. The curve labeled $C$ indicates the line along which the two bubbles collide. In the section on the right, several hyperbolic coordinate patches are generated by the $SO(3, 1)$ symmetry separated by the backward and forward lightcones on $M$. Points along the solid curves are rendered equivalent by this symmetry. These are lines of constant cosmic temperature on our local brane, which cools as the universe expands. In the full $(3 + 1)$-dimensional case, these curves are three-dimensional spacelike hyperboloids of constant negative spatial curvature.

It has been suggested by Coleman and de Luccia that isolated AdS bubbles generically collapse into black holes because of the $SO(4, 1)$ symmetry of the
perfect classical expanding bubble solution. The argument, which is closely related to the perfect refocusing of timelike geodesics emanating from a point described above, is as follows. The universe inside an AdS bubble is a hyperbolic universe that recollapses after a finite amount of time. If the background stress-energy inside the bubble were that of a perfect negative cosmological constant, this would pose no problem. The resulting 'Big Crunch' would be nothing but a coordinate artifact, as indicated in Fig. 5. However, if the scalar field undergoing tunnelling has not reached the true vacuum by the light cone $L$ (which it never does), a singularity in the evolution of the scalar field on the light cone $L'$ generally results. In both cases the behavior of the scalar field near the lightcones is described by a second-order, Bessel-like ordinary differential equation having one regular and one singular solution. On $L$ it is clearly correct to choose the regular solution. This is the initial condition that results from the Euclidean instanton. But unless the potential is extremely finely tuned, upon propagating to $L'$, at least a small admixture of the divergent, irregular solution will be present, causing the scalar field kinetic energy to diverge. In the case of colliding bubbles, however, the underlying symmetry that led to the divergence is broken because the collision generically sends out a wave that spoils the finely tuned convergence of the scalar field that led to black hole formation. Thus the AdS space inside the bubbles is allowed to persist.

To simplify the analysis, we idealize the bubble walls as infinitely thin and assume that upon colliding the bubbles transfer all their available energy onto an infinitely thin brane, with all excess energy converted into radiation and matter confined to the brane. The collision geometry is indicated in Fig. 6. The subsequent evolution of the brane depend on the equation of state on our brane, which we take to be arbitrary, since the considerations presented in this Letter do not depend on its details.

Since bubbles nucleate stochastically, at a rate $\Gamma$ with the dimension of inverse volume inverse time, the proper distance between nucleation centers is a random variable. Consequently, the spatial curvature of the resulting intermediate brane universe varies between bubble pairs. In this scenario it is essential that bubble collisions are rare. A collision with a third bubble would be catastrophic; a wave of energy would move toward us at very nearly the speed of light striking us with essentially no warning. That this has not yet happened is a most trivial application of the anthropic principle. In the case of bubbles expanding in Minkowski space ($M^5$), the nucleation rate $\Gamma$...
does not vary with time, the bubbles will all eventually percolate. Therefore the exterior $M^4$ space could not have persisted infinitely far into the past unless some mechanism, such as a variant of that of extended inflation [15], is postulated to render $M^5$ eternal into the past by making $\Gamma$ vanish in limit of the infinite past. This percolation, however, does not occur for bubbles expanding in $dS^5$ for the small nucleation rates of interest here [11].

3. Quantum corrections: generation of Gaussian cosmological perturbations

In the previous section we demonstrated how a homogeneous and isotropic universe can arise from the collision of two expanding AdS bubbles. We employed the semi-classical ($\hbar \to 0$) limit in which prior to colliding each bubble possesses an exact $SO(4, 1)$ symmetry about its nucleation center, because in the semi-classical limit fluctuations about the configuration of least Euclidean action describing the bubble nucleation process are suppressed as well as the quantum fluctuations of the wall and of the surrounding fields afterward. In this limit one obtains an absolutely homogeneous and isotropic universe, quite unlike the one that we observe. Quantum corrections, however, alter this picture. The leading order corrections in $\hbar$ yield a calculable spectrum of linearized Gaussian fluctuations. These are the usual Gaussian cosmological perturbations.

For calculating the cosmological perturbations, the Bunch–Davies vacuum of de Sitter space (or the Minkowski space vacuum for the case of bubbles nucleating in Minkowski space) define a natural set of initial conditions. The Bunch–Davies vacuum is an attractor, so an initial state deviating from this state evolves to become successively better approximated by the Bunch–Davies vacuum. A full calculation of the perturbations is postponed until a later paper [16]. Here we limit ourselves to a simplified qualitative description ignoring gravitational back reaction and assuming infinitely thin bubbles to illustrate the underlying physical processes.

The quantum state for the fluctuations of a thin wall bubble about the perfect $SO(4, 1)$ expanding bubble solution for a bubble arising from false vacuum decay was first elucidated by J. Garriga and A. Vilenkin [17]. In the thin wall approximation, with the gravitational back reaction of the perturbations ignored, the only available degree of freedom consists of normal displacements of the bubble wall, which may be described as a scalar field localized on the bubble wall itself. We consider the perfect $SO(4, 1)$ symmetric expanding bubble (which has the geometry of de Sitter space). Displacements along the outward normal are described by a free scalar field of mass $m^2 = -4H^2$. The quantum state of this field must obey the same $SO(4, 1)$ symmetry as the classical expanding bubble solution. One might at first sight admit the possibility that bubble nucleation could somehow spontaneously single out a preferred time direction. That this is not possible can be demonstrated by contradiction. Suppose that such a choice of preferred time direction were in fact made. Then all such choices must be equally weighted, according to a Lorentz invariant measure. The calculation of the vacuum decay rate would contain a factor consisting of an integration over the infinite hyperbolic domain (with the geometry of $H^5$) of all such possible choices, thus implying an infinite false vacuum decay rate, a conclusion which is clearly absurd. The $SO(4, 1)$ invariance of the quantum state of the fluctuations suffices to completely fix this state. It is described by the Bunch–Davies vacuum of the de Sitter space of the expanding bubble wall.

Let $\chi_L$ and $\chi_R$ be the scalar fields just described for the two colliding bubbles, using the sign convention that $\chi$ is positive for outward displacements. To analyse how these displacements translate into perturbations of the brane that arises from the bubble collision, it is convenient to consider the linear combinations

$$\chi_+ = (\chi_L + \chi_R)/\sqrt{2},$$
$$\chi_- = (\chi_L - \chi_R)/\sqrt{2} \quad (2)$$

at the instant of collision. The mode $\chi_+$ temporally advances (or retards) the surface on which the bubbles collide leading to under and overdensities. The hyperboloids of constant cosmic temperature are thus warped. This mode translates into scalar density perturbations of the cosmology on the local brane. The mode $\chi_-$, on the other hand, displaces the surface of collision in the normal direction—that is, spatially toward the one or the other bubble. (See Fig. 7).
Although the geometry of the background solution is $Z_2$ symmetric, as in the Randall–Sundrum scenario, the $Z_2$ symmetry here is qualitatively different from the orbifold $Z_2$ symmetry postulated in the Randall–Sundrum proposal. In our proposal, both $Z_2$ even and $Z_2$ odd perturbations are allowed because the degrees of freedom on one side of the brane do not coincide with those on the other side. In the Randall–Sundrum scenario with a single brane there is no bending mode because the relevant degrees of freedom have been decreed not to exist through the orbifold construction. In our case, this mode does in fact exist. The extrinsic curvature (relative to the outward normal) on the two sides need not coincide because twice as many degrees of freedom are present.

4. Concluding remarks

We have demonstrated how the collision of two bubbles filled with AdS space expanding in de Sitter space or Minkowski space can give birth to a brane-world cosmology surrounded by infinite anti-de Sitter space, very similar to the single-brane Randall–Sundrum model. In this colliding bubble scenario well-defined initial conditions naturally arise. The smoothness and horizon problems in $(4 + 1)$ dimensions are absent in this scenario. Although the considerations presented in this Letter apply equally well regardless of the equation of state on the local brane produced after the bubble collision, the fact that inflation on the resulting $(3 + 1)$-dimensional spatially hyperbolic universe can altogether be avoided is intriguing.

If sufficient energy is deposited on this brane after sufficient expansion of the initially nucleated bubble, $\Omega$ today can be very close to one.

We now consider some orders of magnitude. In the Randall–Sundrum scenario (just as in compact five-dimensional Kaluza–Klein models), an effective four-dimensional Planck mass $m_4$ large compared to the five-dimensional Planck mass $m_5$ may be obtained by making the size of the extra dimension $\ell$ large. Here we set $h = c = 1$. In the Randall–Sundrum case $\ell$ is the curvature radius of the AdS bulk. Since $m_2^2 = m_3^3\ell$, $m_4 = m_5(m_5\ell)^{1/2}$.

The five-dimensional Einstein equation and Israel matching condition give $\Lambda = m_3^3\ell^{-2}$ and $\sigma = m_2^2\ell^{-1}$, respectively, where $\Lambda$ is the five-dimensional negative cosmological constant in the bulk and $\sigma$ is the four-dimensional cosmological constant that would be required on the brane for it to have the geometry of four-dimensional Minkowski space ($M^4$). The tension of the wall separating the AdS from the dS phase and that of the local brane in general differ, but for the order-of-magnitude analysis here we take them to coincide. It follows that the approximate size of the critical bubble is $r \approx \sigma / \Lambda \approx \ell$. The vacuum decay rate is approximately $\Gamma = \ell^{-3} \exp[-S_F]$ where $S_F \approx \sigma r^4 \approx (m_4 \ell)^3 \approx (m_4 \ell)^2$. An extra dimension large compared to the Planck scales makes the dimensionless Euclidean action large, leading to an exponentially small bubble nucleation rate. Therefore, a very substantial amount of expansion takes place before bubble pairs collide, and three bubble collisions are rare. The perturbations are of order $1/\sqrt{S_F}$.

We now consider the spatial curvature of the universe on the local brane. The energy density on the
brane produced at the bubble collision is approximately $E_c = (R/r)\sigma$ where $\sigma \approx m_5^4 (m_\xi)^{-1}$ and $R$ is the distance between the nucleation centers of the two bubbles. As long as $(R/r) \lesssim (\ell/\xi) 5^{-1}$ is the five-dimensional Planck length, this energy density is sub-Planckian from the five-dimensional point of view.\(^1\) At the collision $(1 - \Omega_c) \approx (\ell/R)^5$, which is exponentially small. The bubble pair separation $R$ is a random variable differing from pair to pair. The average bubble pair separation $\bar{R}$, however, may be estimated by setting $\Gamma R^5 \approx 1$ assuming bubbles expanding in $M^5$. A more detailed discussion of the probability distribution will appear elsewhere \([18]\). In the case of bubbles expanding in $dS^5$, the interbubble separation (defined as the length of the spacelike geodesic connecting the two nucleation centers) is limited by the size of the horizon of the exterior $dS^5$ space. However, the geometry of colliding bubbles expanding in $dS^5$, which is worked out in detail in a subsequent paper \([16]\), is such that the spatial curvature radius of the surface of brane collision becomes infinite as interbubble separation $R$ defined above approaches its limiting value. The factor $e^{-3\Omega_c}$ provides a natural mechanism to adjust $\Omega_c$ so close to one that $\Omega$ today remains very close to one without resort to unnatural fine tuning.

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\(^1\) The case of trans-Planckian energy densities immediately after the brane collision, however, need not necessarily be discarded, because the analysis of what happens afterward depends little on the details of how the universe cools after the collision. One might regard a brief trans-Planckian epoch after the collision as a sort of black box, much as re-heating at the end of inflation is commonly regarded. One is able to compute the perturbations from inflation with confidence despite our almost total ignorance of how re-heating occurs.
Massive fields temper anomaly-induced inflation: the clue to graceful exit?

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Abstract

A method of calculating the vacuum effective action for massive quantum fields in curved spacetime is outlined. Our approach is based on the conformal representation of the fields action and on the integration of the corresponding conformal anomaly. As a relevant cosmological application, we find that if taking the masses of the fields into account, then the anomaly-induced inflation automatically slows down. The only relevant massive fields for this purpose turn out to be the fermion fields. So in supersymmetric theories this mechanism can be specially efficient, for it may naturally provide the graceful exit from the inflationary to the FLRW phase. Taking the SUSY breaking into account, the anomaly-induced inflation could be free of the well-known difficulties with the initial data and also with the amplitude of the gravitational waves. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Inflation proved very useful in solving numerous problems of the theory of the Early Universe. The conventional approach, which is based on the inflaton field, is extremely helpful for the inflationary phenomenology [1]. At the same time, the origin of the inflaton remains unclear, and it is quite reasonable to look for other approaches yielding similar phenomenological results. Recently, there was an increasing interest for the cosmological applications of quantum field theory in curved spacetime, specially in connection to vacuum effects [2].

An exact calculation of the vacuum effective action is possible only for some special models, usually for massless conformal invariant matter fields and special restrictions of the classical background spacetime. An important example is the $d = 4$ theory on the conformally flat (or similar FLRW with $k = \pm 1$) metric, where the Reigert–Fradkin–Tseytlin effective action [3] is exact and provides the most natural theoretical background for inflation [4–6].

The anomaly-induced action can be applied at high energies, where the masses of the fields are negligi-
ble [5–7]. If, at the high-energy region, there is a supersymmetry (SUSY), one meets the stable version of the anomaly-induced inflation, which starts without any fine tuning and leads to sufficient expansion of the Universe [8]. In the course of inflation, the typical energy scale decreases, and one encounters the non-stable version (Starobinsky model [4]), which provides fast exit to the FLRW phase [5]. The transition from stable to non-stable inflation can be achieved through soft SUSY breaking and the decoupling of the massive sparticles at low energy [9]. The potential importance of supersymmetry is due to the following circumstances. The anomaly-induced inflation may be stable or unstable—depending on the particle content of the underlying quantum field theory. This opens the possibility to interpolate, in a natural way, between the stable regime at the beginning of inflation and the unstable regime at the end of inflation. In particular, the supersymmetric gauge theory may have a particle content corresponding to the stable inflation. The advantage of stable inflation is that it starts independent of the initial data for the conformal factor of the metric, which can emerge after the string phase transition. If, in the course of inflation, the typical gravitational energy scale decreases, and if the sparticles have much bigger masses than the other particles, then they decouple and at the last stage of inflation the number of active degrees of freedom diminishes. This is possible because can be sensibly defined from the value of the Hubble parameter which indeed lessens during the tempered expansion caused by massive fields, as will be shown below. Therefore this mechanism automatically brings inflation into an unstable phase, with the possibility of an eventual transition to the FLRW regime.

Thus, supersymmetry and its breaking may provide a natural qualitative mechanism for the graceful exit from the inflationary phase, without fine-tuning of the parameters of the theory. Furthermore, the spectrum of the gravitational waves in this model [8,11] can be in agreement with the existing CMBR data. Still, in this picture there is an unclear point, namely, the use of the anomaly-induced action for the massive fields is not completely justified. Also, the transition to the FLRW phase may not necessary occur after the exit from the exponential inflation. The behavior of the Universe depends on the initial deviation from the exponential expansion law [5–7,12]. It was established that the universe goes to the FLRW regime if this deviation leads to an expansion slower than exponential, while it goes to the uncontrolled “hyperinflation” [6] if it leads to a faster expansion. Therefore, it would be very nice to learn that the masses of the fields really slow down inflation. If this would be so, the graceful exit to the FLRW phase would not require any suppositions concerning initial perturbations at the instant when SUSY breaks down!

In this Letter we are going to address both mentioned problems. We shall develop a simple but reliable Ansatz for the effective action of massive fields. Our approach to the derivation of the effective action is based on the Cosmon Model, which was developed in [13] for other purposes—see also [14]. The idea is to construct the conformal invariant formulation [13] of the gauge theory (Standard Model or extensions thereof, including GUT’s), and then use the well-known methods to derive the anomaly-induced action. The procedure of “conformization” is known for a long time as applied to General Relativity [15] and Particle Physics [16]. At the classical level, the theory which results from this procedure is always equivalent to the original theory. Nevertheless, in the quantum theory the equivalence will be destroyed by the anomaly, which can be calculated explicitly. Besides the anomalous terms, there are the conformal invariant quantum corrections to the classical vacuum action. However, the complete method of deriving these contributions is not known, just because the effective action cannot be calculated exactly for the massive theories. The idea of our Ansatz is to disregard these contributions because they are, indeed, of higher order with respect to the leading ones we take into account. As we shall see, our results are in perfect agreement with the renormalization group. This provides better understanding of the applicability of our approach.

\[2\] We remark that this mechanism cannot explain the homogeneity and isotropy of the initial state. Perhaps this problem can be solved only in the framework of the string-inspired inflation.
2. Conformization and effective action

Our first purpose is to construct such a formulation of the Standard Model (SM) in curved spacetime which possesses local conformal invariance in $d = 4$. Actually, the procedure can be applied to any gauge theory and we are especially aiming at a realistic supersymmetric gauge theory, providing stability for the anomaly-induced inflation.

The original action of the theory includes kinetic terms for spinor and gauge boson fields, as well as interaction terms, all of them already conformally invariant. As for scalars (e.g., Higgs bosons) we suppose that their kinetic terms appear in the combination $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1/6R\phi^2$, providing the local conformal invariance. The non-invariant terms are the massive ones for the scalar and spinor fields:

\[
\frac{1}{2} \int d^4x \sqrt{-g} m_H^2 \psi^2, \quad \int d^4x \sqrt{-g} m^2 \psi \phi.
\]

Furthermore, there is an action for gravity itself, which is also non-invariant

\[
S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} R.
\]

In all mentioned cases the conformal noninvariance is caused by the presence of dimensional parameters $m_H^2$, $m$, $M_p^2 = 1/G$. The central idea of the Cosmon Model [13] is to replace these parameters by functions of some new auxiliary scalar field $\chi$. For instance, we replace

\[
m_H^2 \rightarrow \frac{m_H^2}{M^2} \chi^2, \quad m \rightarrow \frac{m}{M} \chi,
\]

\[
M_p^2 \rightarrow \frac{M_p^2}{M^2} \chi^2,
\]

where $M$ is some dimensional parameter, e.g., related to a high scale of spontaneous breaking of dilatation symmetry [16]. It is supposed that the new scalar field $\chi$ takes the values close to $M$, especially at low energies. But, there is a great difference between $\chi$ and $M$ with respect to the conformal transformation. The mass does not transform, while $\chi$ does. Then, the action of the new model becomes invariant under the conformal transformation

\[
\chi \rightarrow \chi e^{-\sigma}.
\]

which is performed together with the usual transformations for the other fields

\[
g_{\mu\nu} \rightarrow g_{\mu\nu} e^{2\sigma}, \quad \phi \rightarrow \phi e^{-\sigma}, \quad \psi \rightarrow \psi e^{-3/2\sigma}.
\]

It is easy to see that, in the matter field sector, the terms (1) are replaced, under (3), by Yukawa and quartic scalar interaction terms. These interactions are between physical fields (spinors and scalars) and the new auxiliary scalar $\chi$. Thus, in the matter sector our program of “conformization” is complete.

However, in the gravity sector (2) the conformal symmetry holds only for $\sigma = \text{const}$, i.e., only at the level of global dilatation symmetry, and this is still not what we need. Let us make one more step and require the local conformal invariance. Then the gravity action must be replaced by the expression [15]

\[
S_{EH} = -\frac{1}{16\pi G M^2} \int d^4x \sqrt{-g} \left[ R \chi^2 + 6(\partial\chi)^2 \right].
\]

where $(\partial\chi)^2 = g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$. After setting $\chi \rightarrow M$ the expression (6) becomes identical to the initial one (2). This fixing can be called “conformal unitary gauge” in analogy with the unitary gauge of ordinary gauge theories, and the scale $M$ can be associated to the vacuum expectation value of the spontaneously broken dilatation symmetry at high energies [13,14,16]. But, as far as we consider the spacetime dependence of $\chi$ and define its conformal transformation, the resulting theory exhibits local conformal invariance under (4) and (5) with $\sigma = \sigma(\chi)$. The new conformal symmetry is introduced simultaneously with the new scalar field $\chi$, which absorbs the degree of freedom of the conformal factor of the metric. The new theory satisfies, at the classical level, the conformal Noether identity

\[
\left[ 2 \frac{g_{\mu\nu}}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} - \frac{\chi}{\sqrt{-g}} \frac{\delta}{\delta \chi} + d\Phi_i \frac{\delta}{\delta \Phi_i} \right] S'_c = 0,
\]

where $\Phi_i$ stands for the matter fields of different spins, $d\Phi_i$ denote their conformal weights and $S'_c = S'_c[g_{\mu\nu}, \chi, \Phi_i]$ is the total (classical) action including the modified gravitational term (6).

When we quantize the theory, it is important to separate the quantum fields from the ones which represent a classical background. In order to maintain
the correspondence with the usual formulation of the SM, we avoid the quantization of the field $\chi$ which will be considered, along with the metric, as an external classical background for the quantum matter fields. It is well known (see, e.g., [17]) that the renormalizability of the quantum field theory in external fields requires some extra terms in the classical action of the theory. The list of such terms includes the nonminimal term of the $\int R \varphi^2$-type in the Higgs sector, and the action of external fields with the proper dimension and symmetries. The higher derivative part of the vacuum action has the form

$$S_{\text{vac}} = \int d^4x \sqrt{-g} \left\{ l_1 C^2 + l_2 E + l_3 \nabla^2 R \right\},$$

where, $l_{1,2,3}$ are some parameters, $C^2$ is the square of the Weyl tensor and $E$ is the integrand of the Gauss–Bonnet topological invariant. Now, since there is an extra field $\chi$, the vacuum action should be supplemented by the $\chi$-dependent term. The only possible, conformal and diffeomorphism invariant, terms with dimension 4 are (6) and the $\chi^4$-term. The last contributes to the cosmological constant, which we suppose to cancel and do not consider here in order to keep the discussion clear and compact. The effect of the cosmological constant will be reported elsewhere.

The next step is to derive the conformal anomaly in the theory with two background fields $g_{\mu\nu}$ and $\chi$. Here we follow the strategy used in a similar situation [18]. The anomaly results from the renormalization of the vacuum action [19] including the terms (6) and (8). For the sake of generality, let us suppose that there is also some background gauge field with strength tensor $F_{\mu\nu}$. Then the conformal anomaly has the form

$$\langle T^\mu_\nu \rangle = -\left\{ wC^2 + bE + c\nabla^2 R + dF^2 + f \left[ R \chi^2 + 6(\partial \chi)^2 \right] \right\},$$

where $w, b, c$ are the $\beta$-functions for the parameters $l_1, l_2, l_3$, and $f$ is the $\beta$-function for the dimensionless parameter $1/(16\pi GM^2)$ of the action (6) which will play an essential role in our considerations. Finally, $d$ is the $\beta$-function for the gauge coupling constant, which is standard. The values of $w, b$ and $c$ depend on the particle content of the model and are the following (see, e.g., [16])

$$w = \frac{N_0 + 6N_{1/2} + 12N_1}{120 (4\pi)^2},$$

$$b = -\frac{N_0 + 11N_{1/2} + 62N_1}{360 (4\pi)^2},$$

$$c = \frac{N_0 + 6N_{1/2} - 18N_1}{180 (4\pi)^2}.$$ (10)

Recall that the condition for stable inflation is $c > 0$ [4]. Then one can play with various models; e.g., from the previous equation it follows that the particle content of the SM ($N_0 = 4, N_{1/2} = 24, N_1 = 12$) leads to $c < 0$ (unstable inflation) whereas for the Minimal Supersymmetric Standard Model (MSSM) [20] ($N_0 = 104, N_{1/2} = 32, N_1 = 12$) one has $c > 0$ (stable inflation) etc. On the other hand from direct calculation using the Schwinger–DeWitt method (see, e.g., [17] and references therein) we get

$$f = \sum_i \frac{N_i}{3(4\pi)^2} m_i^2,$$ (11)

where $N_i$ are the number of Dirac spinors with masses $m_i$. We note that bosons do not contribute to $f$.

In order to obtain the anomaly-induced effective action, we put $g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot e^{2\sigma}$ and $\chi = \bar{\chi} \cdot e^{-\sigma}$, where the metric $\bar{g}_{\mu\nu}$ has fixed determinant and the field $\bar{\chi}$ does not change under the conformal transformation. Then, the solution of the equation for the effective action $\bar{\Gamma}$ proceeds in the usual way [3, 17, 18]. Disregarding the conformal invariant term [17] we arrive at the following expression:

$$\bar{\Gamma} = \int d^4x \sqrt{-\bar{g}} \left\{ wC^2 + \left( E - \frac{2}{3}\bar{\nabla}^2 \bar{R} \right) + 2b\sigma \bar{\Delta} + d\bar{F}^2 + f \left[ \bar{R} \bar{\chi}^2 + 6(\partial \bar{\chi})^2 \right] \right\} \sigma - \frac{3c + 2b}{36} \int d^4x \sqrt{-\bar{g}} \bar{R}^2,$$ (12)

which is the quantum correction to the classical action of vacuum.

Let us compare Eq. (12) with the quantum correction from the renormalization group. The expansion of the homogeneous, isotropic universe means a conformal transformation of the metric $g_{\mu\nu}(t) = a^2(\eta)\bar{g}_{\mu\nu}$, where $a(\eta) = \exp \sigma(\eta)$ and $\eta$ is the conformal time. On the other hand, the renormalization group in curved spacetime corresponds to the scale transformation of the metric $g_{\mu\nu} \to g_{\mu\nu} \cdot e^{2\eta}$ simultaneous with the inverse transformation of all dimensional quantities [17,
For any $\mu$ we have $\mu \rightarrow \mu' \cdot e^\epsilon$. Thus, one can compare the dependence of the anomaly-induced effective action (12) on $\sigma$ and the scale dependence of the renormalization-group improved classical action. The last is defined through the solution of the renormalization group equation for the effective action [17, 21]

$$\Gamma[e^{-2\gamma g_{\alpha \beta}, \Phi_i, P, \mu}] = \Gamma[g_{\alpha \beta}, \Phi_i(t), P(t), \mu],$$

(13)

where $\Phi_i$ is, as before, the set of all fields and $P$ the set of all parameters of the theory. In the leading-log approximation one can take, instead of (13), the classical action and replace (for the massless conformal theory) the renormalization group improved classical action. In the leading-log approximation one can take, instead of (13), the classical action and replace (for the massless conformal theory) $P \rightarrow P_0 + \beta P t$. Now, comparing (12) with the result of this procedure, one confirms the complete equivalence of the two expressions in the terms which do not vanish for $\sigma = \text{const}$. In particular, coefficient $f$ is a factor of the $\beta$-function for the Newton constant $G$.

The important general conclusion is that the anomaly-induced effect is a direct generalization of the renormalization group improved classical action. On the other hand, the correspondence in the $f$-term justifies the correctness of our approach and also helps to learn the limits of its validity.

3. The role of masses in tempering inflation

In order to understand the role of the particle masses in the anomaly-induced inflation, let us consider the total action with quantum corrections

$$S_t = S_{\text{matter}} + S_{\text{EH}} + S_{\text{vac}} + \Gamma,$$

(14)

which does not satisfy the Noether identity (7) because of the conformal anomaly. We notice that the account of quantum corrections into the matter sector would be senseless, because matter and radiation can be treated incoherently as a fluid. The only important features of the matter action are the energy density $\rho$, pressure $p$ and their dependence on $\sigma$. Of course, quantum effects may change these dependencies, but we can always choose some model for $\rho(a)$ and $p(a)$ without going into the details of quantum effects. On the other hand, as far as we suppose $\rho \ll M_p^4$ during the inflation period, the matter-radiation content cannot really affect the expansion of the Universe. Since $a(\eta)$ grows very fast during inflation, the energy density greatly decreases in a very short time and cannot play any role. Concerning pressure, its importance is even smaller, because matter is out of equilibrium during the inflation.

One of the approximations we made was to disregard higher loop and non-perturbative effects in the vacuum sector. There is an attractive possibility to consider the strong interacting regime using the AdS/CFT correspondence [22, 23], but this goes beyond the scope of the present Letter. Another approximation is that we take (as the comparison with the renormalization group shows) only the leading-log corrections. Usually, this is justified if the process goes at high energy scale. If the quantum theory has UV asymptotic freedom, the higher loops effects are suppressed, and our approximation is reliable. At the low-energy limit, we suppose that the massive fields decouple and their contributions are not important. Then Eq. (14) can be presented in the form

$$S_t = \int d^4 x \sqrt{-g}\left\{\left(-\frac{M_p^2}{16\pi M^2} + f\sigma\right)
\times \left[\tilde{R}\tilde{\chi}^2 + 6(\partial\tilde{\chi})^2 - \left(\frac{1}{4} - d\sigma\right)f\tilde{F}^2\right]
+ S_{\text{matter}} + \text{high. deriv. terms.}\right\},$$

(15)

One can see that the modifications with respect to the case of free massless fields [6] are an additional $f$-term and the contribution to anomaly due to the background gauge fields.

In order to restore the Hilbert–Einstein term and get the inflationary solution, we fix the conformal unitary gauge and put $\chi = \tilde{\chi} e^\sigma = M$. Furthermore, we can choose the conformally flat metric $\tilde{g}_{\mu
\nu} = \eta_{\mu\nu}$. Then the gravitational part of the action (15) becomes

$$S_{\text{grav}} = \int d^4 x \left\{2b(\partial^2\sigma)^2 - (3c + 2b)[(\partial\sigma)^2 + \partial^2\sigma]^2
- 6M_p^2 e^{2c}(\partial\sigma)^2\left[1 - \frac{16\pi M^2}{M_p^2} f\right]
- \left(\frac{1}{4} - d\sigma\right)f\tilde{F}^2\right\}.\right.$$

(16)

Computing the equation of motion in terms of the physical time $t$ (where $dt = a(\eta)\ d\eta$) we find

$$a^2\dddot{a} + 3a\ddot{a} - \left(5 + \frac{2b}{c}\right)a^2 + a\dot{a}^2 +$$
- \frac{M_p^2}{8\pi c} (a^2 \ddot{a} + aa') + \frac{2f M^2}{c} \ln a (a^2 \ddot{a} + aa')
+ \frac{2f M^2}{c} \frac{\ddot{a}}{a} - \frac{dF^2}{6ca} = 0. \quad (17)

An exact solution of this fourth order non-linear differential equation does not look possible, but it can be easily analyzed within the approximation that \( f \) is not too large. Then the new terms (collected in the second line of Eq. (17)) can be considered as perturbations. Moreover, the last two of them are irrelevant, because during inflation they decrease exponentially with respect to the other terms. Thus, in this approximation, the only one relevant change is the replacement

\[ M_p^2 \rightarrow M_p^2 \left[ 1 - \tilde{f} \ln a(t) \right], \quad (18) \]

where for future convenience we have introduced the dimensionless parameter

\[ \tilde{f} = \frac{16\pi f M^2}{M_p^2} = \sum_i \frac{N_i m_i^2}{3\pi M_p^2}. \quad (19) \]

Notice that \( f \) is given by Eq. (11) and so \( \tilde{f} \) does not depend on the scale \( M \). Since (18) is a slowly varying function, the effect of the masses may be approximated through the modification of the inflation law

\[ a(t) = a_0 e^{H_1 t}, \quad H_1 = \text{const} \quad (20) \]

according to

\[ H_1 = \frac{M_p}{\sqrt{-16\pi b}} \to \frac{M_p}{\sqrt{-16\pi b}} \left[ 1 - \tilde{f} \ln a(t) \right]^{1/2} = H(t). \quad (21) \]

To substantiate our claim, we have solved Eq. (17) directly using the numerical methods. The plots corresponding to the numerical solution of the Eq. (17) using Mathematica [24] are shown in Fig. 1. Since in the first period of inflation masses do not play much role and the stabilization of the exponential inflation proceeds very fast [6], the initial data (in both Eq. (21) and the plots of Fig. 1) were chosen according to the exponential inflation law:

\[ a(0) = 1, \quad \dot{a}(0) = H_1, \quad \ddot{a}(0) = H_1^2, \quad (22) \]

According to the numerical analysis, the total number of e-folds in the “fast phase” of inflation (until the Hubble constant becomes comparable to the SUSY breaking scale) is about \( 10^4 \) for our particular values of the parameters, and at the last stage the expansion essentially slows down. The chosen value of the parameter \( f = 10^{-4} \) in the plot is, as we warned before, independent of the scale \( M \), and it determines where the process of stable inflation finishes as well as the number of e-folds. Following the above considerations, the transition from the stable to the unstable inflation can be associated to a high energy scale which we shall call \( M^* \). Let us remark that this typical scale \( M^* \) may be quite different from the scale of SUSY breaking \( M_{\text{SUSY}} \), in particular \( M^* \) can be some orders of magnitude below \( M_{\text{SUSY}} \). The scale \( M^* \) is such that there is a sufficient number of particles (scalars and fermions) lighter than \( M^* \), so that \( c > 0 \) even well below \( M_{\text{SUSY}} \)—see Eq. (10). As an illustration, let us indicate the unique example of the gauge theory where the spectrum of masses is known: the Standard Model of particle physics. In the SM the symmetry breaking scale is given by the vacuum expectation value, \( v \), of the Higgs field, from which one defines the Fermi scale \( M_F = G_F^{-1/2} \approx 300 \text{ GeV} \). However, most of the particles have masses much below \( M_F \) and \( v \), and even below 1 GeV. One can suppose that a similar situation takes place in the high energy SUSY GUT. As we are going to discuss below, the constraint \( M^* < 10^{14} \text{ GeV} \) provides better properties of the metric perturbations. It is important that this does not put rigid limitations on the value of the scale of supersymmetry breaking which can be some orders of magnitude greater than \( M^* \).

One has to notice that the scale \( M_{\text{SUSY}} \) and corresponding \( M^* \) are not necessarily linked to a high energy SUSY scale (e.g., SUSY-GUT, \( M_X \sim 10^{16} \text{ GeV} \)) but it could just be the SUSY breaking scale of the MSSM at the TeV scale [20]. In the last case, however, the total number of inflation e-folds would be much greater, but this would not lead to any qualita-

---

3 We remind the reader that the coefficient \( b \) is negative for any particle content, see Eq. (10).

Fig. 1. (a) Plot of $\ln a$ versus the physical time $t$ as a result of the numerical analysis of Eq. (17); $t$ is given in units of $16\pi/\sqrt{-16\pi b}$ and we fixed the parameter (19) as $f = 10^{-4}$. In this time interval, inflation does not stop, yet; (b) As in (a), but extending the numerical analysis until reaching an approximate plateau marking the end of stable inflation.

tative change on the shape of the plots of Fig. 1 as can be seen from the analytical structure of Eqs. (17)–(19). The important qualitative point is that for any value of $\tilde{f}$ the approximate plateau eventually appears and signals the end of stable inflation. Also notice from Fig. 1 that the initial evolution is close to the exponential inflation (20), but after that the expansion slows down due to the quantum effects of massive fermions. The general behaviour is close to formula (21). According to the plot in Fig. 1(b), the evolution tends to $H = 0$, but before this there must be a breaking of SUSY and the transition to the unstable phase. Qualitatively and quantitatively, the plot is in a good correspondence with formula (21), especially in the region $\tilde{f}\ln a(t) \ll 1$ where it can be safely used to simplify the analysis. Remember that the effective action (12) has been derived in the leading-log approximation, such that the effect of particles masses has been reduced to the renormalization of the Newton constant. Indeed, this approximation is valid only at high energies, when $H \gg m_i$ for all the fermions. Also, formula (21) is based on treating the $\tilde{f}\ln a(t)$-term as a small perturbation.

4. Graceful exit from anomaly-induced inflation

In order to solve the graceful exit problem in our framework we do not need to insist that the rate of change of the scale factor, $H(t)$, reduces to zero at some point. Recall that $H(t)$ sets the scale $\mu$ of the renormalization group running for the gravitational part. If we consider the SUSY breaking and the corresponding change in the number of active degrees of freedom [9], then the necessary and sufficient condition for the applicability of our approach is that $H(t)$ decreases from the initial value about $M_P/\sqrt{-16\pi b} \sim 10^{18}$ GeV, down to the lower scale $M^*$. The outcome is that the evolution according to (21) lasts until reaching the scale $M^*$, and after that most of SUSY particles decouple, the inflationary solution becomes unstable such that the FLRW phase can start. In fact, the crucial point is the existence of a nonvanishing $f$ as it eventually tempers stable inflation allowing favorable conditions for the universe to tilt into the FLRW phase [5,6,12].

As a result, we arrive at a consistent picture of the graceful exit from the anomaly-induced inflation to the FLRW stage. It is easy to see that this conclusion does not change if we choose another scale for the SUSY breaking. For a lower scale of SUSY breaking (e.g., $10^{10}$ GeV as in the Pati–Salam model, or even 1 TeV as in the MSSM) there is no need to impose the constraint on the SUSY spectrum. In this case the area of applicability of our leading-log approximation is the same as the applicability of Eq. (21) and (much more important!) this approximation is valid until the SUSY breaking scale. The main difference will be that, for a lower $M_{SUSY}$, the total number of $e$-folds

4 In Fig. 1 we have just illustrated a situation where the numerical analysis is sufficiently simple, corresponding to $f$ not too small and so based on a SUSY-GUT scale. For smaller and smaller $f$ the computer time becomes exceedingly long.

5 Notice that $|16\pi b| = O(1)$ in the MSSM, and it is much larger than 1 in any typical SUSY GUT.
will increase dramatically, and that the inflation will consume more time. But, the evolution at the last stage of inflation will be quite similar as can be seen from Eq. (21). Another observation is that, in agreement with (11), only spinor fields contribute to the value of $f$. Therefore, as it was anticipated in the introduction, taking the masses of the fermion fields into account we arrive at a tempered form of inflation; besides, the Universe enters the phase of unstable inflation [5, 9] with such initial conditions that it ends up with the FLRW behavior. Furthermore, according to (11) the result (21) is universal, for it does not depend on the choice of the dilatation symmetry breaking scale $M$. If interpreted physically, one can put constraints on $M$ using the macroscopic forces mediated by the field $\sigma$, demanding that this forces should have the submillimeter range, similarly as in [25].

Finally, for a really successful exit from the inflation phase we need to evaluate the dynamics of $H(t)$ during the last $65$ $e$-folds of inflation. The importance of this calculation is related to the fact that the amplitude of the gravitational waves is consistent with the observable range of anisotropy in the CMBR if, during the last $65$ $e$-folds of the inflation, the Hubble constant $H$ does not exceed $10^{-3}M_P$. This is because the fluctuations in the amplitude $h$ of these waves is evaluated using $\delta h/h = H/M_P$ and, on the other hand, is related to the fluctuations in the temperature of the relic radiation. Thus, it has to satisfy the relation $\delta h/h = \delta T/T = O(10^{-5})$. At the lowest end of the inflation interval this condition corresponds, e.g., in the SUSY-GUT case to a final scale value $H_f = M^* \lesssim 10^{14}$ GeV $\approx 10^{-5}M_P$. We expect that after the onset of the approximate plateau in Fig. 1(b), where the transition to an unstable phase occurs, the universe will take a while before entering the FLRW phase, i.e., the latter will actually initiate at some point well over the plateau. We have numerically checked that $H(t)$ decreases very fast on it. For instance, a $15\%$ increase of the time at the beginning of the plateau amounts $H(t)$ to diminish two orders of magnitude.\textsuperscript{7} So in general $H(t)$ will decrease further below $M_{\text{SUSY}}$, and the difference between $H_f = M^*$ and $M_{\text{SUSY}}$ at the moment of the transition can be significant, say one or two orders of magnitude. Hence $M_{\text{SUSY}}$ can be $10^{16}$ GeV and this does not create problems with CMBR. Next we have to derive the value $H_f$ just some number of $e$-folds $n_e \gtrsim 65$ before the SUSY breaking point $H_f = M^*$, where as usual $n_e$ is defined through $a_f/a_i = \exp[n_e]$. We obtain the following relations:

\begin{align*}
H_i^2 &= H_f^2 + \frac{1}{48\pi^2 b} \sum N_i m_i^2 \ln a_f, \\
H_i^2 &= H_i^2 + \frac{1}{48\pi^2 b} \sum N_i m_i^2 \ln a_i \\
&= H_i^2 - \frac{n_e}{48\pi^2 b} \sum N_i m_i^2. \quad (23)
\end{align*}

Notice that $H_i^2 > H_f^2$ because $b < 0$. However, if we suppose that $H_f = M^*$ and that the sum $\sum N_i m_i^2$ is of the order of $M^*^2$, then $H_i$ is of the same order of magnitude as $H_f$. In other words, the amplitude of the gravitational waves produced by the anomaly-induced inflation can be consistent with the magnitude of the CMBR. Remarkably, this result can be achieved without specifying the details of the gauge model. It is sufficient to make some reasonable suppositions about the mass spectrum of SUSY particles. The numerical analysis confirms the conclusion derived from the approximate formula (23). On the other hand the final conclusion regarding the consistency with the CMBR observations require an explicit derivation and analysis of the metric and density perturbations in the last $65$ $e$-folds of inflation, between $H_i$ and $H_f$. Such study is beyond the scope of the present considerations, and it may require an elaborated analysis of the time dynamics of the Hubble parameter (defining the scale $\mu$ of the gravitational interactions) in the given fundamental theory. Even at the level of our relatively simple effective framework, $H(t)$ is obtained only after numerically solving the non-linear fourth order differential Eq. (17). However, in the gravitational wave sector, one can make some qualitative observations without explicit calculations. The corresponding analysis has

\textsuperscript{6} Let us remind that the spectrum of the gravitational waves in the exponential phase of inflation is almost flat and agrees with all observational data [8].

\textsuperscript{7} This can roughly be compared (as in the original model [1], though of course in a different sense) to the situation in a supercooled phase transition in which energy decreases a lot before the transition really takes place.
been performed, for the case of constant $H$, in [11] and later on in [8] in the effective action framework when all parameter dependences become explicit. According to this work, the perturbation spectrum strongly depends on the parameters of the classical vacuum action and also on the choice of the quantum vacuum state of the induced theory (which was previously discussed in [26] in relation to the analysis of Hawking radiation from the black holes). The general conclusion is that the spectrum is very close to the Harrison–Zeldovich one for the sufficiently small value of the relevant vacuum parameter $a_1$ (consistent with the renormalization group) and with the most natural choice (in comparison to the black hole case) of the quantum vacuum. It is clear that the same possibilities of changing the perturbation spectrum exist for the non-constant $H$. Moreover, since in the phenomenologically important period of inflation the scale factor changes by more than $65 \, e$-folds while $H(t)$ remains to be the same order of magnitude, one can suppose that the constant-$H$ terms will dominate in the equations for the metric perturbations and that the result will not be very different from the one of the constant $H$ in Ref. [8]. Therefore, the anomaly-induced inflation has some predictive power in the description of the perturbations spectrum. But, the small details of this spectrum can be changed by adjusting the parameters of the classical vacuum action and the quantum vacuum. As a result, we may hope to fit with the present and future experimental data within this model. In principle, when the amount of such data will become sufficiently large, one can expect to achieve some additional information concerning the spectrum of the high-energy theory in this framework.

5. Conclusions

In summary, we have considered an effect of particle masses on the anomaly-induced inflation. The method of calculation was based on the local Cosmon Model [13], i.e., the conformal description of the massive fields, and on the conventional method of deriving the anomaly-induced effective action. The output of our approach agrees with the expressions expected from the renormalization group. The cosmological application of our result is that, independent of the details of the particle content of the model, the (spinor) matter fields slow down inflation. Together with the supersymmetry breaking effect [9], this provides the qualitative basis for the graceful exit from the stable anomaly-induced inflation. Furthermore, there is the possibility that under certain assumptions concerning the spectrum of the SUSY GUT, the amplitude of the gravitational waves is consistent with the CMBR constraints. The precise quantitative description will of course require to go into the details of a more fundamental theory (superstring theory or M-theory) underlying this effective approach. In the meantime we see that in the anomaly-induced model there are some indications to a phenomenologically consistent picture of inflation without introducing an ad hoc inflaton and without fine-tuning the parameters and/or the initial data.

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References

Description of supernova data in conformal cosmology without cosmological constant

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Abstract

We consider cosmological consequences of a conformal-invariant formulation of Einstein’s General Relativity where instead of the scale factor of the spatial metrics in the action functional a massless scalar (dilaton) field occurs which scales all masses including the Planck mass. Instead of the expansion of the universe we obtain the Hoyle–Narlikar type of mass evolution, where the temperature history of the universe is replaced by the mass history. We show that this conformal-invariant cosmological model gives a satisfactory description of the new supernova Ia data for the effective magnitude–redshift relation without a cosmological constant and make a prediction for the high-redshift behavior which deviates from that of standard cosmology for $z > 1.7$.

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1. Introduction

The recent data for the luminosity-redshift relation obtained by the supernova cosmology project (SCP)\cite{1} point to an accelerated expansion of the universe within the standard Friedman–Robertson–Walker (FRW) cosmological model. Since the fluctuations of the microwave background radiation\cite{2} provide evidence for a flat universe a finite value of the cosmological constant $\Lambda$ has been introduced\cite{3} which raises to the cosmic coincidence (or fine-tuning) problem\cite{4}. A most common approach to the solution of this problem is to allow a time dependence of the cosmological constant (“Quintessence”\cite{4,5}), the speed of light\cite{6} or the fine structure constant\cite{7}.

The present Letter is devoted to an alternative description of the new cosmological supernova data without a $\Lambda$-term as evidence for Weyl’s geometry of similarity\cite{8}, where Einstein’s theory takes the form of the conformal-invariant theory of a massless scalar field\cite{9–14}.

As it has been shown by Weyl\cite{8} already in 1918, conformal-invariant theories correspond to the relative standard of measurement of a conformal-invariant ratio of two intervals, given in the geometry of simi-
larity\(^1\) as a manifold of Riemannian geometries connected by conformal transformations. This ratio depends on nine components of the metrics whereas the tenth component became the scalar dilaton field that cannot be removed by the choice of the gauge. In the current literature [15,16] (where the dilaton action is the basis of some speculations on the unification of Einstein’s gravity with the standard model of electroweak and strong interactions including modern theories of supergravity) this peculiarity of the conformal-invariant version of Einstein’s dynamics has been overlooked.

The energy constraint converts this dilaton into a time-like classical evolution parameter which scales all masses including the Planck mass. In the conformal cosmology (CC), the evolution of the value of the massless dilaton field (in the homogeneous approximation) corresponds to that of the scale factor in standard cosmology (SC). Thus, the CC is a field version of the Hoyle–Narlikar cosmology [17], where the conformal density, the conformal pressure, the conformal time, the conformal (coordinate) distance, the conformal pressure, etc., using instead of the FRW cosmic scale factor the homogeneous dilaton field which scales all masses in the universe.

\(^1\) The geometry of similarity is characterized by a measure of changing the length of a vector on its parallel transport. In the considered dilaton case, it is the gradient of the dilaton. In the following, we call the scalar conformal-invariant theory the conformal general relativity (CGR) to distinguish it from the original Weyl [8] theory where the measure of changing the length of a vector on its parallel transport is a vector field (that leads to the defect of the physical ambiguity of the arrow of time pointed out by Einstein in his comment to Weyl’s paper [8]).

\[
S_{\text{CGR}} = - \int d^4x \sqrt{-\hat{g}} \frac{1}{6} R(\hat{g}) = \int d^4x \left[ -\sqrt{-g} \frac{w^2}{6} R(g) + w \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu w \right) \right] \tag{1}
\]

with negative sign. The action and conformal-invariant equations of this theory coincide with the ones of Einstein’s general relativity (GR) expressed in terms of the conformal-invariant Lichnerowicz variables \(F(n)\), including the metric \(g\) [19]

\[
\left\| F_{(n)} \right\|_{g}^{-n/6} F(n), \quad \left\| (3) g^{L} \right\| = 1, \quad (ds^{L})^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{2}
\]

where \((3)g_{ij}\) are the 3-dimensional metric components, \(n\) is the conformal weight for a tensor \((n = 2)\), vector \((n = 0)\), spinor \((n = -3/2)\), and scalar \((n = -1)\) field. The role of the dilaton field in GR is played by the scale-metric field

\[
w_g = \left\| (3) g \right\|_{g}^{1/6} M_{\text{Planck}} \sqrt{\frac{3}{8\pi}}. \tag{3}
\]

Therefore, we call this theory the conformal general relativity (CGR).

In contrast to Einstein’s general relativity theory, in Weyl’s conformal relativity we can measure only a ratio of two Einstein intervals that depends only on nine components of the metric tensor. This means that the conformal invariance allows us to remove only one component of the metric tensor using the scale-free Lichnerowicz conformal-invariant field variables (2). We show that the conformal invariance of the action, the variables, and the measurable quantities gives us an opportunity to solve the problems of modern cosmology without inflation by the definition of the observables as conformal-invariant quantities. We introduce the conformal time, the conformal (coordinate) distance, the conformal density, the conformal pressure, etc., using instead of the FRW cosmic scale factor the homogeneous dilaton field which scales all masses in the universe.

2. Conformal general relativity

The principle of relativity of all standards of measurement can be incorporated into the unified theory through the Weyl geometry of similarity as a manifold of conformal-equivalent Riemannian geometries. To escape defects of the first Weyl version of 1918 [8], we use the scalar–tensor conformal invariant \((\tilde{g}_{\mu\nu} = w^2 g_{\mu\nu})\), where \(w\) is a dilaton scalar field described by the Penrose–Chernikov–Tagirov (PCT) action [9]
After the introduction of the CGR for an empty universe we give now to the action of the matter fields in a conformal invariant formulation of the Standard Model (SM)

\[ S_{\text{CSM}} = \int d^4x \sqrt{-g} \left[ \frac{\vert \Phi \vert^2}{6} R(g) + L_{\text{SM}}^0(g, \{v_i\}, \{\psi_j\}, \Phi) + L_{\text{Higgs}}(\vert \Phi \vert, w) \right]. \]  

(4)

where \( L_{\text{SM}}^0(g, \{v_i\}, \{\psi_j\}, \Phi) \) is the SM Lagrangian with the metric tensor \( g \), the Higgs field \( \Phi \), the vector boson fields \( \{v_i\} \), the spinor fields \( \{\psi_j\} \) and the coupling constant \( \lambda \) of the conventional Higgs potential. The latter one has to be replaced by the conformal-invariant one

\[ L_{\text{Higgs}}(\Phi, w) = -\lambda \left[ (\vert \Phi \vert)^2 - C^2(w) \right]^2, \]  

(5)

where the mass term of the Higgs field \( C(w) = \gamma_{\text{Higgs}}w \) is rescaled by the cosmological dilaton \( w \). The conformal-invariant interactions of the dilaton and the Higgs doublet form the effective Newton coupling in the gravitational Lagrangian

\[ \frac{\vert \Phi \vert^2 - w^2}{6} R. \]  

(6)

From this term the necessity becomes obvious to introduce the modulus \( \phi \) and the mixing angle \( \chi \) of the dilaton–Higgs mixing [20] as new variables by

\[ w = \phi \cosh \chi, \quad \vert \Phi \vert = \phi \sinh \chi, \]  

(7)

so that the total Lagrangian of our conformal cosmology model takes the form

\[ L = L_{\text{CGR}} + L_{\text{CSM}} = -\phi^2 R - \partial_\mu \phi \partial^\mu \phi + \phi^2 \partial_\mu \chi \partial^\mu \chi + L_{\text{Higgs}}(\phi, \chi) + \bar{\psi}_e \gamma_5 \phi \sinh \chi \psi_e + \ldots, \]  

(8)

where the Higgs Lagrangian

\[ L_{\text{Higgs}}(\phi, \chi) = -\lambda \phi^4 \left[ \sinh^2 \chi - \gamma_{\text{Higgs}}^2 \cosh^2 \chi \right]^2 \]  

(9)

describes the conformal-invariant Higgs effect of the spontaneous SU(2) symmetry breaking

\[ \frac{\partial L_{\text{Higgs}}}{\partial \chi} = 0 \implies \chi_1 = 0, \]

\[ \sinh \chi_{2,3} = \pm \gamma_{\text{Higgs}} \sqrt{1 - \gamma_{\text{Higgs}}^2} \sim 10^{-17} \]  

(10)

corresponding to the latter pair of solutions \((\chi_{2,3})\). The masses of elementary particles are also scaled by the modulus of the dilaton–Higgs mixing. There are two ways to obtain the Standard Model. The simplest way is to use a scale transformation to convert this modulus into a constant (instead of the Lichnerowicz gauge (2))

\[ \phi(x_0, x) = \tilde{\varphi}_0 = M_{\text{Planck}} \sqrt{\frac{3}{8 \pi}}. \]  

(11)

In this case the Lagrangian (8) goes over into the Einstein–Hilbert one with

\[ \gamma_{\text{Higgs}} = \frac{m_X}{\tilde{\varphi}_0} \sim 10^{-17}. \]  

(12)

In the limit of infinite Planck mass the SM sector decouples from the gravitational one and takes the standard renormalizable form with the Higgs potential

\[ -\lambda (x^2 - m_X^2)^2 + O \left( \frac{1}{M_{\text{Planck}}} \right). \]  

(13)

where the notations \( \varphi_0 \chi = X \) and \( \varphi_0 \gamma_{\text{Higgs}} = m_X \) have been introduced for the Higgs field and its mass term, respectively. However, the gauge (11) violates the conformal symmetry of the equations of motion and introduces an absolute standard of measurement of geometric intervals depending on ten components. This way leads to the standard cosmology.

The second way is to choose the Weyl relative standard of measurement of intervals depending on nine components of the metric tensor in the general case. This way is compatible with the Lichnerowicz gauge (2) that does not violate the conformal symmetry of the equations of motion in the conformal-invariant theory considered. In this case, the equality (11) follows from the energy constraint and means the current (non-fundamental) status of Planck mass [14]. The Weyl relative standard of measurement leads to the conformal cosmology [12].

3. Cosmological solutions for the dilaton–Higgs dynamics

It is well known that the homogeneous and isotropic approximation to GR is described by the metric

\[ ds^2 = g_{00}(x_0) dx_0^2 - a^2(x^0) dx^I dx_I = a^2(x^0) (ds^L)^2, \]  

(14)
where $dt = \sqrt{g_{00}} dx_0$ is the Friedmann time interval.

In this approximation CGR is described by the flat conformal space–time

$$(dx^L)^2 = d\eta^2 - dx_i^2,$$  \hspace{1cm} (15)

where $d\eta = \sqrt{g_{00}} dx_0$ is the conformal time interval and the abbreviation $N_0 = \sqrt{g_{00}}$ will be used. For simplicity we will restrict us here to the discussion of flat space.

The dilaton (17), where the time $\eta$ and the abbreviation $N_0 = \sqrt{g_{00}}$ will be used. For simplicity we will restrict us here to the discussion of flat space.

The field theory reproduces all regimes of the classical SC in their conformal versions. In particular, the theory of the free field describes all the equations of state that are known in the standard cosmology: the rigid state ($\rho_{\text{Rigid}} = \rho_{\text{Rigid}}(\varphi) = \text{const}/\varphi^2$), the radiation state ($\rho_{\text{Radiation}} = \rho_{\text{Radiation}}/3 = \text{const}$), and the matter state ($\rho_{\text{Matter}} = 0$, $\rho_{\text{Matter}} = \text{const} \cdot \varphi$) [13,14]. The origin of the rigid state are excitations of the homogeneous graviton and the dilaton–Higgs field mixing; the radiation state corresponds to excitations of other massless fields and the matter one to those of massive fields.

Now we can ask: what is the best regime for a description of the latest supernova data on the luminosity distance–redshift relation and is this regime compatible with the other cosmological data, like the CMB radiation and element abundances?

### 4. Luminosity distance–redshift relation

Let us establish the correspondence between the SC and the CC determined by the evolution of the dilaton (17), where the time $\eta$, the density $\rho(\varphi)$, and the Hubble parameter $H_0$ are treated as measurable quantities. Let us introduce the standard cosmological definition of the redshift and density parameter

$$(1 + z) \equiv \frac{\varphi_0}{\varphi(\eta)} = \frac{\varphi_0}{\varphi(\eta)}, \quad \Omega(z) = \frac{\rho(\varphi)}{\rho(\varphi_0)},$$  \hspace{1cm} (20)

where $\Omega(0) = 1$ is assumed. The density parameter $\Omega(z)$ is determined in both the SC and the CC as

$$\Omega(z) = \frac{\Omega_{\text{Rigid}}(1 + z)^2 + \Omega_{\text{Radiation}}}{(1 + z) + \Omega_{\text{Lambda}}/4}.$$  \hspace{1cm} (21)

We added here the $\Omega_{\Lambda}$-term that corresponds to the $\lambda \varphi^4$ interaction in the conformal action in order to have the complete analogy with the standard cosmology. Then Eq. (17) takes the form

$$H_0 \frac{dz}{dz} = \frac{1}{(1 + z)^2} \frac{1}{\sqrt{2 \Omega(z)}},$$  \hspace{1cm} (22)

and determines the dependence of the conformal time on the redshift factor. This equation is valid also for
the conformal time–redshift relation in the SC where this conformal time is used for description of a light ray.

A light ray traces a null geodesic, i.e., a path for which the conformal interval \((ds^L)^2 = 0\) thus satisfying the equation \(dr/d\eta = 1\). As a result we obtain for the coordinate distance as a function of the redshift

\[
H_0 r(z) = \int_0^z \frac{dz'}{(1 + z')^2 \sqrt{\Omega(z')}}.
\]  

Eq. (23) coincides with the similar relation between coordinate distance and redshift in SC.

In the comparison with the stationary space in SC and stationary masses in CC, a part of photons is lost. To restore the full luminosity in both SC and CC we should multiply the coordinate distance by the factor \((1 + z)^2\). This factor comes from the evolution of the angular size of the light cone of emitted photons in SC, and from the increase of the angular size of the light cone of absorbed photons in CC.

However, in SC we have an additional factor \((1 + z)^{-1}\) due to the expansion of the universe, as measurable distances in SC are related to measurable distances in CC (that coincide with the coordinate ones) by the relation

\[
\ell = a \int \frac{dt}{a} = \frac{r}{1 + z}.
\]  

Thus we obtain the relations

\[
\ell_{SC}(z) = (1 + z)^2 \ell = (1 + z)r(z),
\]

\[
\ell_{CC}(z) = (1 + z)^2 r(z).
\]

This means that the observational data are described by different regimes in SC and CC. For example, the rigid state (i.e., \(\Omega_{\text{Rigid}} = 1\)) gives the relation

\[
\ell_{CC}(z) = z + \frac{z^2}{2}.
\]

In Fig. 1 we compare the results of the SC and CC for the effective magnitude–redshift relation: \(m(z) = 5 \log[H_0 \ell(z)] + M\), where \(M\) is a constant, with recent experimental data for distant supernovae [1,21].

![Graph](image_url)

**Fig. 1.** \(m(z)\)-relation for a flat universe model in SC and CC. The data points include those from 42 high-redshift type Ia supernovae [1] and that of the recently reported farthest supernova SN1997ff [21]. An optimal fit to these data within the SC requires a cosmological constant \(\Omega_A = 0.7\), whereas in the CC these data require the dominance of the rigid state.
Within the CC model the pure rigid state of dilaton–Higgs dynamics without cosmological constant gives the best description and is equivalent to the SC fit up to the SN1997ff point.

5. Cold universe scenario

In this section we want to discuss the consistency of the here described CC scenario of a nonexpanding Universe, in which the observed redshift of spectra is due to time-dependent elementary particle masses, with other cosmological observations such as the CMB radiation and the distribution of elements.

In the limit of the Early Universe, \( \phi \rightarrow 0 \), the CGR action also gives the most singular rigid state \( \rho/\rho_0 = \Omega_{\text{Rigid}}(z+1)^2 \) and the primordial motion of the dilaton described before

\[
\psi^2(\eta) = \psi_0^2[1 + 2H\eta] = \frac{\psi_0^2}{(1 + z)^2},
\]

\[
H(z) = \frac{\psi'}{\psi} = H_0(1 + z)^2.
\] (28)

At the point of coincidence of the Hubble parameter of this motion with the mass of vector bosons \( m_v(z) \sim H(z) \), there occurs the intensive creation of longitudinal vector bosons, see [23]. Fast thermal equilibration of this boson system takes place since for the inverse relaxation time holds \( \eta_{\text{relaxation}} = \sigma_{\text{scat}} m_v \geq H(z) \), and therefore the density of created vector bosons \( n_v \) defines an equilibrium temperature which appears to be an integral of motion of the cosmic evolution \( T_{\text{eq}} \simeq [m_v^2(z)H(z)]^{1/3} \simeq (m_v^2H_0)^{1/3} = 2.7 \text{ K} \sim H_0 \). This is a surprisingly good agreement of \( T_{\text{eq}} \) with the CMB radiation temperature.

It is worth to emphasize this difference between the CC model and the SC ones: in conformal cosmology, the CMB temperature remains constant (cold scenario) but the masses evolve throughout the history of the universe due to the time dependence of the dilaton field

\[
m_{\text{era}}(z_{\text{era}}) = \frac{m_{\text{era}}(0)}{(1 + z_{\text{era}})} = T_{\text{eq}},
\] (29)

where \( m_{\text{era}}(0) \) is the present-day value of a characteristic energy (mass) scale determining the onset of an era of the universe evolution.

Eq. (29) has the important consequence that all those physical processes which concern the chemical composition of the universe and which depend basically on Boltzmann factors with the argument \( (m/T) \) cannot distinguish between the mass history of conformal cosmology and the temperature history of standard cosmology due to the relations

\[
\frac{m(z)}{T(0)} = \frac{m(0)}{(1 + z)T(0)} = \frac{m(0)}{T(z)}. \tag{30}
\]

This formula makes transparent that in this order of approximation a \( z \)-history of masses with invariant temperatures in the rigid state of CC is equivalent to a \( z \)-history of temperatures with invariant masses in the radiation stage of SC. We expect therefore that the conformal cosmology will be as successful as the standard cosmology in the radiation stage for describing, e.g., the neutron–proton ratio and the primordial element abundances.

An important new feature of the conformal cosmology relative to the standard one is the absence of the Planck era, since the Planck mass is not a fundamental parameter but only the present-day value of the dilaton field [12].

6. Conclusion

We have presented an approach according to which the new supernova data can be interpreted as evidence for a new type of geometry in Einstein’s theory rather than a new type of matter. This geometry corresponds to the relative standard of measurement and to a conformal cosmology with constant three-volume. In this cosmology, the dilaton field scales all masses and its evolution is responsible for observable phenomena like the redshift of spectra from distant galaxies. The evolution of all masses replaces the familiar evolution of the scale factor in standard cosmologies. The infrared dilaton–elementary particle interaction leads to particle creation [23] and in turn to the occurrence of the CMB radiation with a temperature of 2.7 K not changed ever since.

We have defined the cosmological parameters in the conformal cosmology, and we have found that the effective magnitude–redshift relation (Hubble diagram) for a rigid state which originates from the dilaton–Higgs dynamics describes the recent observational...
data for distant (high-redshift) supernovae including the farthest one at $z = 1.7$. While in the standard FRW cosmology interpretation a $\Lambda$-term (or a quintessential analogue) is needed, which entails a transition from decelerated to accelerated expansion at about $z \sim 1.7$, the cosmology presented here does not need a $\Lambda$-term. Both cosmologies make different predictions for the behaviour at $z > 1.7$. Provided that the CSM with a Higgs potential gives a correct description of the matter sector, our findings suggest that new data at higher redshift could discriminate between the alternative cosmological interpretations of the luminosity–redshift relation and answer the question: is the universe expanding or not?

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References

Supernova-neutrino studies with $^{100}\text{Mo}$

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Abstract

We show that supernova neutrinos can be studied by observing their charged-current interactions with $^{100}\text{Mo}$, which has strong spin–isospin giant resonances. Information about both the effective temperature of the electron–neutrino sphere and the oscillation into electron neutrinos of other flavors can be extracted from the electron (inverse $\beta$) spectrum. We use measured hadronic charge-exchange spectra and the Quasiparticle Random Phase Approximation to calculate the charged-current response of $^{100}\text{Mo}$ to electron neutrinos from supernovae, with and without the assumption of oscillations. A scaled up version of the MOON detector for $\beta\beta$ and solar-neutrino studies could potentially be useful for spectroscopic studies of supernova neutrinos as well.

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Neutrinos carry away most of the energy from core-collapse supernovae. Supernova neutrinos (SN-$\nu$'s) can be observed on the earth, and their spectrum contains information about conditions inside the supernova as well as their own properties. Here we aim to show that $^{100}\text{Mo}$, which responds strongly to spin–isospin probes, is useful for studying supernova weak processes and SN-$\nu$ oscillations, and that a good SN-$\nu$ detector can be realized by scaling up the proposed $\beta\beta$ and solar-neutrino detector MOON.

Though there is much we don’t know about supernovae, the consensus of modelers is that SN-$\nu$’s are released roughly thermally from the supernova remnant after diffusing to the surface of last scattering, called the “neutrino sphere”. They therefore escape with an energy corresponding approximately to the thermal energy spectrum at the sphere [1–3]. In this picture there are really three neutrino spheres, one for electron neutrinos ($\nu_e$’s), one for electron antineutrinos ($\bar{\nu}_e$’s), and one for the other flavors ($\nu_x$’s and $\bar{\nu}_x$’s). The $\nu_e$ sphere has the largest radius of these because $\nu_e$’s interact with matter via both charged-and neutral-current reactions. So do $\bar{\nu}_e$’s, but the excess of neutrons over protons in the supernova remnant means that they scatter less frequently through charged-current interactions, so that the radius of their neutrino sphere is smaller. The other neutrinos ($\nu_x$, $\bar{\nu}_x$), with only the neutral-current interactions, decouple deeper within the star. Since the temperature in
the supernova core increases as the radius gets smaller, these last neutrinos will have the highest energy, and the $\nu_e$’s the lowest energy.

The SN-$\nu$ spectrum for a given neutrino species is thought to be roughly \[ S(E_{\nu}) = c T_{\nu}^{-1} \frac{(E_{\nu}/T_{\nu})^2}{\exp(E_{\nu}/T_{\nu} - a) + 1}, \tag{1} \]
where $T_{\nu}$ is the temperature at the neutrino sphere, $a$ is the degeneracy parameter, and $c$ is a normalization constant. Numerical simulations can be approximately reproduced with temperatures $T_{\nu}$ of about 3.5 MeV for $\nu_e$’s, 5 MeV for $\bar{\nu}_e$’s, and 8 MeV for $\nu_x$’s and $\bar{\nu}_x$’s, with the degeneracy parameter $a$ taken to vanish.

Accordingly, the average $\nu$ energies are $\langle E(\nu_e) \rangle \sim 11$ MeV, $\langle E(\bar{\nu}_e) \rangle \sim 16$ MeV, $\langle E(\nu_x) \rangle \sim 25$ MeV, and the spread of SN-$\nu$ energies covers the wide region of $E \sim 5$–70 MeV.

Measuring the $\nu_x$ spectra would provide us information on the electron neutrino sphere, and thus tell us if our supernova models are on the right track. It could also tell us about neutrino oscillations; if our ideas about where the neutrinos leave the supernova are correct, $\nu_x$’s with energies above 30 MeV or so are rarely emitted directly from the supernova. An excess of high-energy $\nu_x$’s reaching the earth would be strong evidence for oscillations from $\nu_x$ to $\nu_e$.

A number of detectors can study neutrinos in the event of a nearby supernova. They have the ability to detect either the charged-current $\nu_e$ ($\bar{\nu}_e$) interaction, which produces electrons (positrons), or the neutral current interaction (for all flavors), which usually results in the production of neutrons and photons, or both. Antineutrinos from SN1987A were observed by the Kamiokande [4] and IMB [5] groups in water Cerenkov detectors via the reaction $p + \bar{\nu}_e \rightarrow n + e^+$. Super-Kamkiokande, with multiplets of water, and the Sudbury Neutrino Observatory (SNO), with kilotons of heavy water, are powerful detectors for SN-$\nu$’s (see Ref. [6]). Super-Kamiokande, however, has a high threshold ($Q \sim 15$ MeV) for the charged-current interaction of $\nu_e$’s with $^{16}$O. The effective threshold energy, including a 5-MeV threshold for detecting an electron produced by the charged-current interaction, is therefore about 20 MeV, well above the average energy of neutrinos emitted from the $\nu_e$ sphere. As a result, while the detector is good for charged-current $\bar{\nu}_e$ interactions, it will have a hard time saying anything about the flux or energy distribution of thermally emitted $\nu_e$’s. Detectors based on liquid scintillator, such as KamLAND [7], also have a high threshold for $\nu_e$ charged current interactions with $^{12}$C—about 17 MeV, with an effective threshold energy of around 20 MeV. They will not be able to study neutrinos from the $\nu_e$ sphere either. SNO, on the other hand, has a low threshold, plus the eventual ability to separately measure charged and neutral current interactions.

Ref. [8] shows that information on SN-$\nu$ energies and oscillations can be obtained by measuring the number of neutrons produced by neutrino scattering from heavy nuclei. The method is very good for getting gross features of the SN-$\nu$ spectra and possible oscillations, and the proposed facilities OMNIS [9], SBNO [10], and LAND [11] are based largely on the detection of neutrons. These detectors cannot easily measure the spectra of charged-current events, however. In addition, the $\nu_e$ cross section on lead is small at low energies because of the extreme concentration of Gamow–Teller (GT) strength in a single resonance at high excitation, so that information about low-energy neutrinos will be hard to obtain.

A low-threshold charged-current detector would therefore add to our ability to study neutrinos from the $\nu_e$ sphere, particularly if the detector could measure the spectrum of electrons from the neutrino interactions and if it were made of a material with a large SN-$\nu$ cross section. If our ideas about the $\nu_e$ sphere are grossly wrong, such a detector would also tell us that. By looking for high-energy $\nu_x$’s, the detector could also complement existing and planned facilities in studying SN-$\nu$ oscillations.

A recent paper [12] argues that MOON (Mo Observatory Of Neutrinos), containing a few tons of $^{100}$Mo, would be useful for studies of both $\beta\beta$ decay (having the ability to detect a neutrino mass as low as $(m_\nu) \sim 0.03$ eV) and real time studies of low energy solar-$\nu$ spectra. In what follows we discuss how $^{100}$Mo and a scaled-up MOON would be useful for studying SN-$\nu$’s as well as low energy solar-$\nu$’s.

The isotope $^{100}$Mo has a threshold ($Q$ value) for the charge-exchange process $\nu_e + ^{100}$Mo $\rightarrow e^- + ^{100}$Tc \[ \tag{2} \]
of only $Q = 0.17$ MeV, much less than other detectors with light nuclei such as $^{12}$C and $^{16}$O. In addition, one expects $^{100}$Mo to exhibit a large response to
charged-current interaction of SN-$\nu$’s because of the large neutron excess (isospin $T_z \equiv (N - Z)/2 = 8$), which enhances the strengths of spin–isospin giant resonances.

Recent measurements of $^{100}$Mo ($^3$He, t)$^{100}$Tc cross sections [13] confirm this expectation. They show that at energies below 50 MeV this reaction (changing neutrons to protons) primarily excites four isospin giant resonances [14]: the isobaric analog resonance (IAR) with $J^\pi = 0^+$, the Gamow–Teller giant resonance (GTR) with $J^\pi = 1^+$, the isovector dipole resonance (IDR) with $J^\pi = 0^-, 1^-, 2^-$. The GTR is accompanied by a low-energy shoulder (GTR$'$) below the main peak. The IAR and IDR are excited by operators in coordinate space (times the isospin-raising operator $\tau_+$) while the GTR and ISDR involve the spin operator $\vec{\sigma}$ as well.

The strength in these resonances are spread over the excitation energy region 5–35 MeV, with the centroid of IAR at 11.6 MeV, the GTR and GTR$'$ centroids at 13.4 MeV and 8 MeV, and the centroid of the combined dipole resonances, which cannot be separated by the experiment, at 21 MeV [13]. This energy range corresponds nicely with that of SN-$\nu$’s, which will therefore also proceed primarily through the resonances, particularly the GTR. The spread of the GT strength down to below 5 MeV together with the low $Q$ value of the charge-exchange process in Eq. (2) make the effective threshold as low as a few MeV, well below the average SN-$\nu_e$ energy. As we discuss next, we can actually use the measured charge-exchange response to calibrate a calculation of SN-$\nu$ cross sections.

Precise expressions for the matrix elements that govern these cross sections are given in Ref. [15]. We use the charge-changing quasiparticle random phase approximation (QRPA) to calculate most of these matrix elements. Our approach is similar to that of Ref. [16] with improvements such as a larger model space (about 20 single-particle levels around the Fermi surface for both protons and neutrons), and a better treatment of the Coulomb interaction of the outgoing electron [17]. The interaction we use has the same $\delta$-function form, with parameters adjusted to fit the observed GTR energy and the low-lying spectrum in $^{100}$Mo. For neutrinos of the energies we consider here, it is sufficient to include multipoles up to $J = 4$.

In the important $1^+$ channel, we replace the QRPA calculation with the measured GT strength. Because the neutrino cross section in this channel is determined mainly by the operator $j_0(qr)\vec{\sigma}\tau_+$, rather than the GT operator $\vec{\sigma}\tau_+$, we must supplement the measured GT strength with a $q$-dependent form factor. We obtain the form factor from the Helm model [18], which takes the strength to be peaked at the nuclear surface. We cannot repeat this procedure for higher multipoles because they are not separated in the measured spin–isospin dipole strength distributions (and the overall normalization is not known). Our theoretical strength distributions, however, reproduce the measured ones quite well, up to the unknown normalization constant. We choose not to artificially quench the strength of the dipole transitions because no clear evidence supports such quenching; muon capture, in fact, argues against it [19]. The use experimental data to calibrate these calculation should make them accurate to within a factor of two at worst.

Fig. 1 shows the calculated cross section for $\nu_e$ scattering on $^{100}$Mo as a function of neutrino energy. The charge-changing flux-averaged SN-$\nu$ cross sections, broken down by multipole, appear in Table 1. We consider two cases, non-oscillating SN-$\nu_e$’s, and SN-$\nu_x$’s (either $\nu_\mu$’s or $\nu_\tau$’s, but not both) that oscillate.

Fig. 1. The calculated cross section for $\nu_e$ charged-current scattering on $^{100}$Mo, as a function of neutrino energy.

---

1 Our cross sections for the highest-energy neutrinos may be slightly too small because of the restrictions on our single-particle space.
Table 1
Calculated flux-averaged neutrino cross sections in units of \(10^{-41}\) cm\(^2\), with contributions from each multipole given separately

<table>
<thead>
<tr>
<th>Multipole</th>
<th>(\nu_e)</th>
<th>(\nu_{ex})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^+)</td>
<td>0.65</td>
<td>8.94</td>
</tr>
<tr>
<td>0(^-)</td>
<td>0.02</td>
<td>0.59</td>
</tr>
<tr>
<td>1(^+)</td>
<td>4.62</td>
<td>32.34</td>
</tr>
<tr>
<td>1(^-)</td>
<td>0.14</td>
<td>11.86</td>
</tr>
<tr>
<td>2(^+)</td>
<td>0.04</td>
<td>4.62</td>
</tr>
<tr>
<td>2(^-)</td>
<td>0.34</td>
<td>14.00</td>
</tr>
<tr>
<td>3(^+)</td>
<td>0.03</td>
<td>3.78</td>
</tr>
<tr>
<td>3(^-)</td>
<td>0.10</td>
<td>1.00</td>
</tr>
<tr>
<td>4(^+)</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>4(^-)</td>
<td>0.07</td>
<td>0.79</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>5.84</strong></td>
<td><strong>78.16</strong></td>
</tr>
</tbody>
</table>

We assume that the SN energy is partitioned equally among all neutrino flavors. The average electron energy of 25 MeV for \(\nu_{ex}\) is about 2.5 times larger than the average energy of 11 MeV for \(\nu_e\), reflecting the ratio of temperatures at the two neutrino spheres. This means that the flux of \(\nu_e\)’s is higher by the same factor, a fact reflected in the count rates.

The large electron energy for \(\nu_{ex}\), together with the large cross section, make a \(\nu_{ex}\) component clearly visible; the observation of a large fraction of the events at relatively high electron energies would be a clear signal of oscillations. But the figure also tells us about the importance of a low threshold. In a large enough detector, the neutrinos from the \(\nu_e\) sphere will clearly be observable if there are no oscillations. If there is a resonant effect that converts all \(\nu_e\)’s into \(\nu_x\)’s then, of course, no detector will tell us anything about the \(\nu_e\) sphere. But if—as in the solution to the solar and atmospheric neutrino problems with large \(\theta_{12}\) and \(\theta_{13} = 0\)—half of the emitted \(\nu_e\)’s oscillate into \(\nu_x\)’s, the number of events from \(\nu_e\) relative to that from \(\nu_{ex}\) will be the same as shown in the figure. At energies below 10 or 15 MeV, a significant fraction of the events would come therefore from the \(\nu_e\) sphere, and one could learn something about the spectrum of emitted \(\nu_e\)’s even in the presence of oscillations.

Fig. 2 shows the calculated spectra (or counts per MeV ton of 100 Mo) of electrons produced by charged-current interactions of both \(\nu_e\) (dashed line) and \(\nu_{ex}\) (solid line), assuming equipartition of SN energy among all flavors. The vertical axis is the number of electrons per MeV per ton of 100 Mo. The proposed MOON detector could be realized either as a supermodule of plastic scintillators with thin natural or enriched molybdenum layers or a liquid scintillator doped with natural or enriched molybdenum.
scintillator, and so could be separated from charged-particles. These particles deposit energy in a large volume of mostly by emitting neutrons and successive $\gamma$ rays. Thus the effective threshold energy ($Q$ value + detector threshold energy) could still as low as 2 MeV, far below the average energy of the $\nu_e$'s.

The cross-section of SN-$\nu_e$'s per unit weight for $^{100}$Mo is about as large as that for $^{208}$Pb because of the large neutron excess ($N - Z)/A = 0.16$ and the small threshold energy. What are the effects of using natural molybdenum rather than $^{100}$Mo? As Table 1 suggests, the non-oscillating $\nu_e$'s mainly excite the GT resonance, so that their cross sections are very roughly given by the product of the GT strength $B$(GT) and a phase space factor $G$. The GT strength is roughly proportional to $T_z$; and $G$ is proportional to $(E_{\nu} - Q_{G})^2$, where $E_{\nu}$ is the effective neutrino energy and $Q_{G}$ the $Q$ value for exciting the GT resonance. $Q_{G}$ has a slight linear dependence on $T_z$ [14,20]. These facts imply that the use of natural Mo with the $T_z \sim 6$ (on average) will reduce the $\nu_e$ count rate by something on the order of 35% from the rate in $^{100}$Mo, which has $T_z = 8$. The $\nu_{\mu}$'s excite all the resonances discussed above, but the strength associated with those also depends linearly on $T_z$. If we assume that the energies of those resonances scale the same way as that of the GT resonance, we find that the count rates in natural molybdenum for the high-energy neutrinos are perhaps 30% smaller than in $^{100}$Mo. The $Q$ values for the ground state transitions are just a few MeV higher for other Mo isotopes than for $^{100}$Mo. Thus a detector with natural Mo can still have a low effective threshold, and efficiencies of the same order as those with $^{100}$Mo. Such a detector could therefore serve our purpose: providing useful information about the spectrum at the electron–neutrino sphere, as well as observing oscillations and measuring the effective temperature at the $\nu_e$ sphere. And if our ideas about the emission of neutrinos by supernovae are wrong, the detector would be sensitive enough to tell us so.

Mo, which has a large neutron excess, is not so sensitive to antineutrinos because most of the GT transitions are Pauli blocked. Neutral-current interactions of SN-$\nu_e$'s would excite the Mo isotopes, which decay mostly by emitting neutrons and successive $\gamma$ rays. These particles deposit energy in a large volume of scintillator, and so could be separated from charged-current events, which have a single electron signal accompanied by several neutron and $\gamma$ signals. But other detectors, such as SK and SNO, would see more neutral current events (and many more antineutrino–charged-current events) than this one would [6,21]. A Mo detector, with its sensitivity to $\nu_e$'s, would therefore not obviate other detectors, but would complement them nicely. Information on the antineutrino spectrum, for example, could strengthen evidence for oscillations that might be observed in the neutrino spectrum.

In summary, $^{100}$Mo and other Mo isotopes have large cross sections for SN-$\nu_e$ and SN-$\nu_{\mu}$. A scaled up version of MOON, which could measure electron energy spectra down to $\sim 2$ MeV, would be useful both for studying neutrino oscillations and for learning about conditions at the electron–neutrino sphere. With the exception of SNO, which has an effective threshold of few MeV, no other detector could do the latter as well. Other heavy nuclei with large $N - Z$ could conceivably be used in place of molybdenum in the liquid-scintillator version of the detector.

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References

Pseudorapidity distributions of charged particles as a function of centrality in Pb–Pb collisions at 158 and 40 GeV per nucleon incident energy

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Abstract

The charged particle distributions $dN_{ch}/d\eta$ have been measured by the NA50 experiment in Pb–Pb collisions at the CERN SPS. Measurements have been done at incident energies of 158 and 40 GeV per nucleon over a broad impact parameter range. Results obtained with two independent centrality estimators, namely the neutral transverse energy $E_T$ and the forward energy $E_{ZDC}$, are reported. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Heavy ion collisions at ultra-relativistic energies are a powerful tool to study nuclear matter under conditions of very high density and temperature. The multiplicity of charged particles produced in the collisions is a global variable that is essential for their characterization, because it quantifies to which extent the incoming beam energy is released to produce new particles. The multiplicity detector of the NA50 experiment, with its good granularity, allows to measure the charged particle multiplicity as a function of pseudorapidity, providing in particular the particle density at midrapidity. This observable gives information about initial conditions such as the energy density, whose knowledge is relevant for the interpretation of the anomalous $J/\psi$ suppression, observed by the NA50 experiment in Pb–Pb collisions [1–3], in terms of the threshold for the creation of a new state of matter.

The pseudorapidity distributions of primary charged particles are presented in this Letter for different classes of events selected according to the centrality of the collision. Two analyses have been performed using two independent centrality-related observables: the energy of the projectile spectator nucleons measured by a Zero Degree Calorimeter and the neutral transverse energy measured by an Electromagnetic Calorimeter. In both cases, the centrality selection has been made using observables which are independent of the multiplicity detector itself, in order to avoid autocorrelations. Preliminary results were reported in [4].

2. Apparatus and data taking conditions

The NA50 apparatus consists of a muon spectrometer equipped with three detectors which measure global observables on an event-by-event basis (namely, charged particle multiplicity, neutral transverse energy and forward energy) and specific devices for beam tagging and interaction vertex identification. The complete detector setup has been presented elsewhere [5]. Here we give some details only on the detectors relevant for the present analysis, as shown in Fig. 1.

The multiplicity measurement is done with a silicon strip Multiplicity Detector (MD) [6–8]. The MD is composed of two identical detector planes called MD1 and MD2, located 10 cm apart. Each of them is a disc of inner radius 6.5 mm and outer radius 88.5 mm (sensitive part), segmented azimuthally in 36 sectors of $\Delta \phi = 10^\circ$ and radially in 192 strips with different sizes in order to have almost constant occupancy per strip. Only digital (hit/no hit) information is provided for each strip. The average granularity is $\Delta \eta \simeq 0.02$. The two planes cover the full $2\pi$ azimuth, while the nominal $\eta$ ranges covered are $1.11 < \eta < 3.51$ for MD1 and $1.61 < \eta < 4.13$ for MD2, for a target located at 11.65 cm from the center of MD1. Since for the analysis presented in this Letter only the 128 innermost strips have been used, the $\eta$ coverage is

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Fig. 1. 1998 experimental setup.

Table 1

<table>
<thead>
<tr>
<th>Beam energy (GeV/nucleon)</th>
<th>Beam intensity (ions/5 s burst)</th>
<th>Target thickness (mm)</th>
<th>Distance (cm)</th>
<th># of events analyzed</th>
</tr>
</thead>
<tbody>
<tr>
<td>158</td>
<td>$3.2 \times 10^6$</td>
<td>3</td>
<td>11.65</td>
<td>48000</td>
</tr>
<tr>
<td>158</td>
<td>$3.9 \times 10^6$</td>
<td>1</td>
<td>9.15</td>
<td>18000</td>
</tr>
<tr>
<td>40</td>
<td>$1 \times 10^6$</td>
<td>3</td>
<td>12.55</td>
<td>35000</td>
</tr>
</tbody>
</table>

restricted to $1.93 < \eta < 3.51$ and $2.47 < \eta < 4.13$ for MD1 and MD2, respectively.

The determination of the centrality of the collision is obtained by means of two different detectors. The Zero Degree Calorimeter (ZDC) [9], made of quartz fibers embedded in tantalum, positioned on the beam axis inside the hadron absorber of the spectrometer, measures the energy $E_{\text{ZDC}}$ of the spectator nucleons travelling in the forward direction, i.e., in the range $\eta \geq 6.3$. The second centrality detector is an electromagnetic calorimeter (EMCAL) made of scintillating fibers embedded into a 14 cm thick lead converter, which measures the neutral transverse energy $E_T$ in the pseudorapidity domain $1.1 < \eta < 2.3$.

The main triggers of the experiment are the dimuon trigger [5] and, more relevant for this analysis, the minimum bias trigger provided by the Zero Degree Calorimeter, generated every time a minimum amount of energy is released in the ZDC. Even for the most central collisions, a few particles produced in the ZDC angular acceptance provide a signal.

Data collected at two different energies of the SPS Pb beam have been used for this analysis: the first data sample was taken in 1998 at 158 GeV per nucleon incident energy, corresponding to $\sqrt{s} = 17.3$ GeV, the second in 1999 at 40 GeV/nucleon energy ($\sqrt{s} = 8.77$ GeV). Special runs taken with the minimum bias trigger at low beam intensity (about 1/10 of the standard intensity used by the experiment) have been used. The average Pb beam intensity, the thickness and the position of the targets are listed in Table 1.
3. Centrality selection

The aim of our analysis is to study the properties of $dN_{ch}/d\eta$ distributions in Pb–Pb collisions as a function of centrality using two independent centrality estimators, namely the forward energy $E_{ZDC}$ and the transverse energy $E_T$. To allow a comparison of the results obtained with these two different variables, centrality intervals have been defined in terms of fractions of the inelastic cross section which was calculated by integrating the Minimum Bias (MB) $dN/dE_{ZDC}$ and $dN/dE_T$ distributions.

Since the MB spectra contain also events from non-interacting Pb projectiles, to obtain the number of interactions ($N_{int}$), the integral of the MB spectra $N_{Pb}$ has to be normalized taking into account the interaction probability $P_{int}$:

$$N_{int} = N_{Pb} \cdot P_{int} = N_{Pb} \cdot (1 - e^{-L_T/\lambda_{int}})$$

where $L_T$ is the target thickness (see Table 1) and $\lambda_{int}$ is the interaction length, which can be expressed as:

$$\lambda_{int} = \frac{A_{targ}}{\rho_{Pb} \cdot N_A \sigma_{inel}},$$

where $A_{targ} = 207.2$ is the atomic mass of the target (Pb) nucleus, $N_A$ is the Avogadro number, $\rho_{Pb} = 11.35$ g/cm$^3$ and $\sigma_{inel}$ is the total inelastic cross section, given by:

$$\sigma_{inel} = \sigma_0 \left( \frac{A_{proj}^{1/3} + A_{targ}^{1/3} - \delta}{A_{targ}^{1/3}} \right)^2.$$  

Assuming $\sigma_0 = 68.8$ mb and $\delta = 1.32$ (obtained interpolating from the values given in [10,11]) one obtains:

$$\sigma_{inel} = 7.62^{+0.54}_{-1.23} \text{ b}, \quad \lambda_{int} = 3.98^{+0.21}_{-0.27} \text{ cm},$$

$$P_{int}(L_T = 3 \text{ mm}) = 7.26^{+0.51}_{-0.35} \times 10^{-2}.$$

3.1. Data at 158 GeV/nucleon

The $E_{ZDC}$ and $E_T$ distributions of MB events at 158 GeV/nucleon beam energy are shown in Fig. 2.

![Graphs showing distributions of $E_{ZDC}$ and $E_T$](image-url)
Table 2

$E_{ZDC}$ and $E_T$ limits for the different centrality classes at 158 GeV/nucleon

<table>
<thead>
<tr>
<th>CLASS</th>
<th>% of c.s.</th>
<th>$E_{ZDC}^{\text{min}}$–$E_{ZDC}^{\text{max}}$ (GeV)</th>
<th>$\langle E_{ZDC} \rangle$ (GeV)</th>
<th>Syst. err. $\langle E_{ZDC} \rangle$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0–5</td>
<td>0–9385</td>
<td>7500</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>5–10</td>
<td>9385–13150</td>
<td>11280</td>
<td>380</td>
</tr>
<tr>
<td>4</td>
<td>15–20</td>
<td>13150–16490</td>
<td>14790</td>
<td>620</td>
</tr>
<tr>
<td>5</td>
<td>20–25</td>
<td>16490–19180</td>
<td>17790</td>
<td>770</td>
</tr>
<tr>
<td>6</td>
<td>25–35</td>
<td>19180–21475</td>
<td>20250</td>
<td>650</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CLASS</th>
<th>% of c.s.</th>
<th>$E_T^{\text{min}}$–$E_T^{\text{max}}$ (GeV)</th>
<th>$\langle E_T \rangle$ (GeV)</th>
<th>Syst. err. $\langle E_T \rangle$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0–5</td>
<td>87.2–140.0</td>
<td>98.3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5–10</td>
<td>71.5–87.2</td>
<td>78.6</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>15–20</td>
<td>58.7–58.7</td>
<td>64.5</td>
<td>2</td>
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<tr>
<td>5</td>
<td>20–25</td>
<td>48.9–40.9</td>
<td>53.4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>25–35</td>
<td>29.6–40.9</td>
<td>44.4</td>
<td>2</td>
</tr>
</tbody>
</table>

The limits of each centrality class have been fixed so as to have classes with a width corresponding to 5% of the total inelastic cross section $\sigma_{\text{inel}}$. When the 5% class would have been too narrow with respect to the $E_{ZDC}$ or $E_T$ resolution giving thus rise to possible biases in the centrality selection, a class with a width corresponding to 10% $\cdot \sigma_{\text{inel}}$ has been defined. In this way, 6 centrality classes have been defined for both centrality estimators. The $E_{ZDC}$ and $E_T$ limits for the different centrality classes are plotted in Fig. 2 and are listed in Table 2 together with the mean $E_{ZDC}$ and $E_T$ value for each class.

The $\simeq 5\%$ uncertainty on the value of $P_{\text{int}}$ is reflected in a $\simeq 5\%$ uncertainty in the evaluation of the total inelastic cross section fraction. This corresponds to uncertainties of $\simeq 300–500$ GeV on the $E_{ZDC}$ limits of each class and $\simeq 2–3$ GeV on the $E_T$ limits, which give the systematic errors on $\langle E_{ZDC} \rangle$ and $\langle E_T \rangle$ quoted in Table 2.

3.2. Data at 40 GeV/nucleon

The same method has been applied to the data sample collected in 1999 at 40 GeV per nucleon incident energy. Due to the worse performance of the ZDC at such a low beam energy, it was not possible to use both centrality estimators and only the analysis with the $E_T$ based centrality selection has been performed. The $E_T$ spectrum of Minimum Bias events is shown in Fig. 3. The limits of the centrality classes (indicated by vertical bars on the spectrum in Fig. 3) correspond to the same intervals of total inelastic cross-section fraction.
used for the 158 GeV/nucleon data sample. Their numerical values can be found in the legend of Fig. 8. At this energy, a larger uncertainty on the centrality intervals is expected as a consequence of the worse resolution of the electromagnetic calorimeter with respect to the 158 GeV/nucleon data sample. Moreover the unavailability of the ZDC information does not allow to make a cross-check with an independent centrality estimator. This affects in particular the most central interval, whose cross-section fraction limit we estimate to be $5^{+1}_{-0.5}$ %.

4. Event analysis

The data analysis has been performed both in the centrality classes defined by $E_T$ and in the ones defined by $E_{ZDC}$, according to the following procedure. First some quality cuts on the data sample have been applied. Then in each centrality class the raw $dN_{ch}/d\eta$ distribution has been calculated. Finally, the primary $dN_{ch}/d\eta$ distribution has been obtained by subtracting the delta electron contribution and then by correcting for secondary processes. Gamma conversions and other processes of secondary particle production or primary particle decay have been evaluated with a complete Monte Carlo simulation based on the VENUS 4.12 [12] event generator and on the GEANT 3.21 [13] package for track propagation and detector response simulation. Below we give some details of the analysis performed for each step. More details can be found in [14].

4.1. Data selection

Events have been selected according to the following criteria. The beam hodoscope recognizes events with two or more ions incident within a 20 ns time window, which could produce pile-up, and are therefore rejected. Interactions occurring upstream from the target are rejected by means of scintillators located close to the beam line. Furthermore, we define as being on-target the events lying in a chosen $E_T$ versus $E_{ZDC}$ correlation band (see, e.g., Fig. 1 of [2]). A further constraint on the vertex position has been applied by requiring the correct geometrical correlation between the hits in the two multiplicity detector planes (MD1, MD2).\footnote{We have verified that this selection (not applied in Figs. 2 and 3) does not affect our centrality classes except for a slight reduction of the number of events in the most peripheral $E_{ZDC}$ class ($21475 < E_{ZDC} < 24790$).}

4.2. Raw multiplicity evaluation

The main difficulty in the extraction of the particle multiplicity from the observed detector occupancy is connected to the presence of many strip clusters, i.e., groups of 2 or more contiguous detector strips firing at the same time.

The origin of clusters could be both physical and instrumental. The physical cluster mechanism is due to particles crossing contiguous strips and to the effects of particle energy deposition inside the detector. By instrumental origin we refer to all clustering sources connected to a deteriorated performance of the detector and front–end electronics. Considering a real system performance, one of the most likely candidates for instrumental clustering effect is the crosstalk between channels in the front–end electronics system. The crosstalk effect gives rise to clustering effects similar to the ones coming from physical processes. Systems having significant crosstalk would show a cluster distribution shifted towards bigger cluster sizes.

Since a VENUS + GEANT Monte Carlo simulation of the full detector system, which includes only clusters of physical origin, does not reproduce the cluster distributions observed in the experimental data, a correction to account for clusters of instrumental origin is needed. The method used to get the true particle occupancy from the measured strip occupancy is based on minimization techniques aimed at reproducing the cluster distribution of the experimental data.

The strip occupancy is simulated generating the particle distributions in the detector and afterwards including the effects responsible for clustering. The particles are generated in each pseudorapidity bin according to a Poissonian distribution whose mean equals the particle occupancy. The clustering mechanisms are modeled by means of coefficients describing the probability that the strips in the vicinity of real particle tracks are firing. Then for a given particle occupancy and probability coefficients, the cluster distribution is
calculated and compared to the observed one. This procedure is applied iteratively, changing the particle occupancy and the probability coefficients, until the generated cluster distribution reproduces the experimental one. The particle occupancy per strip obtained in this way is then used to calculate the raw $dN_{ch}/d\eta$ distributions, taking into account the geometrical acceptance of the strips in each $\eta$ bin.

The stability of reconstructed particle occupancies against the initial values of the parameters has been checked. For testing purposes the method has been applied to VENUS + GEANT generated data in the complete detector system for different centrality classes. It has been found that the reconstructed $dN_{ch}/d\eta$ distributions are in agreement with the generated ones within 5%. For experimental data samples we checked that in each centrality class the reconstructed occupancy for a given pseudorapidity bin agrees within 6% among adjacent azimuthal sectors of the detector. The ratio between the observed detector occupancy and the real particle occupancy ranges from $\sim 1$ for peripheral (low multiplicity) events to $\sim 1.8$ for central (high multiplicity) events.

4.3. Primary particle multiplicity evaluation

The $\delta$ ray contribution to the detector occupancy is evaluated by means of a GEANT 3.21 simulation, taking into account the effects of the target and of all the other materials, including the mechanical support of the detectors. This contribution reaches a maximum of 5% of the true occupancy, in the most peripheral sample considered in this Letter.

The VENUS + GEANT $dN_{ch}/d\eta$ distributions for different centrality classes are reconstructed with the same method as the one used for the experimental data. This is done in order to keep exactly the same treatment for experimental and Monte Carlo data, so that possible systematics are canceled. The secondary/primary correction factors are obtained dividing VENUS + GEANT reconstructed $dN_{ch}/d\eta$ distributions by the primary VENUS $dN_{ch}/d\eta$ distributions. We have run VENUS 4.12 with the default setting for decays of unstable particles, except for neutral pions (whose decay has been taken in charge by GEANT), meaning that charged particles from decays of $K_0$’s, $\Lambda$’s and other hyperons are considered as primary. The correction factors range from 1.2 to 1.8 depending on target thickness, on target position and on the particular multiplicity detector plane. The primary experimental $dN_{ch}/d\eta$ distributions are obtained dividing the raw $dN_{ch}/d\eta$ distributions, after $\delta$ subtraction, by the secondary/primary correction factors.

The resulting $dN_{ch}/d\eta$ particle distributions from MD1 and MD2, being in agreement in their common $\eta$ range, are then merged together, providing a wider $\eta$ coverage. As a final check, the complete procedure is applied to two data samples with different target thickness and position: the results agree, as shown in Fig. 4.

Including other error sources not discussed so far, as uncertainties on the discrimination threshold of the front-end electronics channels, possible misalignments of the detectors and influence of the primary particle composition (depending on the VENUS model) on the Monte Carlo secondary/primary correction factor, we estimate the overall systematic error on the evaluated multiplicity to be below 8%. For the 40 GeV/nucleon data, since only one centrality estimator is used, a larger systematic error (10%) is estimated.

Fig. 4. Comparison of corrected pseudorapidity distributions for the 1 mm (open squares) and the 3 mm (closed circles) targets; the two targets were placed 2.5 cm apart and lead to different $\eta$ regions covered by MD.
5. Experimental results

5.1. Results at 158 GeV/nucleon

The pseudorapidity distributions of charged particles obtained using $E_{ZDC}$ and $E_T$ as centrality estimators (see Table 2) are shown respectively in Figs. 5 and 6. The pseudorapidity coverage is approximately centered at midrapidity and extends over $\sim 2.2$ units, so that the $dN_{ch}/d\eta$ peak is visible in the pseudorapidity distributions without any reflection around midrapidity. The $dN_{ch}/d\eta$ distributions are rather symmetrical around the midrapidity point ($\eta_{\text{max}} \simeq 3.1$ corresponding to $y_{\text{max}} = 2.91$) and their heights increase steadily with increasing centrality.

The $dN_{ch}/d\eta$ distributions have been integrated in the range $2.45 < \eta < 3.65$ (approximately symmetric around midrapidity) and divided by the width of the pseudorapidity interval considered ($\Delta\eta = 1.2$), to obtain the average value of the charged particle pseudorapidity density at midrapidity ($\langle dN_{ch}/d\eta \rangle_{\text{mid}}$).

Depending on the variable ($E_T$ or $E_{ZDC}$) used as centrality estimator, two different values of $\langle dN_{ch}/d\eta \rangle_{\text{mid}}$ have been obtained. The relative difference between these two estimations amounts to $\simeq 1.5\%$ for the five most central classes, while for the most peripheral class it is below $3\%$.

In Fig. 7 the value of $\langle dN_{ch}/d\eta \rangle_{\text{mid}}$ as a function of centrality is plotted, showing an approximately linear dependence of the charged multiplicity on both $E_T$ and $E_{ZDC}$. The average value of charged particle pseudorapidity density for the most central class of events (0–5% of the total inelastic cross section) is:

$$\langle dN_{ch}/d\eta \rangle_{\text{mid}} = 428 \pm 1(\text{stat}) \pm 34(\text{syst})$$

averaged over the values obtained using $E_{ZDC}$ and $E_T$ as centrality estimators. This result can be compared with the corresponding one extracted from a VENUS 4.12 simulation, which is:

$$\langle dN_{ch}/d\eta \rangle_{\text{VENUS}}_{\text{mid}} = 465 \pm 6.$$  

and turns out to be 8% higher than our measured value.

The average primary charged multiplicity in the pseudorapidity range $2.75 < \eta < 3.95$ (approximately equal to the muon spectrometer acceptance) has been calculated by integrating the $dN_{ch}/d\eta$ distributions, obtaining for the 5% most central events:

$$\langle N_{ch} \rangle_{2.75<\eta<3.95} = 508 \pm 2(\text{stat}) \pm 40(\text{syst}).$$
5.2. Results at 40 GeV/nucleon

The same analysis has been performed on data collected at 40 GeV per nucleon beam energy. The $dN_{ch}/d\eta$ distributions are shown in Fig. 8.

The average value of the charged particle pseudorapidity density at midrapidity has been evaluated over the interval $2.15 < \eta < 2.90$, approximately symmetric around the midrapidity value $\eta_{\text{max}} \approx 2.47$, extracted from VENUS. It scales linearly as a function of $E_T$, as it can be seen in Fig. 9. For the most central class (0–5% of the total inelastic cross section) it is:

$$\langle dN_{ch}/d\eta \rangle_{\text{mid}} = 207 \pm 1(\text{stat}) \pm 16(\text{syst})$$
approximately 2 times smaller than the value measured at 158 GeV per nucleon.

6. Conclusions

The charged particle pseudorapidity distributions \( dN_{ch}/d\eta \) in Pb–Pb collisions at 158 GeV per nucleon \( (\sqrt{s} = 17.3 \text{ GeV}) \) and 40 GeV per nucleon \( (\sqrt{s} = 8.77 \text{ GeV}) \) beam energy have been measured in 6 centrality classes defined in terms of fractions of the total inelastic cross section.

Data at 158 GeV/nucleon have been analyzed using two independent centrality estimators, namely, the forward energy \( E_{ZDC} \) and the neutral transverse energy \( E_T \). The results obtained are in agreement within \( \pm 1.5\% \) for the four most central classes of events and within \( 3\% \) for the most peripheral events considered in this analysis.

The average charged particle pseudorapidity density in a \( \eta \) interval symmetric around the peak has been evaluated for each centrality class and shows an approximately linear correlation with both measured centrality-related variables \( (E_{ZDC} \text{ and } E_T) \). No saturation or enhancement of the charged multiplicity is observed up to the most central \( E_T \) or \( E_{ZDC} \) interval considered in this analysis. This conclusion is also valid for the 40 GeV per nucleon data sample for which only the analysis in terms of \( E_T \) has been performed.

From the comparison of the data collected at the two energies it results that the charged multiplicity increases by a factor of \( \approx 2 \) when going from \( \sqrt{s} = 8.77 \text{ GeV} \) to \( \sqrt{s} = 17.3 \text{ GeV} \).

A further analysis of these results as a function of the number of participant nucleons and of the number of binary nucleon–nucleon collisions is presented in [15].

Acknowledgements

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References

Scaling of charged particle multiplicity in Pb–Pb collisions at SPS energies

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Abstract

The charged particle multiplicity distribution $dN_{ch}/d\eta$ has been measured by the NA50 experiment in Pb–Pb collisions at the CERN SPS. Measurements were done at incident energies of 40 and 158 GeV per nucleon over a broad impact parameter range. The multiplicity distributions are studied as a function of centrality using the number of participating nucleons ($N_{\text{part}}$), or the number of binary nucleon–nucleon collisions ($N_{\text{coll}}$). Their values at midrapidity exhibit a power law scaling behaviour given by $N_{\text{part}}^{1.00}$ and $N_{\text{coll}}^{0.75}$ at 158 GeV. Compatible results are found for the scaling behaviour at 40 GeV. The width of the $dN_{ch}/d\eta$ distributions is larger at 158 than at 40 GeV/nucleon and decreases slightly with centrality at both energies. Our results are compared to similar studies performed by other experiments both at the CERN SPS and at RHIC. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

The pseudorapidity distributions of charged particles in Pb–Pb collisions at the CERN SPS have been measured with the multiplicity detector of the NA50 experiment [1]. The charged multiplicity information may help constrain different models of particle production, and quantify the relative importance of soft versus hard processes in the particle production mechanism at different energies. On this respect, an important test for models of particle production in heavy ion reactions is the study of its scaling properties with respect both to the number of participant nucleons ($N_{\text{part}}$) and to the number of binary collisions ($N_{\text{coll}}$).

A scaling with $N_{\text{part}}$ is expected in scenarios dominated by soft processes, when the produced particles undergo a strong rescattering in the final state and the memory of the exact history of multiple collisions is lost. Then the participant nucleons can be assumed to contribute with the same amount of energy to particle production, and the scaling with $N_{\text{part}}$ is approximately linear. This kind of law has already been shown to work at the SPS energies in describing the charged particle multiplicity in $p$–A [2] and in Pb–Pb collisions [3–5], and also the $E_T$ measured in oxygen and sulphur induced reactions [6] and in Pb–Pb collisions [7].

On the contrary, a scaling with $N_{\text{coll}}$ arises naturally in a scenario where nuclear collisions are modeled as a superposition of binary nucleon–nucleon collisions. Such a scaling is expected to be observed in a regime of nuclear reactions where hard processes dominate over the soft particle production, as could be reached at RHIC and LHC energies. First results from the experiments at RHIC [8–10], showing evidence of a large contribution of hard processes to particle production, seem to indicate that such a regime has already been reached at RHIC energies.

In this Letter the particle density at midrapidity ($dN/d\eta$|$_{\text{max}}$) is studied versus the centrality of the collision using values of $N_{\text{part}}$ and of $N_{\text{coll}}$ calculated in the framework of the Glauber model [11].

Another information relevant for constraining particle production models is provided by the study of the scaling of charged particle multiplicity versus $\sqrt{s}$. Results from the analysis presented in this Letter, at the two beam energies corresponding to $\sqrt{s} = 8.77$ and 17.3 GeV, are important to enrich the pattern outlined by the results of other experiments at the SPS and RHIC.

2. Experimental setup and data taking conditions

In this Letter we only refer to data collected with heavy ion collisions. For those runs, the NA50 apparatus [12] consists of a muon spectrometer, equipped with three centrality detectors (a Zero Degree Calorimeter, an Electromagnetic Calorimeter and a Multiplicity...
Detector) and specific devices for beam tagging and interaction vertex identification.

The data presented in this Letter are extracted from special runs taken with the Minimum Bias (MB) trigger, which requires a non-zero energy deposit in the ZDC, at low beam intensity (about 1/10 of the standard intensity used by the experiment).

The multiplicity and the angular distribution of charged particles in a wide acceptance window is measured by a silicon strip Multiplicity Detector (MD) [13–15].

The determination of the centrality of the collision is obtained by means of a Zero Degree Calorimeter (ZDC) [16] which measures the energy $E_{ZDC}$ of the spectator nucleons travelling in the forward direction and by an electromagnetic calorimeter (EMCAL) which measures the neutral transverse energy $E_T$ in the pseudorapidity range $1.1 < \eta < 2.3$.

Data collected at two different energies of the SPS Pb beam have been used: the first data sample was taken in 1998 at 158 GeV per nucleon beam energy, the second in 1999 at 40 GeV per nucleon energy. The data selection is described in Section 4.1 of Ref. [1].

3. Evaluation of $N_{\text{part}}$ and $N_{\text{coll}}$

The aim of the present analysis is the study of the scaling properties of the charged particle production in Pb–Pb collisions as a function of centrality expressed in terms of the number of participant nucleons $N_{\text{part}}$ and of binary collisions $N_{\text{coll}}$.

The NA50 apparatus allows to study the $dN_{ch}/d\eta$ distributions in Pb–Pb collisions as a function of the centrality of the collision, estimated using two independent observables (namely, the neutral transverse energy $E_T$ and the forward energy $E_{ZDC}$) as explained in [1]. Centrality classes for the 158 GeV data sample have been defined in terms of cross-section fractions using both $E_T$ and $E_{ZDC}$. For each cross-section interval, the average values of $N_{\text{part}}$ and $N_{\text{coll}}$ have been estimated in the framework of the Glauber model.

![Graph 1](image1.png)

![Graph 2](image2.png)

Fig. 1. Distributions of the forward energy $E_{ZDC}$ and of the neutral transverse energy $E_T$ in Pb–Pb collisions at 158 GeV per nucleon incident energy. Predictions of the Glauber model are superimposed (hatched histograms).
Table 1
Number of participant nucleons and of binary collisions calculated for the 158 GeV/nucleon data sample for the different centrality classes defined by the two independent ($E_{ZDC}$ and $E_T$) centrality variables

<table>
<thead>
<tr>
<th>Class</th>
<th>% of c.s.</th>
<th>$E_{ZDC}^{min} - E_{ZDC}^{max}$ (GeV)</th>
<th>$\langle N_{part} \rangle$</th>
<th>RMS $N_{part}$</th>
<th>$\langle N_{coll} \rangle$</th>
<th>RMS $N_{coll}$</th>
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<tr>
<td>1</td>
<td>0–5</td>
<td>0–9385</td>
<td>354</td>
<td>22</td>
<td>802</td>
<td>66</td>
</tr>
<tr>
<td>2</td>
<td>5–10</td>
<td>9385–13150</td>
<td>294</td>
<td>23</td>
<td>634</td>
<td>63</td>
</tr>
<tr>
<td>3</td>
<td>10–15</td>
<td>13150–16490</td>
<td>246</td>
<td>25</td>
<td>501</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>15–20</td>
<td>16490–19180</td>
<td>205</td>
<td>26</td>
<td>395</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>20–25</td>
<td>19180–21475</td>
<td>173</td>
<td>28</td>
<td>316</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>25–35</td>
<td>21475–24790</td>
<td>129</td>
<td>35</td>
<td>214</td>
<td>74</td>
</tr>
</tbody>
</table>

Table 2
Number of participant nucleons and of binary collisions calculated for the 40 GeV/nucleon data sample for the different centrality classes

<table>
<thead>
<tr>
<th>Class</th>
<th>% of c.s.</th>
<th>$E_T^{min} - E_T^{max}$ (GeV)</th>
<th>$\langle N_{part} \rangle$</th>
<th>RMS $N_{part}$</th>
<th>$\langle N_{coll} \rangle$</th>
<th>RMS $N_{coll}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0–5</td>
<td>87.2–140.0</td>
<td>352</td>
<td>25</td>
<td>796</td>
<td>73</td>
</tr>
<tr>
<td>2</td>
<td>5–10</td>
<td>71.5–87.2</td>
<td>294</td>
<td>26</td>
<td>632</td>
<td>72</td>
</tr>
<tr>
<td>3</td>
<td>10–15</td>
<td>58.7–71.5</td>
<td>245</td>
<td>23</td>
<td>498</td>
<td>61</td>
</tr>
<tr>
<td>4</td>
<td>15–20</td>
<td>48.9–58.7</td>
<td>203</td>
<td>20</td>
<td>392</td>
<td>52</td>
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<tr>
<td>5</td>
<td>20–25</td>
<td>40.9–48.9</td>
<td>169</td>
<td>19</td>
<td>309</td>
<td>44</td>
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<tr>
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<td>25–35</td>
<td>29.6–40.9</td>
<td>127</td>
<td>20</td>
<td>213</td>
<td>45</td>
</tr>
</tbody>
</table>

assuming that $E_T$ is proportional to the number of participants and $E_{ZDC}$ to the number of projectile spectators. Smearing effects due to the experimental resolution of the calorimeters have also been included in our calculation. In Fig. 1 we show a comparison between the $E_T$ and $E_{ZDC}$ Minimum Bias spectra calculated with the Glauber model and the experimentally measured ones with and without the vertex constraint based on the geometrical correlation between the two MD planes. The agreement between the data and the model is remarkable.

In the Glauber calculations the nuclear density $\rho$ has been parametrized by a Woods–Saxon distribution:

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-r_0)/C}}$$

with parameters $r_0 = 6.624$ fm, $C = 0.549$ fm and $\rho_0 = 0.16$ fm$^{-3}$ [17].

The results concerning $N_{part}$ and $N_{coll}$ at 158 GeV are reported in Table 1 where it can be seen that the $E_T$ and $E_{ZDC}$ based calculations are generally in good agreement.

For the 40 GeV data sample, due to the worse performance of the ZDC at such a low beam energy, only the analysis with the $E_T$ based centrality selection has been performed resulting in a larger uncertainty on the centrality interval definition. The $E_T$ distribution at 40 GeV/nucleon (as it can be seen in Fig. 3 of Ref. [11]) does not exhibit a sharp knee, as it does at 158 GeV. This can be connected to the calorimeter resolution and to the fact that not all the data sample cleaning cuts

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7 The vertex constraint rejects the non interacting Pb ions whose contribution dominates the spectra at high $E_{ZDC}$-low $E_T$. 


were applied. In particular, it was not possible to use the $E_T$ vs. $E_{ZDC}$ correlation cut and the halo counter cut. For this reason, we decided to use the $N_{\text{part}}$ distributions calculated without including the experimental smearing extracted from the fit to the observed $E_T$ distributions of MB events. The values of $\langle N_{\text{part}} \rangle$ obtained in this way turn out to be in good agreement with the ones obtained at 158 GeV/nucleon energy where the calorimeter resolution is taken into account. The average values of $N_{\text{part}}$ and $N_{\text{coll}}$ at 40 GeV per nucleon incident energy are reported in Table 2.

4. Data analysis

4.1. Properties of the pseudorapidity distributions

The pseudorapidity distributions of the primary charged particles for the different centrality classes have been calculated following the procedure explained in [1].

The $dN_{\text{ch}}/d\eta$ distributions thus obtained have been fitted with gaussian functions, to obtain an estimate of the charged particle pseudorapidity density at the peak ($dN_{\text{ch}}/d\eta|_{\text{max}}$), of the peak position ($\eta_{\text{max}}$) and of the gaussian width ($\sigma_{\text{gaus}}$). We emphasize that, thanks to the wide $\eta$ coverage ($\sim 2.2$ units) approximately symmetric around the peak, we do not need to fix the mid-rapidity point $\eta_{\text{max}}$ at the theoretical value. Instead, we leave it as a free parameter of the fit.

The results of the fits for the 158 GeV data sample with the two independent centrality selections are shown in Fig. 2 ($E_{ZDC}$ selection) and in Fig. 3 ($E_T$ selection), together with tables listing the resulting fit parameters.

The results obtained with the two independent centrality estimators are in agreement within 1.5% except for the most peripheral class where the difference
between the $dN_{ch}/d\eta|_{\text{max}}$ values amount to $\simeq 2.5\%$, confirming the results presented in [1].

The midrapidity values resulting from the gaussian fits are compatible with the value $\eta_{\text{max}} \simeq 3.1$ extracted from VENUS [18].

In Fig. 4, the particle pseudorapidity distributions obtained for the data collected at 40 GeV per nucleon beam energy are shown, as well as the values of $\eta_{\text{max}}$, $\sigma_{\text{gaus}}$ and $dN_{ch}/d\eta|_{\text{max}}$ resulting from the gaussian fits. The $\eta_{\text{max}}$ value expected from VENUS is 2.47 and is compatible with our results.

The comparison of our results on $dN_{ch}/d\eta|_{\text{max}}$ with those of other SPS experiments is reported in Table 3 for the 158 and for the 40 GeV/nucleon data samples. The systematic error on our multiplicity evaluation amounts to 8% at 158 GeV/nucleon and to 10% at 40 GeV/nucleon [1].

The width of the gaussian fit to our pseudorapidity distribution is lower at 40 than at 158 GeV/nucleon, reflecting the fact that the available phase-space in rapidity increases with the center-of-mass energy. In Fig. 5 we compare our results on the width in central Pb–Pb collisions with existing data at SPS and AGS energies. First, we present the evolution with $\sqrt{s}$ of the gaussian width of $dN_{ch}/d\eta$ for our most central class (0–5%) of Pb–Pb collisions at 40 and 158 GeV per nucleon, together with the width measured in central Au–Au collisions by the E877 Collaboration [24] at 10.8 GeV/c per nucleon. The fit of our results, taken together with the E877 one, to the simple scaling law $\sigma_\eta = a + b \times \ln \sqrt{s}$ [25,26], gives $\sigma_\eta = (0.58 \pm 0.09) + (0.32 \pm 0.04) \times \ln \sqrt{s}$ (solid line), confirming the already observed fact that the width of the pseudorapidity distribution in central ion–ion coll-
sions at AGS–SPS energies appears to follow a simple logarithmic scaling law independent of system size. It is interesting to note [32] that at 158 GeV/nucleon the width of the rapidity distributions is about twice as large as the one expected from a single thermal source located at midrapidity.

A similar scaling law actually holds also for the width of the pion rapidity distribution measured in $p$–$p$ collisions for the same energy range [27–30]: $\sigma_{y}(\pi^{+}) \approx 0.59 \times \ln \sqrt{s}$ and $\sigma_{y}(\pi^{-}) \approx 0.54 \times \ln \sqrt{s}$. The gaussian width of $dN_{ch}/d\eta$ for particles having $\beta > 0.85$ (thus excluding slow protons) measured in $p$–$p$ collisions at 100 and 200 GeV/c beam momentum [31] is within errors compatible with $\sigma_{y}(\pi^{+})$ at the same energies, confirming a close relationship between pseudorapidity and rapidity widths. So, $p$–$p$ collisions show a width (for $\pi^{+}$ and $\pi^{-}$ rapidity distributions) which is similar to the one of central ion–ion collisions at AGS energy, but then rises slightly faster with $\sqrt{s}$, reaching $\approx 20\%$ higher values at the highest SPS energy.

We have also reported in Fig. 5 the widths of the rapidity distributions of identified $\pi^{+}$, $\pi^{-}$, $K^{+}$ and $K^{-}$ measured by the E802 Collaboration [33–35] in the 3–5% most central Au–Au collisions at 11.6 GeV/c per nucleon and by the NA49 Collaboration [20,23,36] in the 7% (respectively 5%) most central Pb–Pb collisions at 40 (respectively 158) GeV per nucleon (dashed lines connect $\sigma_{y}$ values for the same particle species). We note that the widths of the rapidity distributions for identified produced hadrons follow the same scaling vs. $\sqrt{s}$ as the width of the pseudorapidity distribution, and furthermore $\sigma_{\eta} \approx \sigma_{y}(\pi^{+})$, with $\sigma_{y}(\pi^{+}) > \sigma_{y}(\pi^{-}) > \sigma_{y}(K^{+}) > \sigma_{y}(K^{-})$.

Finally, we have reported in Fig. 5 the width of the pseudorapidity distribution in the $\approx 10\%$ most central O–Ag/Br collisions (between 14.6 and 200 GeV/c per nucleon) measured by the EMU01 Collaboration [37] (triangles and dashed-dotted line), which have the

<table>
<thead>
<tr>
<th>Class</th>
<th>% of c.s.</th>
<th>$\eta_{\text{max}}$</th>
<th>$\sigma_{\text{gaus}}$</th>
<th>$dN_{ch}/d\eta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0–5</td>
<td>2.43 ± 0.03</td>
<td>1.27 ± 0.02</td>
<td>211 ± 4</td>
</tr>
<tr>
<td>2</td>
<td>5–10</td>
<td>2.43 ± 0.03</td>
<td>1.29 ± 0.02</td>
<td>165 ± 3</td>
</tr>
<tr>
<td>3</td>
<td>10–15</td>
<td>2.45 ± 0.03</td>
<td>1.31 ± 0.02</td>
<td>138 ± 2</td>
</tr>
<tr>
<td>4</td>
<td>15–20</td>
<td>2.42 ± 0.03</td>
<td>1.35 ± 0.03</td>
<td>114 ± 2</td>
</tr>
<tr>
<td>5</td>
<td>20–25</td>
<td>2.45 ± 0.03</td>
<td>1.31 ± 0.03</td>
<td>96 ± 1</td>
</tr>
<tr>
<td>6</td>
<td>25–35</td>
<td>2.38 ± 0.08</td>
<td>1.39 ± 0.06</td>
<td>73 ± 1</td>
</tr>
</tbody>
</table>

Fig. 4. Pseudorapidity distributions of charged particles in 40 GeV/nucleon Pb–Pb collisions obtained using $E_{T}$ as centrality estimator.
same slope as our data but slightly smaller absolute values (note that slow protons are excluded from the EMU01 measurement).

We further observe a decrease of the width $\sigma_{\eta}$ of $\approx 10\%$ at both 158 and 40 GeV/nucleon when going from our most peripheral class to our most central one. The narrowing of the shape of pseudorapidity distributions with increasing centrality has been observed by several other experiments, among which NA35 [38,39], WA80 [40,41], NA34/2 [42], HELIOS-Emulsion [43] and E802 [44]. This narrowing can be associated with the higher degree of stopping reached in the interaction [19], and is mostly due to the decreasing contribution of protons from target and projectile fragmentation. In fact, emulsion experiments, which report the distribution of shower particles ($\beta > 0.7$) excluding therefore slow protons from the target fragmentation, usually find a weaker dependence of $\sigma_{\eta}$ on centrality (see, e.g., [45]).

4.2. Centrality dependence of charged particle production

To evaluate the centrality dependence of particle production, the scaling behaviour of the $dN_{ch}/d\eta|_{\text{max}}$ as a function of the number of participant nucleons $N_{\text{part}}$ has been parametrized with the usual power law behaviour:

$$\left(\frac{dN_{ch}}{d\eta}\right)_{\text{max}} \propto N_{\text{part}}^{\alpha}.$$ 

The fit has been performed with the technique explained in [46] to take into account also the error on the independent variable $N_{\text{part}}$. The error on the average value of the number of participants has been assumed to be proportional to the RMS of the distribution (quoted in Tables 1 and 2), and tuned on the basis of the deviations observed on $\langle N_{\text{part}} \rangle$ when varying the smearing parameters in the Glauber calculations. We thus assume $\delta N_{\text{part}} = 0.2 \cdot \text{RMS for the 158 GeV sample and } \delta N_{\text{part}} = 0.4 \cdot \text{RMS for the 40 GeV sample.}$

The value of the scaling exponent for the 158 GeV/nucleon data sample results to be $\alpha = 1.00 \pm 0.01$ with both the $E_T$ and the $E_{ZDC}$ centrality.
Fig. 5. Gaussian width of pseudorapidity or rapidity distributions as a function of center-of-mass energy for ion–ion collisions. See text for explanation.

Fig. 6. Pseudorapidity density of $N_{ch}$ at midrapidity as a function of the number of participants ($N_{\text{part}}$) in 158 GeV per nucleon Pb–Pb collisions with the two independent centrality selections. Power-law fits are superimposed.

selections, as it can be seen in Fig. 6. The systematic error on the $\alpha$ exponent has been estimated by a Monte Carlo simulation where the values of $\langle N_{\text{part}} \rangle$ have been varied inside their error bars independently for the 6 centrality classes. A systematic error of 0.04 on the exponent has thus been obtained. We can therefore conclude that both centrality selections lead to a scaling of the charged particle production with the number of participants characterized by an exponent:

$$\alpha_{158} = 1.00 \pm 0.01 \pm 0.04 \text{(syst)}.$$  

This value of $\alpha$ is in agreement with the Wounded Nucleon Model assumption that the average multiplicity in a collision is proportional to the number of participant (wounded) nucleons.
It has to be stressed that the value of the exponent \( \alpha \) is strongly dependent on the value of \( \langle N_{\text{part}} \rangle \) and may vary significantly as a consequence of slight variations of \( \langle N_{\text{part}} \rangle \). Also the \( \langle N_{\text{part}} \rangle \) definition plays an important role [47,48]. For this reason we performed also power law fits using different \( \langle N_{\text{part}} \rangle \) evaluations. If the values of \( \langle N_{\text{part}} \rangle \) from VENUS 4.12 [18] are used, we obtain \( \alpha = 1.08 \) with the \( E_T \) centrality selection and \( \alpha = 1.05 \) with the \( E_{ZDC} \) selection. For the \( E_{ZDC} \) centrality selection we performed also the fit using a straightforward \( \langle N_{\text{part}} \rangle \) evaluation, namely, \( \langle N_{\text{part}} \rangle = 2 \cdot 208 \cdot (1 - E_{ZDC}/E_{\text{beam}}) \) for which we obtain \( \alpha = 1.02 \).

Our results for the \( \alpha \) exponent of the power law fit to the \( N_{\text{part}} \) dependence of \( dN_{\text{ch}}/d\eta|_{\text{max}} \) can be compared with the results of the WA98 [3] and WA97/NA57 [4] experiments at the SPS. The WA97/NA57 experiment uses a Glauber calculation of the number of participants and finds \( \alpha = 1.05 \pm 0.05 \), which is compatible with our result. The WA98 result (\( \alpha = 1.08 \pm 0.03 \)) has been obtained using a VENUS based estimation of \( N_{\text{part}} \) and is in agreement with what we find using \( N_{\text{part}} \) extracted from VENUS.

A fit to the power law \( dN_{\text{ch}}/d\eta|_{\text{max}} \propto N_{\text{coll}}^d \) has also been performed, obtaining for the exponent the values \( \beta = 0.74 \) and \( \beta = 0.76 \) with \( E_{ZDC} \) and \( E_T \) centrality selections, respectively. Therefore, we can conclude that \( N_{\text{coll}} \) is well suited to describe the scaling of particle production with the centrality of the collision and that a scaling like \( N_{\text{coll}} \) is not observed at this energy.

Finally, a fit with the function \( dN_{\text{ch}}/d\eta|_{\text{max}} = A \times N_{\text{part}} + B \times N_{\text{coll}} \) has been done, in order to verify the possible presence of a term proportional to the number of collisions. The results of the fits for both centrality selections lead to values of \( B \) compatible with zero, indicating that the contribution from hard processes to charged particle production is negligible at this energy.

The data sample collected at 40 GeV per nucleon has also been fitted with \( N_{\text{part}}^p \) leading to \( \alpha = 1.02 \pm 0.02 \). The systematic uncertainty on \( \alpha \), coming both from the same Monte Carlo evaluation used for the 158 GeV/nucleon data sample and from neglecting the experimental smearing in the Glauber calculations, is estimated to be 0.06. Therefore, the scaling exponent at 40 GeV/nucleon is determined to be:

\[ \alpha_{40} = 1.02 \pm 0.02 \text{(stat)} \pm 0.06 \text{(syst)} \]

compatible with the value found at 158 GeV per nucleon. If the values of \( \langle N_{\text{part}} \rangle \) from VENUS 4.12 [18] are used instead, we obtain \( \alpha = 1.10 \), which confirms the already known fact [48] that VENUS gives an \( \alpha \) value systematically \( \sim 0.08 \) higher than the analytical Glauber calculation.

4.3. Energy dependence of charged particle production

In order to examine the energy dependence of charged particle production and to compare our results with the ones obtained for other colliding systems, we study the charged particle pseudorapidity density at midrapidity per participant pair. The results are plotted in Fig. 7 as a function of \( N_{\text{part}} \) (evaluated with Glauber calculations). The error bars take into account the statistical error on \( dN_{\text{ch}}/d\eta|_{\text{max}} \) as well as the uncertainty on \( \langle N_{\text{part}} \rangle \), while the 8% (respectively 10% at 40 GeV) systematic error on the multiplicity evaluation is not included.

In particular, at 158 GeV per nucleon for the 0–5% centrality range we obtain:

\[ \left( \frac{dN_{\text{ch}}/d\eta|_{\text{max}}}{0.5 \cdot \langle N_{\text{part}} \rangle} \right)_{0.5 \cdot \langle N_{\text{part}} \rangle} = 2.49 \pm 0.03 \text{(stat)} \pm 0.20 \text{(syst)} \]

which is the average of the values obtained with the \( E_T \) and \( E_{ZDC} \) centrality selections. The systematic error accounts for the 8% systematic uncertainty on the multiplicity evaluation.

At 40 GeV per nucleon, for the 0–5% centrality range we obtain:

\[ \left( \frac{dN_{\text{ch}}/d\eta|_{\text{max}}}{0.5 \cdot \langle N_{\text{part}} \rangle} \right)_{0.5 \cdot \langle N_{\text{part}} \rangle} = 1.18 \pm 0.03 \text{(stat)} \pm 0.17 \text{(syst)} \]

The large systematic error bar is due both to the 10% systematic error on the multiplicity and to the uncertainty (\( \approx 10\% \)) on the evaluation of \( \langle N_{\text{part}} \rangle \) for the most central band.

The yield per participant pair thus obtained can be compared to the ones measured at higher energies by RHIC experiments PHOBOS [8,49], BRAHMS [10] and PHENIX [9]. Since RHIC measurements are performed in the center-of-mass frame, to make a quantitative comparison we need to convert our results, obtained in the laboratory frame, to the center-of-mass frame. For our data at 158 GeV, assuming pions, kaons and protons relative yields as measured by
Fig. 7. Pseudorapidity density of $N_{ch}$ at midrapidity per participant pair as a function of the number of participants $N_{\text{part}}$ in 158 and 40 GeV/nucleon Pb–Pb collisions.

NA49 [5,20] and using the formula:

$$\frac{dN_{ch}}{dp_Td\eta} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{dN_{ch}}{dy}$$

the measured yield of 2.49 translates into 2.14 ± 0.17. At 40 GeV we use the relative yields as obtained with VENUS 4.12 since the proton fraction has not been yet measured. We estimate that the measured yield of 1.18 translates into 0.97 ± 0.14 in the center-of-mass frame. In Fig. 8, the pseudorapidity density per participant pair in the center-of-mass frame for the most central ion–ion collision at SPS and RHIC is shown together with some fits to $p\bar{p}$ data. It is important to point out that the yield per participant pair depends on the $N_{\text{part}}$ calculation and therefore the comparison of our results with other experiments, which may use different models for the evaluation of
$N_{\text{part}}$, is very delicate, even if $N_{\text{part}}$ is not strongly model dependent for central collisions [47].

When comparing our results with the charged particle pseudorapidity density measured in nucleon–nucleon collisions we observe that our result at 40 GeV/nucleon is in agreement with the fit to data of inelastic $p\bar{p}$ interactions obtained by the UA5 experiment [50] assuming a logarithmic energy dependence, $dN/d\eta |_{\text{max}} = (0.01 \pm 0.14) + (0.22 \pm 0.02) \ln s$. It is also compatible with the UA5 fit obtained assuming a power law energy dependence, $dN/d\eta |_{\text{max}} = (0.74 \pm 0.04)x^{0.105\pm0.006}$. Therefore, we can conclude that the charged particle yield per participant pair at 40 GeV/nucleon is compatible with the one observed in nucleon–nucleon interactions at similar energies.

On the opposite our result at 158 GeV/nucleon is more than 50% higher than any of the mentioned fits for the corresponding center-of-mass energy. In addition our result at 158 GeV is also $\sim 20\%$ higher than the fit ($dN/d\eta |_{\text{max}} = 2.5 - 0.25 \ln s + 0.023 \ln^2 s$) of the yield obtained by CDF [51] in $p\bar{p}$ non-single diffractive interactions for much higher energies. The isospin effect ($\sim 10\%$ among $pp$, $pn$ and $nn$ interactions [52]) cannot account for such a discrepancy as it can be argued also from the fact that our measurement at 158 GeV/nucleon results higher than the one obtained in $pp$ interactions at 200 GeV [53] ($dN/d\eta_{\text{CM}} \approx 1.55 \pm 0.04$ after conversion from the measured $dN/d\eta$). This comparison suggests a steep increase of particle production in central ion–ion collisions between 40 and 158 GeV which cannot be described by a simple energy scaling as observed in nucleon–nucleon collisions. Therefore, the particle production at 158 GeV/nucleon, although it scales approximately linearly with the number of participants, cannot be explained as an ordinary superposition of nucleon–nucleon interactions.

5. Conclusions

The charged particle pseudorapidity distributions $dN_{\text{ch}}/d\eta$ have been studied as a function of the number of participant nucleons $N_{\text{part}}$ and of binary nucleon–nucleon collisions $N_{\text{coll}}$ in Pb–Pb collisions at two different beam energies, namely, 158 GeV per nucleon ($\sqrt{s} = 17.3$ GeV) and 40 GeV per nucleon ($\sqrt{s} = 8.77$ GeV).

The maximum of the $dN_{\text{ch}}/d\eta$ distributions has been estimated by means of gaussian fits. The results obtained indicate a steep increase of particle production at SPS energies, which amounts to approximately a factor of 2 when going from $\sqrt{s} = 8.77$ GeV to 17.3 GeV.

The charged particle pseudorapidity density at mid-rapidity scales as $N_{\text{part}}$ with $\alpha = 1.00 \pm 0.01$ (stat) $\pm 0.04$ (syst) at 158 GeV per nucleon beam energy, in agreement with the Wounded Nucleon Model predictions. The presence of a contribution scaling like $N_{\text{coll}}$ is not observed, so that hard processes seem to play a negligible role in charged particle production at 158 GeV per nucleon. This is also supported by the fact that the value of the $\alpha$ exponent is compatible with the one obtained from the data at 40 GeV per nucleon ($\alpha = 1.02 \pm 0.02 \pm 0.06$) where no contribution from hard processes is expected.

The increase of charged particle production at midrapidity between 40 and 158 GeV/nucleon cannot be described by the simple energy scaling observed in nucleon–nucleon collisions at similar energies.

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Search for $\gamma\gamma \rightarrow \eta_b$ in $e^+e^-$ collisions at LEP 2

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Abstract

A search for the pseudoscalar meson η_b is performed in two-photon interactions at LEP 2 with an integrated luminosity of 699 pb$^{-1}$ collected at $e^+e^-$ centre-of-mass energies from 181 GeV to 209 GeV. One candidate event is found in the six-charged-particle final state and none in the four-charged-particle final state, in agreement with the total expected background of about one event. Upper limits of $\Gamma_{\gamma\gamma}(\eta_b) \times \text{BR}(\eta_b \rightarrow 4 \text{ charged particles}) < 48$ eV, $\Gamma_{\gamma\gamma}(\eta_b) \times \text{BR}(\eta_b \rightarrow 6 \text{ charged particles}) < 132$ eV are obtained at 95% confidence level, which correspond to upper limits of 9.0% and 25% on these branching ratios.

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1. Introduction

The $b\bar{b}$ ground state, the $\eta_b$ meson, has not yet been observed. Because of their initial state, two-photon collisions are well suited for the study of pseudoscalar mesons, for which $J^{PC} = 0^{-+}$. The high $\gamma\gamma$ cross section and the high LEP luminosity and energy, as well as the low background from other processes, make LEP 2 a good environment to search for this meson.

Theoretical estimates (from perturbative QCD and lattice nonrelativistic QCD) of the mass difference, $\Delta m$, between the $\eta_b$ and the $\Upsilon$ ($m_{\Upsilon} = 9.46$ GeV/c$^2$) are summarized in Table 1 and those of the partial decay width of the $\eta_b$ into two photons, $\Gamma_{\gamma\gamma}(\eta_b)$, in Table 2. For the former, values ranging from $\Delta m = 34$ to 141 MeV/c$^2$ are obtained. For the latter, a value of $\Gamma_{\gamma\gamma}(\eta_b) = 557 \pm 85$ eV, chosen in this Letter, is obtained from the average of the first order estimates (488 eV) shifted by 69 eV at the second order in $\alpha_s$. It yields an exclusive $\eta_b$ production cross section of $0.304 \pm 0.046$ pb in $e^+e^-$ collisions at $\sqrt{s} = 197$ GeV. The branching ratios of the $\eta_b$ into four and six charged particles are estimated as in Ref. [1] to be 2.7% and 3.3%, respectively. (The same estimate gives 9.9% for the $\eta_c$ decay branching fraction into four charged particles, in agreement with the measured value of 9.3 ± 1.8% [2].) Six and seven exclusive $\eta_b$ are therefore expected to be produced in the 699 pb$^{-1}$ of data collected by ALEPH above the WW threshold, in the four- and six-charged-particle final states, respectively.

A measurement of the $\eta_b$ mass and of its decay modes would therefore provide a test of pQCD and NRQCD [3–5]. Searches have already been conducted by the CUSB and CLEO Collaborations in the cascade decay of the $\Upsilon(3S)$: the CUSB Collaboration finds for the product of the branching ratios $\text{BR}(\Upsilon(3S) \rightarrow \pi^+ \pi^- h_b) \times \text{BR}(h_b \rightarrow \gamma \eta_b) < 0.45\%$ at 90% C.L. for an $\Upsilon$–$\eta_b$ splitting between 50 and 110 MeV/c$^2$ [6]. The CLEO Collaboration has published a 90% C.L. upper limit on the product of the branching ratios $\text{BR}(\Upsilon(3S) \rightarrow \pi^+ \pi^- h_b) \times \text{BR}(h_b \rightarrow \gamma \eta_b)$ of about 0.1% for the $\eta_b$ mass range from 9.32 to 9.46 GeV/c$^2$ with a photon energy ranging from 434 to 466 MeV and the $h_b$ mass restricted to 9.900 ± 0.003 GeV/c$^2$ [7].

In this Letter, a search is presented for the $\eta_b$ meson via its decay into four and six charged particles. The search is performed in quasireal two-photon interactions where the meson is produced exclusively. This Letter is organized as follows. A description of the ALEPH detector is given in Section 2. The data analysis with event selection, efficiency calculation, background estimate and systematic uncertainty determination is described in Section 3. The results of the search are presented in Section 4. Finally, in Section 5 a summary is given.

2. ALEPH detector

A detailed description of the ALEPH detector and its performance can be found in Ref. [19]. The central part of the ALEPH detector is dedicated to the reconstruction of the trajectories of charged particles. The trajectory of a charged particle emerging from the interaction point is measured by a two-layer silicon strip vertex detector (VDET), a cylindrical drift

Table 1
Estimates for the mass splitting $\Delta m = m(\Upsilon) - m(\eta_b)$ from QCD calculations

<table>
<thead>
<tr>
<th>Calculation</th>
<th>$\Delta m$ [MeV/c$^2$]</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice NRQCD</td>
<td>45–100</td>
<td>[3,8,9]</td>
</tr>
<tr>
<td>Lattice potential</td>
<td>60–110</td>
<td>[10]</td>
</tr>
<tr>
<td>1/m expansion</td>
<td>34–114</td>
<td>[12]</td>
</tr>
<tr>
<td>Potential model</td>
<td>60–141</td>
<td>[13–15]</td>
</tr>
</tbody>
</table>

Table 2
Estimates for the two-photon width $\Gamma_{\gamma\gamma}(\eta_b)$

<table>
<thead>
<tr>
<th>$\Gamma_{\gamma\gamma}(\eta_b)$ [eV]</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates $O(\alpha_s)$</td>
<td></td>
</tr>
<tr>
<td>Potential model</td>
<td>500 ± 30</td>
</tr>
<tr>
<td>Potential model, $\Gamma_{e^+e^-}(\Upsilon)$</td>
<td>490 ± 40</td>
</tr>
<tr>
<td>NRQCD</td>
<td>460</td>
</tr>
<tr>
<td>NRQCD, $\Gamma_{e^+e^-}(\Upsilon)$</td>
<td>501</td>
</tr>
<tr>
<td>Estimates $O(\alpha_s^2)$</td>
<td></td>
</tr>
<tr>
<td>NRQCD, $\Gamma_{e^+e^-}(\Upsilon)$</td>
<td>570 ± 50</td>
</tr>
<tr>
<td>Used in this Letter</td>
<td>557 ± 85</td>
</tr>
</tbody>
</table>
chamber (ITC) and a large time projection chamber (TPC). The three tracking detectors are immersed in a 1.5 T axial magnetic field provided by a superconducting solenoidal coil. Together they measure charged particle transverse momenta with a resolution of $\delta p_t/p_t = 6 \times 10^{-4} p_t \oplus 0.005$ (p$_t$ in GeV/c). The TPC also provides a measurement of the specific ionization $dE/dx_{\text{meas}}$. An estimator may be formed to test a particle hypothesis, $\chi^2 = (dE/dx_{\text{meas}} - dE/dx_{\exp,h})/\sigma_{\exp,h}$, where $dE/dx_{\exp,h}$ and $\sigma_{\exp,h}$ denote the expected specific ionization and the estimated uncertainty for the particle hypothesis $h$, respectively.

Photons are identified in the electromagnetic calorimeter (ECAL), situated between the TPC and the coil. The ECAL is a lead/proportional-tube sampling calorimeter segmented in $0.9^\circ \times 0.9^\circ$ projective towers read out in three sections in depth. It has a total thickness of 22 radiation lengths and yields a relative energy resolution of $0.18/\sqrt{E} + 0.009$, with $E$ in GeV, for isolated photons. Electrons are identified by their transverse and longitudinal shower profiles in ECAL and their specific ionization in the TPC.

The iron return yoke is instrumented with 23 layers of streamer tubes and forms the hadron calorimeter (HCAL). The latter provides a relative energy resolution of charged and neutral hadrons of $0.85/\sqrt{E}$, with $E$ in GeV. Muons are distinguished from hadrons by their characteristic pattern in HCAL and by the muon chambers, composed of two double-layers of streamer tubes outside HCAL.

The information from the tracking detectors and the calorimeters are combined in an energy-flow algorithm [19]. For each event, the algorithm provides a set of charged and neutral reconstructed particles, called energy-flow objects in the following.

3. Analysis

3.1. Event selection

The search is performed in the four- and six-charged-particle modes, where four (or six) charged energy-flow objects with a net charge zero are required. In order to keep the efficiency high, loose selection cuts are chosen. No attempt is made to reconstruct $K_S$ mesons at this stage. The $dE/dx$ measurement, when available, must be consistent with the pion or kaon hypothesis ($\chi^2_{\pi,K} < 9$); the more likely hypothesis is used for mass assignment. When no $dE/dx$ information is available the pion mass is assigned to the particle. No neutral energy-flow object with $E > 1$ GeV must be present within 20$^\circ$ of the beam axis. No muon and no electron (as defined by the ECAL) must be observed. Events are also excluded if a photon conversion is detected, where the electron and positron are identified by requiring $\chi^2 < 9$, and the pair invariant mass is smaller than 25 MeV/c$^2$.

The total transverse momentum of charged particles in the event ($\sum p_t$) must be smaller than 250 MeV/c. The energy-flow objects in the event are boosted into their centre-of-mass frame and the thrust is computed in this frame. The thrust axis must form an angle $\theta_{\text{thr}}$ larger than 45$^\circ$ with respect to the beam axis to reject events from the $\gamma\gamma$ continuum background. The $\gamma\gamma \rightarrow \tau^+\tau^-$ background is reduced to a negligible fraction by the rejection of events in which both hemispheres, as defined by the thrust axis, have a net charge of ±1 and an invariant mass less than 1.8 GeV/c$^2$.

3.2. Signal efficiency

Selection and reconstruction efficiencies are studied with events generated with PHOTO2 [20] in which the $\eta_b$ mass is set to 9.4 GeV/c$^2$ and the total width to 7 MeV/c$^2$. The width is calculated under the assumption that the two-gluon decay is dominant [2,21,22]. Four samples of 2500 events each are generated for the final state with four charged particles ($2(\pi^+\pi^-), \pi^+\pi^-K^+K^-, 2(K^+K^-), K_SK^+\pi^-$). Four other samples of 2500 events each are generated for the final state with six charged particles ($3(\pi^+\pi^-), 2(\pi^+\pi^-)K^+K^-, \pi^+\pi^-2(K^+K^-), 3(K^+K^-)$). For the decays, it is assumed that the momenta are distributed according to phase space. The event samples are passed through the detector simulation and reconstruction programs. The mass resolution of the selected events is about 0.14 GeV/c$^2$ and is dominated by wrong mass assignment from $\pi^-K$ misidentification. A signal region between 9.0 and 9.8 GeV/c$^2$ is chosen. The event selection efficiencies averaged over the four decay channels are found to be 16.7% and 9.3% for the four- and six-charged-track channels, respectively.
3.3. Systematic uncertainties

The lack of knowledge of the decay modes and kinematics of the $\eta_b$ meson is the source of the dominant systematic uncertainties in the analysis. The uncertainty on the selection efficiency due to the unknown decay mode of the $\eta_b$ meson is estimated from the spread of the efficiencies of the four simulated decay modes. The relative uncertainty is 7.5% and 20.4% for the four- and six-charged-particle final states. In order to check the effect of the selection efficiency due to the assumption of phase space decays, the $\eta_b$ is forced to decay into a pair of $\phi$ mesons, each giving two charged kaons. In this case a relative increase of 10% in the detection efficiency is found.

Further studies are performed without the final cut on neutral energy or with modified cuts on $\sum p_{t,i}$, $\theta_{\text{brust}}$, and hemisphere mass. An uncertainty of 5.5% is estimated. The limited statistics of simulated events contribute an uncertainty of 2.4% and 3.2% for the two decay modes, respectively.

A total relative uncertainty of 9.7% (21.4%) on the selection efficiency is calculated for the four- (six-) charged-track channel.

3.4. Background estimate

The background estimate suffers from the low statistics of the simulated events selected and from the poor description of the shape of the invariant mass spectra. The background, dominated by $\gamma\gamma$ continuum production, is therefore estimated from data by means of a fit to the ratio of the mass spectra after all cuts are applied and before the final cuts on $\sum p_{t,i}$, $\theta_{\text{brust}}$, and hemisphere mass are applied. The ratio is fitted with an exponential function up to $m = 6 \text{ GeV}/c^2$ ($m = 7 \text{ GeV}/c^2$) for the four- (six-) charged-particle topology. The average of the values of this function at $m = 6 \text{ GeV}/c^2$ ($m = 7 \text{ GeV}/c^2$) and at $m = 9.4 \text{ GeV}/c^2$ is then multiplied by the number of events in the signal region before the final cuts to obtain the background estimate. Half of the difference between these two values is taken as the systematic uncertainty on the estimate. The background in the signal region is determined to be 0.30 ± 0.25 (0.70 ± 0.34) events for the four- (six-) charged-particle topology.

4. Results

Invariant mass spectra of the selected events are shown in Fig. 1. A total of 33727 (3432) events is selected in the four- (six-) charged-particle final states. In the signal region, only one event is found in the six-prong topology.

4.1. Cross section upper limit

From the knowledge of the background $b$ and the efficiency $\varepsilon$, the observed number of events $n$ is converted into an upper limit on the signal events $\mu$ into a confidence level $\alpha$ given by

$$1 - \alpha = \frac{\int g(b) f(\varepsilon) \sum_{i=0}^{n} P(i | \mu \varepsilon + b) \, db}{\int g(b) \sum_{j=0}^{n} P(j | b) \, db},$$

where $P(j | x)$ is the Poisson probability that $j$ events be observed, when $x$ are expected. The probability density functions for the background $g(b)$ and the efficiency $f(\varepsilon)$ are assumed to be Gaussian, but restricted to the range where $b$ and $\varepsilon$ are positive. Upper limits of 3.06 (4.69) events at 95% confidence level are calculated for the four- (six-) prong topology. This translates into the upper limits

$$\Gamma_{\gamma\gamma}(\eta_b) \times \text{BR}(\eta_b \rightarrow 4 \text{ charged particles}) < 48 \text{ eV},$$

$$\Gamma_{\gamma\gamma}(\eta_b) \times \text{BR}(\eta_b \rightarrow 6 \text{ charged particles}) < 132 \text{ eV}.$$ 

With a two-photon width of $557 \pm 85 \text{ eV}$, upper limits on the branching ratios $\text{BR}(\eta_b \rightarrow 4 \text{ charged particles}) < 9.0\%$ and $\text{BR}(\eta_b \rightarrow 6 \text{ charged particles}) < 25\%$ are derived.

4.2. Mass of the candidate

The raw reconstructed mass of the candidate, as obtained from the measured momenta of the six charged particles and with masses assigned according to the $dE/dx$ measurement, is 9.45 GeV/c$^2$. The mass estimate can be refined with additional information visible from the event display shown in Fig. 2. Two of the six tracks form a secondary vertex compatible with the decay of a $K_S$ into $\pi^+\pi^-$. This hypothesis is supported by the presence of a third track compatible with a $K^{-}$ ($\chi^2 = 6.0$ and $x^2_K = 3.8 \times 10^{-8}$). The secondary vertex is therefore fitted to this hypothesis, and the five particles (three charged pions,
one charged kaon and one K_S are forced to originate from a common primary vertex. A mass of $9.30 \pm 0.02 \pm 0.02 \text{ GeV}/c^2$ is derived from these constraints.

A control sample of $\eta_c$ mesons is selected in the $K_S K^+ \pi^-$ decay mode, without the final cuts but that on the total transverse momentum, which is relaxed to $\sum \vec{p}_{t,i} < 500 \text{ MeV}/c$. The analysis is repeated with
Fig. 2. An \( r \phi \) view of the \( \eta_c \rightarrow K_S K^- \pi^+ \pi^- \pi^+ \) candidate event with the reconstructed mass of \( 9.30 \pm 0.02 \pm 0.02 \) GeV/c\(^2\), selected in the signal region. The track coordinates recorded in the VDET and the ITC are shown. The tracks are appropriately labeled. The plot to the right shows an \( r z \) view of the ALEPH apparatus. Information is given for each track: particle type, momentum (GeV/c), momentum error (GeV/c), azimuthal and polar angle (degrees), transverse and longitudinal impact parameter (cm).

this control sample for the study of the systematic uncertainty on the mass determination. The mass of the \( \eta_c \) meson is fitted and is found consistent with the world average value \([2]\) within its statistical accuracy of 4.7 MeV/c\(^2\). A systematic uncertainty of the same size is assigned. The total uncertainty is then rescaled with the mass ratio \( m(\text{candidate})/m(\eta_c) \) and a systematic uncertainty of 21 MeV/c\(^2\) is obtained for the mass estimate of the \( \eta_c \) candidate. The \( \eta_c \) signal is shown in Fig. 3 together with the \( D^0 \) signal as observed in its \( K^- \pi^+ \) decay mode. The fitted \( D^0 \) mass agrees with the world average value \([2]\) within its statistical accuracy of 0.9 MeV/c\(^2\). The number of observed \( \eta_c \) mesons is consistent with previous measurements \([2,22,24]\).

5. Summary

With an integrated luminosity of 699 pb\(^{-1}\) collected at e\(^+\)e\(^-\) centre-of-mass energies between 181 and 209 GeV, the pseudoscalar meson \( \eta_b \) is searched for in its decays to four and six charged particles. One candidate is retained in the decay mode into six charged particles, while no candidate is found in the four-charged-particle decay mode. The candidate \( \eta_b \) has a reconstructed invariant mass of \( 9.30 \pm 0.02 \pm 0.02 \) GeV/c\(^2\). The observation of one event is consistent with the number of events expected from background.

Upper limits on \( \Gamma_{\gamma\gamma}(\eta_b) \times \text{BR} \) of 48 and 132 eV, corresponding to limits on the branching ratios.
BR(\eta_b \rightarrow 4 \text{ charged particles}) < 9.0\% and BR(\eta_b \rightarrow 6 \text{ charged particles}) < 25\%, are obtained at a confidence level of 95\%.

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Self energies of the pion and the $\Delta$ isobar from the $^3\text{He}(e,e'\pi^+)^3\text{H}$ reaction


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Abstract

In a kinematically complete experiment at the Mainz microtron MAMI, pion angular distributions of the $^3\text{He}(e,e'\pi^+)^3\text{H}$ reaction have been measured in the excitation region of the $\Delta$ resonance to determine the longitudinal ($L$), transverse ($T$), and the $LT$ interference part of the differential cross section. The data are described only after introducing self-energy modifications of the pion and $\Delta$-isobar propagators. Using Chiral Perturbation Theory (ChPT) to extrapolate the pion self energy as inferred from the measurement on the mass shell, we deduce a reduction of the $\pi^+$ mass of $\Delta m_{\pi^+} = (-1.7^{+1.7}_{-2.1})$ MeV/c$^2$ in the

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neutron-rich nuclear medium at a density of \( \rho = (0.057^{+0.085}_{-0.057}) \text{ fm}^{-3} \). Our data are consistent with the \( \Delta \) self energy determined from measurements of \( \pi^0 \) photoproduction from \( ^4\text{He} \) and heavier nuclei. © 2002 Elsevier Science B.V. All rights reserved.

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**Keywords:** Pion electroproduction; Longitudinal-transverse separation; Few-body system; \(^3\text{He} \); Medium effects; Delta resonance region; Self energy

1. Introduction

A basic question in hadronic physics concerns the properties of constituents as they are embedded in a nuclear medium. Such medium effects are commonly treated in terms of self energies from which effective masses and decay widths are deduced. Electroproduction of charged pions from \(^3\text{He} \) represents a viable testing ground to study the influence of the nuclear medium on the production and propagation of mesons and nucleon resonances such as the pion and the \( \Delta \) resonance. As a simple composite nucleus, \(^3\text{He} \) is amenable to precise microscopic calculations of the wave function and other ground state properties [1] and offers the great advantage that effects of final state interaction are expected to be much smaller than in heavier nuclei. Moreover, the mass-three nucleus may already be considered as a medium. In this Letter, we present the results of an experiment which allows the determination of the self energies of the pion and the \( \Delta \) isobar from the analysis of the longitudinal and transverse cross section components, respectively. These self-energy terms are the subject of theoretical descriptions in the framework of the \( \Delta \)-hole model [2] and Chiral Perturbation Theory (ChPT) [3].

2. Measurements

To this end, we have measured the \(^3\text{He}(e,e'\pi^+)\(^3\text{H} \)) reaction in a kinematically complete experiment at the high-resolution three-spectrometer facility [4] of the A1 collaboration at the 855 MeV Mainz microtron (MAMI). The specific experimental arrangements of the present experiment, including that of the cryogenic gas target and the data acquisition and analyses methods are described in detail in [5]. The very high missing mass resolution of \( \delta M \approx 700 \text{ keV}/c^2 \) (FWHM) is quite adequate [5–8] to clearly isolate the coherent channel \((\pi^0\pi^+)\) from the three- and four-body final states \((\pi\pi\pi\pi)\) and \((\pi\pi\pi\pi\pi)\).

The three-fold differential pion electroproduction cross section with unpolarized electron beam and target can be written as [9]

\[
\frac{d^3\sigma}{d\Omega_e\, dE_e \, d\Omega_\pi} = \Gamma \frac{d\sigma_V}{d\Omega_\pi}(W, Q^2, \theta_\pi; \phi_\pi, \epsilon),
\]

with

\[
\frac{d\sigma_V}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon \frac{d\sigma_L}{d\Omega_\pi} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi_\pi \frac{d\sigma_{LT}}{d\Omega_\pi} + \epsilon \cos 2\phi_\pi \frac{d\sigma_{TT}}{d\Omega_\pi}.
\]

Here the quantities \( \epsilon \) and \( \Gamma \) denote the polarization and flux of the virtual photon. The indices \( T, L, LT, \) and \( TT \) refer to the transverse and longitudinal components and their interferences, respectively. The explicit dependence of \( d\sigma_V/d\Omega_\pi \) on the azimuthal pion angle \( \phi_\pi \) and the polarization \( \epsilon \) is used for a separation of the response functions.

The measurements were carried out at two four-momentum transfers \( Q^2 = 0.045 \) and \( 0.100 \text{ (GeV/c)}^2 \), referred to as kinematics 1 and 2, respectively. The energy transfer in the laboratory frame has been chosen at \( \omega = 400 \) and 394 MeV, respectively, i.e., in the \( \Delta \) resonance region. At each \( Q^2 \), three measurements in parallel kinematics with various values of \( \epsilon \) were made to determine the \( L \) and \( T \) cross sections (Rosenbluth separation). Parallel kinematics implies that the pion is detected in the direction of the three momentum of the virtual photon. We have also measured the in-plane pion angular distribution (i.e., \( \phi_\pi = 0^\circ \) or \( 180^\circ \), respectively) for the second kinematics at \( \epsilon = 0.74 \) to determine the \( LT \) term. Parts of the experimental results together with model interpretations have already been presented elsewhere [5,8]. In this Letter, we offer a combined analysis of the entire data set of the experiments in the two kinematics and draw definitive con-
conclusions about medium effects, which are especially well understood for the pion.

3. Results and discussion

The results of the Rosenbluth separation are shown in Fig. 1 where the cross sections are displayed as a function of the virtual photon polarization. The longitudinal cross section is identified as the slope, while the transverse one is given by the intercept with the axis at $\epsilon = 0$. Also shown in Fig. 1 are the fit results for the $L$ and $T$ components with statistical errors. The systematic errors amount to 10% (8%) for kinematics 1 (2), respectively. The theoretical calculations are based on the most recent elementary pion production amplitude in the framework of the so-called Unitary Isobar Model [9,10]. In Plane-Wave Impulse Approximation (PWIA), the amplitude includes the Born terms as well as $\Delta$- and higher resonance terms. For the mass-three nuclei, realistic three-body Faddeev wave functions are employed. In the Distorted-Wave (DWIA) calculations, the final state interaction due to pion rescattering is included [11]. As is seen in Fig. 1, the DWIA calculations underestimate the longitudinal component and overestimate the transverse component, each by about a factor of two. Since the longitudinal component is dominated by the pion-pole term and a large part of the transverse part arises from the $\Delta$ resonance excitation, both the pion and the $\Delta$ propagators have to be modified (see also [12]). In parallel kinematics the pion-pole term only contributes to the longitudinal part of the cross section, while the $\Delta$ excitation is almost purely transverse. Therefore, the pion-pole and the $\Delta$ contribution essentially decouple in the longitudinal and transverse channel and can be studied separately. We next discuss the estimate of these terms.

3.1. Modification of the pion

The inadequacy of the DWIA to account for the longitudinal response (cf. Fig. 1) is remedied by replacing the free pion propagator in the t-channel pion-pole term of the elementary amplitude, $[\omega_{\pi}^2 - \vec{q}_{\pi}^2 - m_{\pi}^2]^{-1}$, by a modified one, $[\omega_{\pi}^2 - \vec{q}_{\pi}^2 - m_{\pi}^2 - \Sigma_{\pi}(\omega_{\pi}, \vec{q}_{\pi})]^{-1}$, where $\Sigma_{\pi}(\omega_{\pi}, \vec{q}_{\pi})$ denotes the pion self energy in the nuclear medium [13]. For the two values of $Q^2$, the energy $\omega_{\pi}$ and the momentum $\vec{q}_{\pi}$ of the virtual pion are fixed as $\omega_{\pi} = 1.7$ (4.1) MeV and $|\vec{q}_{\pi}| = 80.9$ (141.2) MeV/c, such that two experimental numbers for $\Sigma_{\pi}$ can be determined from a fit to the respective longitudinal cross sections. The best-fit values result in $\Sigma_{\pi} = -(0.22 \pm 0.11) m_{\pi}^2$ for kinematics 1 and $\Sigma_{\pi} = -(0.44 \pm 0.10) m_{\pi}^2$ for kinematics 2. Close to the static limit, i.e., for $\omega_{\pi} \approx 0$, appropriate for the kinematical conditions of the present experiment, the pion self energy can be written as

$$\Sigma_{\pi}(0, \vec{q}_{\pi}) = -\frac{\sigma_{N}}{f_{\pi}^2} (\rho_{p} + \rho_{n}) - \frac{\alpha}{\omega_{\pi}} \chi(0, \vec{q}_{\pi}),$$

(2)

where $\rho_{p}$ and $\rho_{n}$ denote the proton and neutron densities, $\sigma_{N} = 45$ MeV the $\pi N$ sigma term [14], $f_{\pi} = 92.4$ MeV the pion decay constant, and $\chi(0, \vec{q}_{\pi})$ the p-wave pionic susceptibility. Since the virtual $\pi^+$ propagates in a triton-like medium, we have $\rho_{n} = 2 \rho_{p}$. In
infinite nuclear matter with Fermi momentum $p_F$, the p-wave pionic susceptibility $\chi(0, \vec{q}_\pi)$ can be approximated by a constant for $|\vec{q}_\pi| \lesssim p_F$, and we will assume that this is also the case here, although a local density approximation for such a small nucleus may be questionable. With the two values for $\Sigma_\pi$ given above, we immediately obtain $\chi = 0.31 \pm 0.22$. On the other hand, a standard calculation with particle-hole (ph) and $\Delta$-hole ($\Delta h$) susceptibilities (see, e.g., [13]) for infinite isospin-asymmetric nuclear matter results already at small densities in much higher values for $\chi$. For example, with $\rho_p + \rho_n = \frac{1}{2}\rho_0 \left( \rho_0 = 0.17 \text{ fm}^{-3} \right)$ being the saturation density, $\rho_0 = 2\rho_p$ and the Migdal parameters $g'_{NN} = 0.8$ and $g'_{\Delta N} = g'_{\Delta \Delta} = 0.6$, we find $\chi \approx 0.8$ (Fig. 2). This is principally due to the large contribution of the ph Lindhard function which, at $\omega_N = 0$, is proportional to $p_F$ and therefore does not change appreciably if one reduces the density within a reasonable range. One obvious improvement is the use of an energy gap in the ph-spectrum at the Fermi surface. It accounts in an average way for the low-lying excitation spectrum of a finite nucleus [15]. Using a gap of 8.5 MeV, appropriate for the continuum threshold of the triton, leads to a reduction of $\chi$ in $N \neq Z$ nuclear matter but is still not able to describe the slope of $\Sigma_\pi$ inferred from the measurement (Fig. 2). This indicates that the use of the bulk-matter Lindhard function is not appropriate for such a small nucleus and the kinematics probed in the experiment. Therefore, we do not attempt to calculate $\chi$ but rather use the above value $\chi = 0.31 \pm 0.22$ from experiment. This allows an extrapolation of the self energy to $\vec{q}_\pi = 0$ and to determine the mean density experienced by the virtual pion, with the result $\rho = \rho_p + \rho_n = (0.057^{+0.085}_{-0.057}) \text{ fm}^{-3} \approx \frac{1}{2}\rho_0$, albeit with a large error. The self energy corresponding to the best fit is displayed in Fig. 2.

For further physical interpretation of the measurement we use guidance from ChPT to infer the effective $\pi^+$ mass at the density probed in the experiment. Given the above mentioned uncertainties in the use of the local density approximation for the medium modification of the pion in very light nuclei these results should be regarded as qualitative. The effective mass can be obtained from an extrapolation of the pion self energy to the mass shell. Up to second order in $\omega_\pi$ and $m_\pi$, the self energy of a charged pion in homogeneous, spin-saturated, but isospin-asymmetric nuclear matter in the vicinity of $\omega_\pi \approx m_\pi$ and for $\vec{q}_\pi = 0$ is given by the expansion

\[
\Sigma_\pi^{(\pm)}(\omega_\pi, 0) = \left( -\frac{2(c_2 + c_3)\omega^2}{f_\pi^2} - \frac{\sigma_N}{f_\pi^2} \right) \rho + \frac{3}{4\pi^2} \left( \frac{3\pi^2}{2} \right)^{1/3} \frac{\omega^2}{4f_\pi^2} \rho^{4/3} + \frac{\omega_\pi}{2f_\pi^2} (\rho_p - \rho_n) + \cdots , \tag{3}
\]

where the $+/-$ signs refer to the respective charge state of the pion (see also [16]). The low-energy constants (LEC’s) $c_2$ and $c_3$ of the Chiral Lagrangian and the $\pi N$ sigma term $\sigma_N$ characterize the $\pi N$ interaction and are related to the $\pi N$ scattering lengths. We use $(c_2 + c_3) \times m_\pi^2 = -26 \text{ MeV}$ [14], but one should remark here that third-order corrections may change the LEC’s somewhat [17]. The pion self energy in Eq. (3) consists of two isoscalar parts proportional to $\rho$ and $\rho^{4/3}$, respectively, and an isovector part proportional to $(\rho_p - \rho_n)$. The latter is known as the "Tomozawa–Weinberg term" [18]. Based on PCAC arguments, it reflects the isovector dominance of the $\pi N$ interaction at $\omega_\pi = m_\pi$, where the isoscalar scattering length as given by the first coefficient in Eq. (3) vanishes at leading order. The second isoscalar term proportional to $\rho^{4/3}$ is caused by $s$-wave pion scattering from correlated nucleon pairs [19]. The sign of the Tomozawa–Weinberg term depends on the isospin.

![Fig. 2. The pion self energy as a function of $\vec{q}_\pi^2$ near $\omega_\pi \approx 0$. The data points with the error bars are from the longitudinal cross sections. The dotted line corresponds to $\chi \approx 0.8$ from the Lindhard function with $\rho = 0.057 \text{ fm}^{-3}$. The dashed line results after taking into account a gap of 8.5 MeV for the ph excitation energy, i.e., the binding energy of $^3\text{H}$. The solid line results from a fit of $\chi$ and $\rho$ to the data according to Eq. (2).](image-url)
asymmetry of the nuclear medium. In the present case of a virtual \( \pi^+ \) propagating in a triton-like medium with \( \rho_p - \rho_n = -\frac{1}{2} \rho \), the isovector term becomes attractive.

The effective \( \pi^+ mass \) \( m_{\pi^+}^* \) is deduced from the pole of the pion propagator at \( \tilde{q}_\pi = 0 \) which is determined by the solution of \( \omega_2^2 - m_{\pi^+}^* - \Sigma_{\pi}(\omega_\pi, 0) = 0 \) with the self energy as given by Eq. (3). Using \( \rho = (0.057_{-0.057}^{+0.060}) \text{ fm}^{-3} \), one obtains a mass shift \( \Delta m_{\pi^+} = m_{\pi^+}^* - m_{\pi^+} = (-1.7_{-2.1}^{+1.7}) \text{ MeV/c}^2 \) when the \( \pi^+ \) propagates in \(^3\text{H}\). It is interesting to compare the determined negative mass shift \( \Delta m_{\pi^+} \) with a positive mass shift \( \Delta m_{\pi^-} \) derived from deeply bound pionic states [20,21] in \(^{207}\text{Pb}\) and \(^{205}\text{Pb}\) with \( N/Z \approx 1.5 \). Itahashi et al. [21] have reported a strong repulsion of 23 to 27 MeV due to the local potential \( U_{-\pi}(r) \) for a deeply bound \( \pi^- \) in the center of the neutron-rich \(^{207}\text{Pb}\) nucleus. Evaluating Eq. (3) for this case with \( \rho_p + \rho_n = \rho_0 \) and \( \rho_n/\rho_p = N/Z \approx 1.5 \) one calculates \( U_{-\pi}(0) = \Sigma_{\pi}(m_{\pi^-}, 0)/(2m_{\pi^-}) \approx 18 \text{ MeV} \). This is in good agreement with the findings of Ref. [16]. Yet, there remains the problem of a “missing repulsion” in the interpretation of the pionic atom data.

Fig. 3 shows the contributions to the pion mass shift in \(^3\text{H}\): the two isoscalar contributions to \( \Sigma_{\pi} \) are both repulsive and increase the pion mass. One thus notices from Eq. (3) that at \( \omega_{\pi^-} = m_{\pi^-}^* \neq m_{\pi^-} \) already the isoscalar contribution to the self energy is sizeable. For a neutron-rich nucleus the isovector \( \pi N \) interactions are attractive (repulsive) for \( \pi^+(\pi^-) \) giving rise to a splitting of the mass shifts (contribution 3 in Fig. 3). In \(^3\text{H}\), the isoscalar and isovector terms are compensating each other to a large extent, resulting in the very small decrease of the \( \pi^+ \) mass.

3.2. Modification of the \( \Delta \)

Most of the DWIA overestimate in the transverse channel (cf. Fig. 1) is removed by a medium modification of the \( \Delta \) isobar. The in-medium \( \Delta \) propagator is written [9] as \( [(\sqrt{3} - M_{\Delta} + i\Gamma_{\Delta}/2 - \Sigma_{\Delta}]^{-1} \), where one introduces a complex self-energy term \( \Sigma_{\Delta} \) in the free \( \Delta \) propagator. Besides this explicit medium modification of the production amplitude also the DWIA formalism for the pion-nucleus rescattering effectively accounts for a \( \Delta \) modification in the medium [11]. The quantity \( \Sigma_{\Delta} \) has been deduced from an energy-dependent fit to a large set of \( \pi^0 \) photoproduction data [9,22] from \(^4\text{He}\) and also consistently describes recent photoproduction data from \(^{12}\text{C},^{40}\text{Ca}\) and \(^{208}\text{Pb}\) [23]. The fitting procedure reported in [9] has been redone with the unitary phase excluded from the propagator in accordance with prescriptions often used in the \( \Delta \)-hole model [24]. The resulting \( \Delta \) self energy exhibits a dependence on the photon energy. Evaluated for the kinematics 1 and 2, which correspond to the photon equivalent energies \( k_{\gamma}^T \approx 392 \) and 376 MeV, respectively, the real and imaginary parts are Re \( \Sigma_{\Delta} \approx 50 \) and 39 MeV and \( \text{Im} \Sigma_{\Delta} \approx -36 \) and \(-29 \text{ MeV} \). Although quite large values are obtained in view of the small density \( \rho \approx 4\rho_0 \), one should stress that the on-shell \( \Delta \) self energy at resonance position is numerically considerably smaller [6,25]. As a result, the agreement with the transverse cross section is significantly improved, although the experimental values are still overestimated by about 30%. The remaining discrepancy may be due to additional theoretical uncertainties. For example, the Fermi motion of the nucleons is effectively accounted for by a factorization ansatz [9]. An exact treatment might reduce the prediction of the transverse cross section by about 10%. A second uncertainty of the order of 10% concerns the knowledge of the elementary \( \pi^+ \) production amplitude at \( \theta_\pi = 0^\circ \). This kinematical region is not probed in photoproduction but may be accessible in the future with appropriate electroproduction data from the photon. Assigning the entire \( \Delta \) self energy to a mass shift \( \Delta M_{\Delta} \) and a width change \( \Delta \Gamma_{\Delta} \), we deduce an increase by 40 to 50 MeV and 60 to 70 MeV, respectively. These values seemingly differ from our earlier results [5], where we have employed the parameterization from Ref. [26] which did not include the \( \Delta \)-hole interaction, giving Re \( \Sigma_{\Delta} \approx -14 \text{ MeV} \) for
a mean $^3$He density of $\rho = 0.09$ fm$^{-3}$. On the other hand, the self-energy term of the present work is an effective parameter which incorporates the influence of the $\Delta$-spreading potential, Pauli- and binding effects as well as the $\Delta$-hole interaction including the Lorentz–Lorenz correction. This finally leads to the positive sign.

The effects of the medium modifications were also examined in the angular distribution of the produced pions in kinematics 2. The data along with the model calculations are shown in the l.h.s. of Fig. 4. The asymmetry of the combined distribution is due to a finite LT interference term. From the azimuthal dependence on $\phi_\pi$ for three polar angle bins $\theta_\pi$ we extract the LT interference term as a function of the pion emission angle $\theta_\pi$, as shown in the r.h.s. of Fig. 4 along with the comparison to the model calculations. It is obvious that only the full calculation, incorporating the medium modifications in the pion and $\Delta$ propagators, is able to reproduce the angular distributions.

4. Summary

In summary, in a kinematically complete experiment, we have measured the longitudinal and transverse cross section as well as the LT interference term for the first time in the $^3$He(e,e'\pi$^\pm$)$^3$H reaction. The high sensitivity of the electroproduction cross section shows clear evidence for self-energy corrections in both the pion and $\Delta$-isobar propagators and complements the large body of previous results from pion-nucleus data. Using ChPT we have extrapolated the pion self energy determined from the present experiment to the mass shell to deduce the effective $\pi^+$ mass in $^3$H. Although qualitative, the results appear to be consistent with the theoretical analysis of deeply bound pionic atoms and the deduced effective $\pi^-$ mass [16]. In the transverse channel, the medium modification of the $\Delta$ isobar is also evident and the self-energy modifications inferred from the present measurements conform with $\pi^0$ photoproduction data over a wide mass range.

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References


High-spin g factors and proton alignment in $^{180,182,184}$Pt

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Abstract

The average $g$ factors of high-spin states in $^{180,182,184}$Pt were measured by the transient-field technique. In all three isotopes the quasicontinuum $g$ factor at an angular momentum of $\sim 20\hbar$ is $\langle g \rangle \sim 0.37$. This contrasts with similar measurements on other nuclei that have $70 \leq Z \leq 80$, where typical values of $\langle g \rangle \sim 0.22$ have been attributed to the influence of quasineutron alignments. Evidently proton and neutron configurations are about equally important at high spin in the Pt isotopes near mid-shell. This inference is consistent with the discrete spectroscopy, including the contention, supported by $g_K - g_R$ values, that $h_9/2$ proton pairs align along with the $i_{13/2}$ neutrons at rotational frequencies of $\hbar\omega \approx 0.3\ MeV$ in $^{184}$Pt. Links between the quasicontinuum $g$ factors and features in the discrete spectroscopy are explored by comparing and contrasting the behavior of $^{184}$Pt and $^{166}$Hf.

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The $g$ factors of high-spin states depend mainly on the relative contributions of aligned protons and/or neutrons [1]. An average high-spin $g$ factor can be measured by the transient-field technique following heavy-ion fusion–evaporation reactions. By measuring the precessions inherited by the discrete states at lower spin, this technique [2–10] determines the average ‘quasicontinuum’ $g$ factor at high spin ($I \gtrsim 20\hbar$) in the discrete bands and the damping region of the continuum. There have been relatively few such measurements, mainly in the rare-earth region: $^{155}$Dy [6], $^{153-156}$Dy [4,5], $^{162}$Yb [3] and $^{162-166}$Hf [7,8]. Recently, $g$ factors of the quasicontinuum feeding several normal-deformation bands in $^{193,194}$Hg were measured along with the $g$ factors of three superdeformed bands [9,10]. With the exception of an early measurement on $^{154}$Dy [4], the quasicontinuum measurements all indicate a dominant role for neutron configurations at spins between about $20\hbar$ and $40\hbar$.

In this Letter we report measurements of the average high-spin $g$ factors in $^{180}$Pt, $^{182}$Pt and $^{184}$Pt,
where, because of the higher proton Fermi surface, quasiproton configurations may have a greater influence than in the cases previously studied. We also explore connections between the quasicontinuum g factor and features of the discrete spectroscopy, which can be near ‘complete’ up to about 1 MeV above the yrast band when studied with modern detector arrays (i.e., almost all of the predicted bands can be observed). On the experimental side, we have developed a rigorous formulation to analyze perturbed directional correlation data obtained with a multidetector array [11–13], which should broaden the scope for future moment measurements.

The platinum isotopes $^{180,182,184}$Pt lie in a transitional region where the nuclei are soft and show co-existing structures that can be associated with different deformations. These structures are well characterized at low spin ($I \lesssim 10h$), as discussed, for example, in Ref. [14]. However, the shapes of these nuclei also change as a function of spin and as different quasiparticle configurations are excited. As a consequence, the unusual quasiparticle alignment characteristics of bands in these nuclei and their odd-$A$ neighbors are subject to debate [15–18]. One question of long-standing interest is whether the alignment properties of bands in the Ir, Pt and Au nuclides around $N = 106$ include a $(\pi h_{9/2})^2$ band crossing at spins between $10h$ and $20h$, along with the $(\nu i_{13/2})^2$ crossing, or whether there is some other process, perhaps due to the softness of the nuclear shape. While the present experiment focuses on states in the quasicontinuum having spins near the upper end of this range, the presence or absence of a $(\pi h_{9/2})^2$ alignment must have an effect on the measured g factor. We will return to this question below.

The neutron-deficient platinum isotopes from $^{180}$Pt to $^{184}$Pt were populated simultaneously by 145 MeV $^{29}$Si induced reactions on a 6 mg/cm$^2$ thick natural Gd foil which served as both the target and the ferromagnetic host. Because the plane of the target is at $65^\circ$ to the beam axis, the effective Gd thickness is 6.6 mg/cm$^2$. The Pt isotopes were produced mainly in the front 2 mg/cm$^2$ of the target. Since the beam energy drops below the Coulomb barrier after traversing $\sim 3$ mg/cm$^2$ of Gd, the recoiling Pt nuclei have energies between 22.5 MeV and 20 MeV, with a range of approximately 2.5 mg/cm$^2$. The Gd foil was backed by a 17.5 mg/cm$^2$ evaporated layer of Pb (to stop the beam) and, for improved thermal conduction, attached to a 12 µm thick Cu foil using a flashing of indium as adhesive. Beams were delivered by the Australian National University 14UD Pelletron accelerator. The multilayered target was cooled to approximately 90 K by flowing liquid nitrogen. Two-fold coincidences between the de-exciting $\gamma$ rays were detected with an array CAESAR [19] configured with seven Compton suppressed Ge detectors in the same plane as the beam. A magnetic field of 0.09 T was applied perpendicular to the detection plane by means of a compact electromagnet. The field direction was reversed automatically every 15 min during the measurement. The $\gamma$-ray energies and relative times, together with a field-direction tag, were written to disk, event-by-event, for subsequent off-line analysis.

In order to extract the average precession angle of the nuclear spin, a rigorous formulation [12] of the Perturbed $\gamma$–$\gamma$ Directional Correlations from the reaction-Oriented nuclei (PDCO) was used. The unperturbed DCO must be determined first. For the short-lived states of interest, the experimental unperturbed DCO can be obtained by adding the data for the two field directions, so that the perturbation cancels out. Some of the unperturbed DCO data are shown in Fig. 1. To make the measured precession angles independent of the detector efficiencies, and to minimize systematic errors, double ratios [6–8] were formed for each two-detector combination. The precessions were then extracted by a simultaneous fit to the 21 independent double ratios involving all two-detector combinations in the CAESAR array. A more detailed account of the formalism and analysis procedures is presented in Refs. [11–13,20]. Comparisons with other techniques are included in Refs. [11,13].

The magnetization of the Gd target foil was measured as a function of temperature and polarizing field, from which it was inferred that $M = 0.18$ T under the conditions of the experiment. The transient-field strength was calibrated using the Rutgers parametrization [21] because it agrees very well with independent measurements of the field strength for Pt in Gd [22,23]. These measurements cover the velocity range of present interest, i.e., they include thin-foil measurements that sample the field for Pt ions with energies down to $\sim 6$ MeV [22], and measurements on $\sim 9$ MeV $^{194,196,198}$Pt ions that stop in Gd hosts [23]. Of particular importance, the low-velocity transient-
field effect inferred from the data in Ref. [23] is in excellent agreement with the Rutgers parametrization. The stopping powers of Ziegler et al. [24] were used consistently in the present and previous calibration work [22,23]. Since our transient-field calibration is based on experimental data for Pt in Gd, we estimate that it can be uncertain by no more than 10%.

The extracted transient-field precession angles and inferred $g$ factors are summarized in Table 1. Like previous quasicontinuum $g$-factor measurements [2,4–10], the inferred $g$ factors here correspond to the average precessions shown by the probe transitions. Since the transient-field precession is accumulated at high spin, largely in the quasicontinuum E2 cascades, the observed precession angles are independent of the discrete transitions used to measure them. (See also the discussion in Ref. [10].)

Within errors, the average quasicontinuum $g$ factors in $^{180}$Pt, $^{182}$Pt and $^{184}$Pt are the same and are larger than the measured values of $g(2^+_1)$ [20,25,26]. (For example, in $^{184}$Pt, $g(2^+_1) = 0.28(3)$ [25].) This is in marked contrast with the previous measurements [3,7,8,10] of quasicontinuum $g$ factors for even nuclei with $70 \leq Z \leq 80$, as summarized in Fig. 2. The smaller average $g$ factors at high spin, usually in the range between 55% and 60% of $Z/A$, and with typical values of $\langle g \rangle \sim 0.22$, have been attributed to the rotational alignment of quasineutron configurations [3,7,8,10]. The values for the Pt isotopes point to contributions from quasiproton configurations, along with the usual neutron configurations.

Since $^{180}$Pt has a larger experimental uncertainty, the following discussion will apply mainly to $^{182}$Pt and $^{184}$Pt. In these two nuclides the strongly populated bands near the yrast lines have been assigned the same—or closely related—configurations [16,27], and the similar values measured for the quasicontinuum $g$ factors reflect the fact that, on average, similar configurations remain near the Fermi surface.
In order to interpret the experimental data, it is useful to have an estimate of the spin range and rotational frequencies that the average $g$ factors sample. This can be obtained by calculating the accumulating transient-field precession in a ‘generic’ rotational band representative of the bands in the continuum (cf. Ref. [3]). A moment of inertia parameter, $h^2 I / 2\pi = 8.6$ keV, and transition quadrupole moment, $Q_T = 500$ fm$^2$, which approximate the data [16,28] above $I = 16\hbar$ in the ground-state band of $^{184}$Pt, were chosen. The moment of inertia is also representative of most of the other discrete bands in the spin range of interest and is consistent with the spacing between the ridges observed in our total $\gamma - \gamma$ matrix. The population of the band immediately after the reaction was distributed as a function of spin to reflect the average angular momentum in the evaporation residues, taking into account the energy loss of the beam in the target. The evolution of the population and the transient field precession accumulated in each state was then tracked until the recoiling nuclei come to rest. Fig. 3 shows how the transient-field precession, calculated with the Rutgers parametrization [21] for unit $g$ factors, samples different spins. Although the level lifetimes below about spin 16 are not very realistic in this model, the distribution of transient-field precessions is not sensitive to these lifetimes because they are all of the order of, or longer than, the time for which the transient field acts. A calculation is also shown for $Q_T = 700$ fm$^2$, which is representative of the yrast states near spin 8 in $^{182}$Pt and $^{184}$Pt [27,28]. In this case the mean spin sampled decreases from $\langle I \rangle = 20.0\hbar$ to $\langle I \rangle = 18.2\hbar$. Thus substantial changes in the $Q_T$ value lead to relatively small shifts in the distribution of spins sampled by the transient-field interaction. These calculations imply that the average $g$ factor mainly samples states in the range $I \sim 15\hbar$ to $\sim 26\hbar$ with $\langle I \rangle \sim 20\hbar$. The corresponding rotational frequencies are in the range from $\hbar\omega \sim 0.35$ to $\sim 0.45$ MeV, which is near the upper end of the frequency range in which pairs of quasiparticles begin to align in the discrete bands.

Before discussing the magnitude of the quasicontinuum $g$ factors in more detail, we must return to the question concerning the alignments in the discrete bands of $^{184}$Pt. The situation has been summarized recently by Mueller et al. [18]. According to blocking arguments based on comparisons with bands in neighboring nuclei [15,16], the alignment patterns in the yrast and near-yrast bands of $^{184}$Pt suggest that both $(\pi h_{9/2})^2$ and $(\nu_{11/2})^2$ alignments occur at frequencies near $\hbar\omega = 0.3$ MeV ($I \sim 14\hbar$, depending on the band), with the $\pi h_{9/2}$ alignment probably occurring first. However the extensive cranked shell model analysis in Ref. [16] suggests that it is two $\nu_{13/2}$ pairs that align sequentially and that the $\pi h_{9/2}$ alignment is at higher frequencies. From another theoretical perspective, calculations using a generator coordinate method [17] predict that the $\nu_{11/2}$ pair starts to align first, but this is followed by a sharp $(\pi h_{9/2})^2$ alignment, and the second $(\nu_{13/2})^2$ alignment occurs at higher frequencies ($\hbar\omega \geq 0.45$ MeV).

Further experimental evidence for $(\pi h_{9/2})^2$ alignments in the discrete bands of $^{184}$Pt comes from the $g_K - g_R$ values in the $K = 8^-$ two-quasineutron band. (To our knowledge these have not been discussed in the literature, although the experimental $\gamma$-ray branching ratios were given in Ref. [16].) As shown in Fig. 4, the $g_K - g_R$ values above the band crossing at $\hbar\omega \simeq 0.26$ MeV ($I \sim 13\hbar$), evaluated using data from Ref. [16], are in excellent agreement with a $(\pi h_{9/2})^2$ alignment, but do not support a $\nu(i_{13/2})^2$ alignment. This shows that if the first $\nu(i_{13/2})^2$ alignment is blocked there can be a low-frequency proton alignment in $^{184}$Pt. It therefore supports the interpretation that both protons and neutrons align near $\hbar\omega = 0.3$ MeV in the yrast band.

In relation to the measured quasicontinuum $g$ factor we note, on one hand, that the shorter-lived discrete
In contrast, in 166 Hf, quasicontinuum states are only a subset of the states that experience the transient field, and hence the measured quasicontinuum g factor cannot distinguish between details of the alignment scenarios proposed for the states at lower spin. But, on the other hand, the aligned configurations observed in the discrete spectroscopy must be present in the quasicontinuum. In the following discussion we therefore explore parallels between features in the discrete spectroscopy and the magnitude of the observed quasicontinuum g factor. To broaden the scope of the discussion we compare the present results for 184 Pt with 166 Hf, where the discrete spectroscopy has been studied thoroughly [29] and the quasicontinuum g factor has been measured for three spin ranges between 15 and 35 h (see Fig. 2 and Ref. [7]).

In 182 Pt and 184 Pt the influence of proton excitations is evident in the discrete spectroscopy. Aside from the $(\pi h_{9/2})^2$ alignment in 184 Pt near $\hbar \omega = 0.3$ MeV discussed above, there is a two-quasiproton band with configuration $\pi h_{9/2} \otimes i_{13/2}$ that becomes yrast at $I \sim 25\hbar$ in both isotopes, after a pair of $i_{13/2}$ neutrons have aligned [16,27]. In contrast, in 166 Hf, two- and four-quasineutron configurations dominate at lower spins and no two-quasiproton bands have been observed. Quasiproton bands appear at higher spins, but they are then associated with multiparticle excitations that include neutrons [29]. For example, the two signatures of a four-quasiparticle (two-proton, two-neutron) band come within 500 keV of yrast only above spin 30, and while many of the bands show evidence of quasiproton alignment near $\hbar \omega = 0.5$ MeV ($I \sim 40\hbar$), they are then four-neutron two-proton excitations.

To illustrate how the g factors in different types of bands may contribute to the average quasicontinuum g factor, and to highlight the differences between rare-earth nuclei like 166 Hf and the Pt isotopes, schematic calculations of g factors were made as a function of spin for the discrete bands in 166 Hf and 184 Pt which have assigned configurations. The g factors were evaluated using a generalization of the semiclassical formulation of Dönau and Frauendorf [1] where the single-particle g factors were evaluated with Nilsson model wavefunctions at deformation $\epsilon = 0.2$ and with the spin g factors quenched to 0.6 of the free nucleon values. The alignments and crossing frequencies were taken from Refs. [16,29]. A spin-independent rotational g factor of $g_R = 0.3$ was adopted for 184 Pt, consistent with the measured g factors of the low-spin states and systematics [25]. In 166 Hf, where $g(2^+)$ has not been measured, we took $g_R = 0.4 \sim Z/A$. (For the present purposes the signature splitting contributions are ignored, as in Ref. [11].) These calculations are shown as the dotted lines in Fig. 5.

A rigorous calculation of the quasicontinuum g factor is not a simple task. It requires an evaluation of the excitation energies, lifetimes and g factors of the states in the quasicontinuum, together with a modeling of the time-evolution of their population through the time interval over which the transient field acts, so that the appropriate average g factor can be evaluated. The construction of such a model is beyond the scope of the present work. However, as a first step, we show in Fig. 5 the result of assuming that the quasicontinuum g factor can be estimated from a smoothed average of the calculated g factors of the observed discrete states, with equal weighting given to all bands, irrespective of their observed population. These calculations are shown as the thick solid lines.

The rationale for this simple approach stems from the observation that at high spin the g factors depend on the single-particle characteristics in such a way that the deviations from $g_R$ are predominately due to the relative contributions from aligned protons and/or neutrons. If it is assumed that the discrete spectroscopy
identifies the configurations near the Fermi surface—including the relative numbers of quasiproton versus quasineutron bands and the rotational frequencies at which pairs of particles begin to align—then it follows that the distribution of $g$ values for each spin in the high-spin discrete spectroscopy is approximately representative of that in the quasicontinuum where it can be assumed that all bands are about equally populated.

Clearly, a more sophisticated model is required for rigorous, quantitative comparisons with experiment. Nevertheless the simple approach already suggests that the reduced value of $\langle g \rangle$ compared with $Z/A$ near spin 16 in $^{166}$Hf can be associated with the presence of many bands, in the relevant part of the quasicontinuum, that include two-quasineutron excitations. The further reduced $\langle g \rangle$ value near spin 24 is probably associated with the sampling of a greater number of four-quasineutron excitations, while the relative increase in $\langle g \rangle$ in the higher spin range ($\sim 30\hbar$) could be linked to the first proton pair alignment, which effectively cancels one of the neutron pair alignments.

Calculations for $^{184}$Pt, similar to those for $^{166}$Hf, are shown in the right-hand panel of Fig. 5. The increases in the calculated $g$-factor values due to proton alignments at comparatively low spin and the presence of the two-quasiproton band with large $g$ factors make a striking contrast with $^{166}$Hf. In $^{184}$Pt the estimated quasicontinuum $g$ factors must be treated with some caution. Apart from the fact that relatively few discrete bands have been identified, Total Routhian Surface (TRS) calculations [16] predict shape changes as a function of spin and configuration that cannot necessarily be anticipated from the discrete bands. For example, oblate bands involving $\pi h_{11/2}$ configurations could contribute to the $\langle g \rangle$ value observed in $^{184}$Pt. Despite these caveats, a good qualitative agreement with experiment is obtained, particularly in relation to the different behavior of $^{184}$Pt and $^{166}$Hf, since for $^{184}$Pt we obtain $\langle g \rangle \sim g_R$, which contrasts with $^{166}$Hf, where $\langle g \rangle \sim 0.5g_R$. (See Fig. 5.)

The calculated $\langle g \rangle$ values in $^{184}$Pt are perhaps somewhat smaller than the experimental value. Although exact agreement cannot be expected for such a simple approach, it is worth noting that the calcu-
lated quasicontinuum $g$ factor would increase with an increased contribution from quasiproton bands and/or if $g_R$ were closer to $Z/A$ than to $g(2^+_1)$. This latter possibility draws attention to the fact that changes in pairing correlations and the consequent changes in $g_R$ should be considered along with particle alignments in the evaluation of high-spin $g$ factors.

The thick broken line in the right-hand panel of Fig. 5 shows the estimated quasicontinuum $g$ factors for the ‘theoretical scenario’ of Ref. [16], which ascribes both alignments near $\hbar\omega = 0.3$ MeV in $^{184}$Pt to $i_{13/2}$ neutrons. These calculated $\langle g \rangle$ values are similar to those in $^{166}$Hf and as such are almost a factor of two smaller than experiment. This contrast again supports the contention that both protons and neutrons align at low rotational frequencies in $^{184}$Pt.

Overall, the comparisons in Fig. 5 suggest that there are strong correlations between the quasicontinuum $g$ factors and features in the discrete spectroscopy. This observation invites a more quantitative evaluation. The present experimental work is the first to combine the use of transient fields and a rigorous analysis of the perturbed $\gamma-\gamma$ directional correlations observed in a multidetector array. Further measurements of $g$ factors at high spin are warranted as these observables can give unique insights into nuclear structure between the yrast line and the quasicontinuum.

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References

Strange resonance production: probing chemical and thermal freeze-out in relativistic heavy ion collisions

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Abstract

The production and the observability of $\Lambda(1520)$, $K^0(892)$ $\Phi$ and $\Delta(1232)$ hadron resonances in central Pb + Pb collisions at 160 $A$ GeV is addressed. The rescattering probabilities of the resonance decay products in the evolution are studied. Strong changes in the reconstructable particle yields and spectra between chemical and thermal freeze-out are estimated. Abundances and spectra of reconstructable resonances are predicted. © 2002 Elsevier Science B.V. All rights reserved.

Strange particle yields and spectra are key probes to study excited nuclear matter and to detect the transition of (confined) hadronic matter to quark–gluon-matter (QGP) [1–8]. The relative enhancement of strange and multi-strange hadrons, as well as hadron ratios in central heavy ion collisions with respect to peripheral or proton induced interactions have been suggested as a signature for the transient existence of a QGP-phase [1].

Unfortunately, the emerging final state particles remember relatively little about their primordial source, since they had been subject to many rescatterings in the hadronic gas stage [9–11]. This has given rise to the interpretation of hadron production in terms of thermal/statistical models. Two different kinds of freeze-outs are assumed in these approaches:

1. A chemical freeze-out, were the inelastic flavor changing collisions processes cease, roughly at an energy per particle of 1 GeV [12].
2. Followed by a later kinetic/thermal freeze-out were also elastic processes have come to an end and the system decouples.

Chemical and thermal freeze-out happen sequentially at different temperatures ($T_{ch} \approx 160–170$ MeV, $T_{th} \approx 120$ MeV) and thus at different times.

To investigate the sequential freeze-out in heavy ion reactions at SPS the Ultra-relativistic Quantum Molecular Dynamics model (UrQMD 1.2) is applied [13,14]. UrQMD is a microscopic transport approach based on the covariant propagation of constituent quarks and diquarks accompanied by mesonic and baryonic degrees of freedom. It simulates multiple interactions of ingoing and newly produced particles, the excitation and fragmentation of color strings and the formation and decay of hadronic resonances. At present energies,
the treatment of sub-hadronic degrees of freedom is of major importance. In the UrQMD model, these degrees of freedom enter via the introduction of a formation time for hadrons produced in the fragmentation of strings [15–17]. The leading hadrons of the fragmenting strings contain the valence-quarks of the original excited hadron. In UrQMD they are allowed to interact even during their formation time, with a reduced cross section defined by the additive quark model, thus accounting for the original valence quarks contained in that hadron [13,14]. Those leading hadrons therefore represent a simplified picture of the leading (di)quarks of the fragmenting string. Newly produced (di)quarks do, in the present model, not interact until they have coalesced into hadrons—however, they contribute to the energy density of the system. For further details about the UrQMD model, the reader is referred to Ref. [13,14].

Let us start by asking whether a microscopic non-equilibrium model can support the ideas of sequential chemical and thermal break-up of the hot and dense matter? To analyze the different stages of a heavy ion collision, Fig. 1 shows the time evolution of the elastic and inelastic collision rates in Pb + Pb at 160 A GeV. The inelastic collision rate (full line) is defined as the number of collisions with flavor changing processes (e.g., $\pi\pi \rightarrow K\overline{K}$). The elastic collision rate (dashed dotted line) consists of two components, true elastic processes (e.g., $\pi\pi \rightarrow \pi\pi$) and pseudo-elastic processes (e.g., $\pi\pi \rightarrow \rho \rightarrow \pi\pi$). While elastic collision do not change flavor, pseudo-elastic collisions are different. Here, the ingoing hadrons are destroyed and a resonance is formed. If this resonance decays later into the same flavors as its parent hadrons, this scattering is pseudo-elastic.

Indeed the main features revealed by the present microscopic study do not contradict the idea of a chemical and thermal break-up of the source as shown in Fig. 1 (top). However, the detailed freeze-out dynamics is much richer and by far more complicated as expected in simplified models:

1. In the early non-equilibrium stage of the AA collision ($t < 2$ fm/c) the collision rates are huge and vary strongly with time.
2. The intermediate stage ($2 < t < 6$ fm/c) is dominated by inelastic, flavor and chemistry changing processes until chemical freeze-out.

3. This regime is followed by a phase of dominance of elastic and pseudo-elastic collisions ($6 < t < 11$ fm/c). Here only the momenta of the hadrons change, but the chemistry of the system is mainly unaltered, leading to the thermal freeze-out of the system.

Finally the system breaks-up ($t > 11$ fm/c) and the scattering rates drop exponentially.

Fig. 1 (bottom) depicts the average energy\(^1\) per particle at midrapidity ($|y - y_{cm}| \leq 0.1$). One clearly observes a correlation between chemical break-up in

\(^1\) To compare to thermal model estimates, the energies are calculated from interacted hadrons with the assumption $p_T^2 = (p_T^x + p_T^y)/2$. This assures independence of the longitudinal motion in the system and the chosen rapidity cut.
terms of inelastic scattering rates and the rapid decrease in energy per particle. Thus, the suggested phenomenological chemical freeze-out condition of \( \approx 1 \text{ GeV/particle} \) is also found in the present microscopic model analysis.

To verify this scenario, we exploit the spectra and abundances of \( \Lambda (1520), \bar{K}^0(892) \) and other resonances which unravel the break-up dynamics of the source between chemical and thermal freeze-out. In the statistical model interpretation of heavy ion reactions the resonances are produced at chemical freeze-out. If chemical and thermal freeze-out are not separated—e.g., due to an explosive break-up of the source—all initially produced resonances are reconstructable by an invariant mass analysis in the final state. However, if there is a separation between the different freeze-outs, a part of the resonance daughters rescatter, making this resonance unobservable in the final state. Thus, the relative suppression of resonances in the final state compared to those expected from thermal estimates provides a chronometer for the time period between the different reaction stages. Even more interesting, inelastic scatterings of the resonance daughters (e.g., \( \bar{K}p \to \Lambda \)) might change the chemical composition of the system after ‘chemical’ freeze-out by as much as 10% for all hyperon species. While this is not in line with the thermal/statistical model interpretation, it supports the more complex freeze-out dynamics encountered in the present model.

To answer these questions we address the experimentally accessible hadron resonances: at this time \( \Phi \) and \( \Lambda (1520) \) have been observed in heavy ion reactions at SPS energies [18,19] following a suggestion that such a measurement was possible [20]. SPS [19] and RHIC experiments [21] report measurement of the \( \bar{K}^0(892) \) signal, and RHIC has already measured both the \( K^0 \) and the \( \bar{K}^0 \). In the SPS case, the \( \Lambda (1520) \) abundance yield is about 2.5 times smaller than expectations based on the yield extrapolated from nucleon–nucleon reactions. This is of highest interest in view of the \( \Lambda \) enhancement by factor 2.5 of in the same reaction compared to elementary collisions.

As an explanation for this effective suppression by a factor 5, we show that the decay products (\( \pi, \Lambda \), etc.) produced at rather high chemical freeze-out temperatures and densities have rescattered. Thus, their momenta do not allow to reconstruct these states in an invariant mass analysis. However, even the question of the existence of such resonance states in the hot and dense environment is still not unambiguously answered. Since hyperon resonances are expected to dissolve at high energy densities (see, e.g., [22]) it is of utmost importance to study the cross section of hyperon resonances as a thermometer of the collision.

The present exploration considers the resonances, \( \Delta (1232), \Lambda^+ (1520), \bar{K}^0(892) \) and \( \Phi \). The properties of these hadrons are depicted in Table 1.

<table>
<thead>
<tr>
<th>Particle</th>
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<th>Width (MeV)</th>
<th>( \tau ) (fm/c)</th>
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<td>1.6</td>
</tr>
<tr>
<td>( \Lambda^+ (1520) )</td>
<td>1520</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>( \bar{K}^0(892) )</td>
<td>893</td>
<td>50</td>
<td>3.9</td>
</tr>
<tr>
<td>( \Phi (1020) )</td>
<td>1019</td>
<td>4.43</td>
<td>44.5</td>
</tr>
</tbody>
</table>

Fig. 2. Rapidity densities for \( \Delta (1232), \Lambda^+ (1520), \bar{K}^0(892) \) and \( \Phi \) in Pb(160 A GeV)Pb, \( b < 3.4 \) fm collisions. Fig. 2 (left), shows the total amount of decaying resonances. Here, subsequent collisions of the decay products have not been taken into account—i.e., whenever a resonance decays during the systems evolution it is counted. However, the additional inter-
action of the daughter hadrons disturbs the signal of the resonance in the invariant mass spectra. This lowers the observable yield of resonances drastically as compared to the primordial yields at chemical freeze-out. Fig. 2 (right) addresses this in the rapidity distribution of those resonances whose decay products do not suffer subsequent collisions—these resonances are in principle reconstructable from their decay products. Note that reconstructable in this context still assumes reconstruction of all decay channels, including many body decays.

At SPS energies, the rapidity distributions $dN/dy$ may be described by a Gaussian curve

$$dN/dy(y) = A \times \exp\left(-\frac{y^2}{2\sigma^2}\right),$$

with parameters given in Table 2. However, in this approximation the details of the rapidity distributions are lost. Especially the dip in the rapidity distributions of the $\Lambda(1520)$ is not accounted for.

The rescattering probability of the resonance decay products depends on the cross section of the decay product with the surrounding matter, on the lifetime of the surrounding hot and dense matter, on the lifetime of the resonance and on the specific properties of the daughter hadrons in the resonance decay channels. This leads to different ‘observabilities’ of the different resonances: rescattering influences the observable $\Phi$ and $\Lambda^*$ yields only by a factor of two, due to the long life time of those particles. In contrast strong effects are observed in the $K^*$ and $\Delta$ yields, which are suppressed by more than a factor three.

One can use the estimates done by [23] in a statistical model, and try to relate the result of the present microscopic transport calculation to thermal freeze-out parameters. The ratios of the $4\pi$ numbers of reconstructable is $\Lambda(1520)/\Lambda = 0.024$ and $K^0(892)/K^* = 0.25$. In terms of the analysis by [23], the microscopic source has a lifetime below 1 fm/c and a freeze-out temperature below 100 MeV. Thus, the values obtained from UrQMD seem to favor a scenario of a sudden break-up of the initial hadron source, in contrast to the time evolution of the chemical and thermal decoupling as shown in Fig. 1. However, note that these numbers are based on the above mentioned thermal scenario. In fact, fitting hadron ratios of UrQMD calculations for central Pb+Pb interactions at 160 A GeV with a Grand Canonical ensemble yields a chemical freeze-out temperature of 150–160 MeV [24].

In fact, the rescattering strength depends on the phase space region studied. Fig. 3 addresses the longitudinal momentum distribution of the rescattering strength. The probability $R$ to observe the resonance

Table 2  
Gaussian fit parameters for the rapidity densities of all and reconstructable resonances

<table>
<thead>
<tr>
<th>Particle</th>
<th>$A$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>181</td>
<td>1.49</td>
</tr>
<tr>
<td>$\Lambda^*$</td>
<td>0.89</td>
<td>1.20</td>
</tr>
<tr>
<td>$K^*$</td>
<td>22.0</td>
<td>1.23</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>2.22</td>
<td>1.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Particle</th>
<th>$A$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^*$</td>
<td>46.3</td>
<td>1.60</td>
</tr>
<tr>
<td>$\Lambda^*$</td>
<td>0.42</td>
<td>1.27</td>
</tr>
<tr>
<td>$K^*$</td>
<td>6.95</td>
<td>1.34</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>1.62</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Fig. 3. Rapidity dependent ratio $R$ of reconstructable resonances over all resonances of a given type as they decay. For $\Delta(1232)$, $\Lambda^*(1520)$, $K^0(892)$ and $\Phi$ in Pb(160 A GeV) + Pb, $b < 3.4$ fm collisions.
$H$ is given by

$$R = \frac{H \to h_1 h_2 (\text{reconstructable})}{H \to h_1 h_2 (\text{all})}$$

(2)

The 'observability' decreases strongly towards central rapidities. This is due to the higher hadron density at central rapidities which increases the absorption probability of daughter hadrons drastically. Unfortunately, in the case of the $\Phi$ meson this decrease of the observable yield increases the discrepancies between data and microscopic model predictions (e.g., [2]).

The absorption probability of daughter hadrons is not only rapidity dependent but also transverse momentum dependent. The decay products are rescattered preferentially at low transverse momenta. Fig. 4 depicts the invariant transverse momentum spectra of reconstructable $\Delta(1232)$, $\Lambda^*(1520)$, $K^0(892)$ and $\Phi$ in Pb(160 A GeV)Pb, $b < 3.4$ fm collisions.

Fig. 5 directly addresses the $p_t$ dependence of the observability of resonances. The present model study supports a strong $p_t$ dependence of the rescattering probability. This effects leads to a larger apparent temperature (larger inverse slope parameter) for resonances reconstructed from strongly interacting particle. A similar behavior has been found for the $\Phi$ meson in an independent study by [25]. This effective heating of the $\Phi$ spectrum might explain the different inverse slope parameter measured by NA49 ($\phi \to K^+ K^-$) as compared to the NA50 ($\phi \to \mu^+ \mu^-$) Collaboration [2,7,25,26].

In conclusion, central Pb + Pb interactions at 160 A GeV are studied within a microscopic non-equilibrium approach. The calculated scattering rates exhibit signs of a chemical and a subsequent thermal freeze-out. The time difference between both freeze-outs is explored with hadronic resonances. The rapidity and transverse momentum distributions of strange and non-strange resonances are predicted. The observability of unstable (strange) particles ($\Lambda^*(1520)$, $\phi$, etc.) in the invariant mass analysis of strongly interacting decay products is distorted due to rescattering of decay products from chemical to thermal freeze-out. Approximately 25% of $\phi$’s and 50% of $\Lambda^*$’s are not directly detectable by reconstruction of the invariant mass spectrum. The rescattering strength is strongly rapidity dependent. Rescattering of the decay products alters the transverse momentum spectra of reconstructed resonances. This leads to higher apparent temperatures for resonances. Inelastic collisions of antikaons from decayed strange resonance al-
ter the chemical composition of strange baryons by up to 10%.

Appendix A

Tables 3–6 show the integrated yields and the data points of the rapidity spectra shown in Fig. 2.

Table 3
4π yields of all decaying resonances in Pb(160 A GeV) + Pb, b < 3.4 fm

<table>
<thead>
<tr>
<th>y (GeV)</th>
<th>dσ/dy(Δ)</th>
<th>dσ/dy(Λ(1520))</th>
<th>dσ/dy(K0(892))</th>
<th>dσ/dy(Φ)</th>
</tr>
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<tr>
<td>0.125</td>
<td>0.78</td>
<td>21.36</td>
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<tr>
<td>0.375</td>
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<td>20.36</td>
<td>2.05</td>
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<td>0.625</td>
<td>0.79</td>
<td>19.20</td>
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<td>0.875</td>
<td>0.7</td>
<td>17.56</td>
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<td></td>
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<tr>
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<td>0.58</td>
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<tr>
<td>1.375</td>
<td>0.42</td>
<td>12.83</td>
<td>1.10</td>
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<td>1.625</td>
<td>0.36</td>
<td>10.31</td>
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<td>1.875</td>
<td>0.26</td>
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<td>0.48</td>
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<td>0.16</td>
<td>4.815</td>
<td>0.26</td>
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<td>0.09</td>
<td>2.378</td>
<td>0.09</td>
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<td>0.802</td>
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Table 4
Rapidity density of all decaying resonances in Pb(160 A GeV) + Pb, b < 3.4 fm

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<td>K0(892)/event</td>
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<tr>
<td>Φ/event</td>
<td>5.917</td>
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Table 5
4π yields of all reconstructable resonances in Pb(160 A GeV) + Pb, b < 3.4 fm

<table>
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<td>Λ/event</td>
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<td>K0(892)/event</td>
<td>22.61</td>
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<tr>
<td>Φ/event</td>
<td>4.35</td>
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Table 6
Rapidity density of reconstructable resonances in Pb(160 A GeV) + Pb, b < 3.4 fm

<table>
<thead>
<tr>
<th>y (GeV)</th>
<th>dσ/dy(Δ)</th>
<th>dσ/dy(Λ(1520))</th>
<th>dσ/dy(K0(892))</th>
<th>dσ/dy(Φ)</th>
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<tr>
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References


[22] M.F. Lutz, C.L. Korpa, nucl-th/0105067.


Note on finite temperature sum rules for vector and axial-vector spectral functions ✤

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Abstract

An updated analysis of vector and axial-vector spectral functions is presented. The resonant contributions to the spectral integrals are shown to be expressible as multiples of $4\pi^2 f_\pi^2$, encoding the scale of spontaneous chiral symmetry breaking in QCD. Up to order $T^2$ this behavior carries over to the case of finite temperature. © 2002 Elsevier Science B.V. All rights reserved.

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Keywords: Spontaneous symmetry breaking; Chiral symmetries; Sum rules; Finite temperature; Meson properties

Quantum Chromodynamics (QCD) has an approximate chiral symmetry which is spontaneously broken. This Nambu–Goldstone realization of chiral symmetry manifests itself through the presence of a quark condensate $\langle \bar{q}q \rangle$, pseudoscalar Goldstone bosons and a non-vanishing pion decay constant, $f_\pi \simeq 92.4$ MeV, which sets the characteristic chiral “gap” scale, $4\pi f_\pi \sim 1$ GeV. The symmetry breaking pattern is also evident in the difference between the spectral functions of vector ($J^{PC} = 1^{--}$) and axial-vector ($J^{PC} = 1^{++}$) channels. These spectral functions would be identical if chiral symmetry were realized in the (unbroken) Wigner–Weyl mode.

At high temperatures, $T > T_c \simeq 0.2$ GeV, chiral symmetry is expected to be restored. The vector and axial-vector channels should become degenerate as $T$ approaches $T_c$. The leading temperature dependence of the corresponding current correlators has been derived in Refs. [1,2]. The result is that vector and axial-vector modes mix at finite $T$ as a consequence of their couplings to the pionic heat bath (see also Ref. [3]), but that their masses remain unchanged to leading order, $O(T^2)$.

The $T = 0$ vector ($V$) and axial-vector ($A$) spectral functions have recently been determined with high accuracy up to invariant masses $\sqrt{s} = m_\tau \simeq 1.78$ GeV (the tau lepton mass) by the ALEPH [4] and OPAL [5] collaborations.
Collaborations. These data serve as a reliable basis for all sorts of sum rule analysis (see, e.g., Ref. [6]), at least for the lowest moments of the $V$ and $A$ spectral distributions.

The purpose of our present note is twofold. First we follow up on a previous study [7], using finite distributions. An interpretation of how this scale is encoded in the spectral sum rules (see also Ref. [8]). We give an interpretation of how this scale is encoded in the pattern of the measured $V$ and $A$ spectral distributions and (as a reminder) in the Weinberg ($V - A$) sum rules. Second we study the case of finite temperature $T$. The change of the chiral “gap” with temperature is determined by the temperature dependence of the $V$ and $A$ correlators and examine their consistency with the Eletsky–Ioffe result [1,2] to or-

\[ T \]

The pion pole term is

\[ \Pi_{\mu
u}(q) = i \int d^4x e^{iqx} \langle 0 | T \left[ J^{\mu}(x) J^{\nu}(0) \right] | 0 \rangle \]  

(1)

can be decomposed as

\[ \Pi_{\mu
u}(q) = (g^{\mu\nu} g^{\mu\nu} - q^2 g^{\mu\nu}) \Pi^{(1)}(q^2) + q^\mu q^\nu \Pi^{(0)}(q^2), \]  

(2)

in terms of their spin-0 and spin-1 parts, $\Pi^{(0)}$ and $\Pi^{(1)}$, respectively. The spin-0 part is relevant only for the axial-vector current where it represents the induced pseudoscalar (pion pole) contribution. The $V$ and $A$ spectral functions are defined as

\[ v_1(q^2) = 2\pi \text{ Im} \, \Pi_V^{(1)}(q^2), \]  

(3)

\[ a_{0,1}(q^2) = 2\pi \text{ Im} \, \Pi_A^{(0,1)}(q^2) \quad (q^2 \equiv s \geq 0). \]  

(4)

The pion pole term is

\[ a_0(q^2) = 4\pi^2 f_\pi^2 \delta(q^2 - m_\pi^2). \]  

(5)

Given the analytical properties of $\Pi_V^{(1)}(s)$ and $\Pi_A^{(0)}(s)$ + $\Pi_A^{(1)}(s)$ in the complex $s$-plane, we now write FESR relations for their spectral functions [7,13]. Assume that the spectral continuum is described in terms of perturbation theory at $s \gg s_c$ and that only the first few mass dimensions are relevant in the operator product expansion (OPE) of the current correlators evaluated at $|q^2| = s_c$. Choose a closed path which surrounds the cut along the real $s$-axis and joins a circle of radius $s_c$. The Cauchy integral around this closed path includes the integration along the circle which can be estimated using the OPE.\(^2\) By means of the optical theorem the integral along the positive, real axis is evaluated using experimental cross sections directly or by considering a model motivated by observation. For details see Ref. [7]. As far as quark mass corrections in the OPE (mass perturbation in the perturbative part and the term involving the quark condensate) are concerned, we neglect them, which is justified by their numerical smallness.\(^3\)

A compendium of the OPE for vector and axial-vector correlators can be found in Ref. [14]. As a result of this procedure one finds the following sum rules for the lowest moments of $v_1$ and $a_0 + a_1$:

\[ \int ds \, v_1(s) = \int ds \, [a_0(s) + a_1(s)] = \frac{s_c}{2}(1 + \delta_0), \]  

(6)

with the pion pole contribution

\[ \int ds \, a_0(s) = 4\pi^2 f_\pi^2, \]  

(7)

and

\[ \int ds \, s v_1(s) = \int ds \, a_1(s) \]

\[ = \frac{s_c^2}{4}(1 + \delta_1) - \pi^2 \frac{\alpha_s}{6} \]  

(8)

\(^{2}\) If $\sqrt{s_c} \gg \lambda^{-1}$, where $\lambda$ denotes a typical correlation length characterizing gauge invariant two point functions, which generalizes the local condensates, then there are no oscillations in the Minkowski-like and no exponential suppression in the euclidean-like domains [9].

\(^{3}\) At $T > 0$ there are to order $T^2$ corrections to the gluon condensate and contributions from $O(3)$ invariant operators proportional to $m_\pi^2$ [10] which hence vanish in the chiral limit. However, if the vacuum state is allowed to be affected by the heat bath (not addressed in this work) then the gluon condensate does exhibit a $T$ dependence [11].
The right-hand sides of Eqs. (6), (8) include the radiative corrections $\delta_{0,1}$ computed in perturbative QCD. Their explicit expressions up to order $\alpha_s^3$ are given in Ref. [7]. The first moments (8) introduce the dimension 4 gluon condensate $\langle (\alpha_s/\pi)G^2 \rangle_{1^+} \sim (0.36 \text{ GeV})^4$. Higher moments involve condensates of correspondingly higher dimensions and will not be considered here. The Weinberg sum rules (WSR) [15] follow immediately:

$$\int_0^{s_c} ds \left[ v_1(s) - a_1(s) \right] = 4\pi^2 f_{\pi}^2,$$

$$\int_0^{s_c} ds \left[ v_1(s) - a_1(s) \right] = 0,$$

where the limit $s_c \to \infty$ can be taken since the high-energy continuum parts of $v_1$ and $a_1$ are identical. In practice this asymptotic behaviour is reached at $s_c \approx 5 \text{ GeV}^2$.

Before turning to the temperature dependent spectral functions, let us introduce a model of the vector spectrum was shown to be

$$v_1(s) = 8\pi^2 f_{\pi}^2 \delta(s - m_{\rho}^2) + \frac{1}{2} \theta(s - s_0),$$

with the $\rho$ meson mass $m_{\rho}$ and the continuum threshold $s_0$ both expressed in terms of the chiral scale as $\sqrt{2m_{\rho}} = \sqrt{s_0} = 4\pi f_{\pi}$. The contribution of the $\rho$ meson to the WSR (9) is then $8\pi^2 f_{\pi}^2$, twice that of the pion pole (7). This is what is seen in the data [4,5] when taking the WSR integral up to $s \approx 1 \text{ GeV}^2$, covering the $\rho$ resonance. The phenomenological bookkeeping seems to follow a pattern in which the $n$-pion sectors of the spectrum each contribute $n$ units of $4\pi^2 f_{\pi}^2$ to the spectral integral, with the $\rho$ meson collecting the strength in the $n = 2$ sector, for example. This conjecture suggests a parametrization

$$v_1(s) = 4\pi^2 f_{\pi}^2 \left[ 2d_{\rho}(s) + 4d_{\rho'}(s) \right] + \text{continuum},$$

$$a_0(s) + a_1(s) = 4\pi^2 f_{\pi}^2 \left[ d_{\pi}(s) + 3d_{\pi}(s) \right] + \text{continuum},$$

where the distributions $d_n(s)$ are normalized as $\int ds \times d_n(s) = 1$. In comparison with the model of Ref. [7], we have now included an explicit $\rho'$ contribution which replaces part of the continuum in the $V$ channel. As a consequence, the continuum threshold $s_c$ is shifted upward from the one used in [7] and Eq. (11).

In the “zero width” (large $N_c$) limit we expect $d_n(s) = \delta(s - m_n^2)$ with $m_1 = m_{\pi}$, $m_2 = m_{\rho} = \sqrt{2 \cdot 2\pi f_{\pi}^2}$, $m_3 = m_{\omega} = 4\pi f_{\pi}$, $m_4 = m_{\phi} = \sqrt{2 \cdot 4\pi f_{\pi}^2}$. With these masses the sum rules for the first moments, Eq. (8), are satisfied. The first WSR (9) is fulfilled in this limit if we model the spectral functions with a perturbative continuum starting at $3s_0$ and $2s_0$ for $v_1(s)$ and $a_1(s)$, respectively.

The actual, finite width resonances used in our analysis follow this scheme, but with their widths fitted to the data (see Appendix A). The resulting spectral functions $v_1(s)$ and $a_1(s)$ are shown in Fig. 1(a), (b). By construction these model spectra satisfy the two WSR’s (9), (10).

The point to be emphasized is that the spectral strength in the resonance region is well described by a pattern of localized distributions, all of which integrate to even (for vector channels) or odd (for axial-vector channels) multiples of $4\pi^2 f_{\pi}^2$. Turning to finite temperature $T$, we will now show that this statement survives to order $T^2$.

At finite $T$ the correlators are expressed in terms of their Gibbs averages

$$\Pi_{V,V}^{\mu\nu}(q; T) = \frac{i \sum_n \int d^4 x e^{iqx} (n|T[J^\mu(x)J^\nu(0)]e^{-H/T}|n)}{\sum_n \langle n|e^{-H/T}|n \rangle},$$

The primary effect at low $T$ is a mixing of $V$ and $A$ modes through their coupling to thermal pions. It was shown in Ref. [1] that, to leading order in $T^2$, $\Pi_{V,V}^{\mu\nu}(q; T) = (1 - \epsilon)\Pi_{V,V}^{\mu\nu}(q; T = 0) + \epsilon \Pi_{A,A}^{\mu\nu}(q; T = 0)$, $\Pi_{A,A}^{\mu\nu}(q; T) = (1 - \epsilon)\Pi_{A,A}^{\mu\nu}(q; T = 0) + \epsilon \Pi_{V,V}^{\mu\nu}(q; T = 0)$.

Note that the vector dominance model would imply $v_1(s) = (4\pi^2 m_{\rho}^2/g^2)d_{\rho}(s)$ with $g \approx 6$. Our scheme has $g = 2\pi$. The Weinberg relation $m_{\omega} = \sqrt{3/2}m_{\rho}$ is evident.

Exact fulfillment of the second WSR (10) requires a minor tuning of the $\rho'$ mass, $m_{\phi} = \sqrt{3/2}4\pi f_{\pi}^2$. 

5 Exact fulfillment of the second WSR (10) requires a minor tuning of the $\rho'$ mass, $m_{\phi} = \sqrt{3/2}4\pi f_{\pi}^2$. 

4 Note that the vector dominance model would imply $v_1(s) = (4\pi^2 m_{\rho}^2/g^2)d_{\rho}(s)$ with $g \approx 6$. Our scheme has $g = 2\pi$. The Weinberg relation $m_{\omega} = \sqrt{3/2}m_{\rho}$ is evident.
victions components, we derive for their thermal spectral func-
change. Reducing Eq. (16) to its spin-0 and spin-1 relators remain at their positions; only their residues by the common factor $\epsilon$ with similar).

ALEPH data [4] (the comparison with OPAL data [5] looks very
by the parametrization (12), (13) and Appendix A, compared with
Fig. 1. Vector (a) and axial-vector (b) spectral functions as given

$$T$$

Moreover, the reduction of strengths in
the case of the pion pole, the $V$-correlator now
receives an induced spin-0 (pion pole) contribution.

$${v}_0(s; T) = 2\pi \mathrm{Im} \Pi^{(0,1)}_V(s; T)$$

and $${a}_{0,1}(s; T) = 2\pi \mathrm{Im} \Pi^{(0,1)}_A(s; T):$$

$${v}_0(s; T) = \varepsilon \alpha_0(s; T = 0),$$

$${a}_0(s; T) = (1 - \varepsilon) \alpha_0(s; T = 0),$$

$${v}_1(s; T) = \varepsilon [v_3(s; 0) - \alpha_1(s; 0)],$$

$${a}_1(s; T) = \alpha_1(s; 0) + \varepsilon [v_3(s; 0) - \alpha_1(s; 0)].$$

Note that due to the coupling of the vector current to pions in the heat bath, the thermal $V$-correlator now receives an induced spin-0 (pion pole) contribution. The thermal $V$–$A$ mixing is evident in Eq. (17). Moreover, the reduction of strengths in $\alpha_0$, $v_1$ and $a_1$ by the common factor $(1 - \varepsilon)$ is fully consistent with our previous discussion: to order $T^2$ the resonances again contribute to the spectral integral as multiples of $f^2(T) = f^2(1 - T^2/4T^2)[12]$! Reversing the argument in the case of the pion pole, the $T$ dependence of $f^2$ to

this order was read off in Ref. [1] by defining $f^2(T)$ to be the residue at $T > 0$.

For $s \geq s_c \simeq 5$ GeV$^2$ the continuum parts of the $V$ and $A$ spectra do not change with $T$ to order $T^2$: since $v_1(s) = \alpha_1(s)$ at $T = 0$ for $s \geq s_c$, this is immediately evident from Eq. (17).

Considering a given invariant (note that there are thermally induced pion pole terms in the vector channel), the FESR’s for the lowest moments of the $V$ and $A$ spectral distributions are not affected to order $T^2$. For example, consider

$$\left(\frac{4q_{\mu}q_{\nu}}{3q^4}\right)\Pi^{\mu\nu}(q; T) = \Pi^{(0)}(q^2; T) + \Pi^{(1)}(q^2; T),$$

we have then

$$\int_{0}^{s_c} ds \left[v_0(s; T) + v_1(s; T)\right] = \int_{0}^{s_c} ds v_1(s; 0)$$

$$- \varepsilon \int_{0}^{s_c} ds \left[v_1(s; 0) - \alpha_1(s; 0) - \alpha_0(s; 0)\right].$$

Using (7), the $T$ dependent part vanishes by virtue of the first Weinberg sum rule (9). An analogous statement holds for $\alpha_0 + \alpha_1$. Furthermore, it is easily seen that

$$\int_{0}^{s_c} ds \left[v_0(s; T) + v_1(s; T)\right] = \int_{0}^{s_c} ds v_1(s; 0),$$

$$\int_{0}^{s_c} ds \left[a_0(s; T) + \alpha_1(s; T)\right] = \int_{0}^{s_c} ds a_1(s; 0).$$

when using the second WSR, Eq. (10). To order $T^2$ the presence of the heat bath causes a mere redistribution of spectral strength in both the $V$ and $A$ sectors but such that the lowest moments of the spectral functions remain unchanged. We illustrate these features by an explicit calculation at $T = 140$ MeV with the results shown in Fig. 2. It is amusing that already order $\varepsilon$ suggests the coincidence of vector and axial-vector spectra at $T \simeq 160$ MeV ($\varepsilon = 1/2$) which is not far from the critical temperature $T_c \simeq 170$ MeV of the chiral phase transition determined by two-flavour lattice QCD [16].
To summarize, based on the new experimental data we have shown that the resonant contributions to the spectral integrals in the $V$ and $A$ channels can be written as multiples of the square of the spontaneous chiral symmetry breaking scale. Up to order $T^2$ this pattern carries over to the case of finite temperature.

Acknowledgements

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Appendix A

To parametrize the resonances according to Eqs. (12), (13) we use the following functional forms for $d_n$:

$$d_n = a_n \gamma_n^2(s) / [(s - m_n^2)^2 + \gamma_n^2(s)],$$

where $\gamma_n(s) = [b_n + c_n(s - e_n)]^2 [b_n + c_n(s - e_n)^2].$ with $n = 2$ for $\rho$, $n = 3$ for $a_1$ and $n = 4$ for $\rho'$. The parameter sets which reproduce the empirical spectra [4,5] are presented in Table 1. Note that the mass parameters $m_n$ deviate only marginally (by less than 10% for $\rho$ and 3% for $a_1$, $\rho'$) from the expected “large $N_c$” pattern (see text), and the normalization of the $d_n(s)$ required to reproduce the empirical data is equal or close to one.

References

Consistent treatment for valence and nonvalence configurations in semileptonic weak decays

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Abstract

We discuss the semileptonic weak decays of \( P \to P \) (\( P \) denotes a pseudoscalar meson). In these timelike processes, the problem of the nonvalence contribution is solved systematically as well as the valence one. These contributions are related to the light-front quark model (LFQM), and the numerical results show the nonvalence contribution of the light-to-light transition is larger than of the heavy-to-light one. In addition, the relevant CKM matrix elements are calculated. They are consistent with the data of Particle Data Group.

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The study of exclusive semileptonic decays has attracted much interest for a long time. Heavy-to-heavy semileptonic decays, such as \( B \to D l \nu \), provide an ideal testing ground for heavy-quark symmetry and heavy-quark effective theory (for a review, see [1]). On the other hand, heavy-to-light and light-to-light weak decays are much more complicated theoretically since there exists no guiding symmetry principle. Nevertheless, it is essential to understand the reaction mechanisms of these decay modes, because they are the main sources of information on the CKM mixing matrix between heavy and light quarks.

Hadronic matrix elements of weak \( P \to P \) transition is described by two form factors. Phenomenologically, the hadronic form factors can be evaluated in various models, including the popular quark model. However, since usual quark-model wave functions best resemble meson states in the rest frame, or where the meson velocities are small, the form factors calculated in the nonrelativistic quark model are, therefore, trustworthy only when the recoil momentum of the daughter meson relative to the parent meson is small. As the recoil momentum increases (corresponding to decreasing \( q^2 \)), we have to consider relativistic effects seriously.

It is well known that the LFQM [2,3] is a relativistic quark model in which a consistent and fully relativistic treatment of quark spins and the center-of-mass motion can be carried out. This model has many advantages. For example, the light-front (LF) wave function is manifestly Lorentz invariant as it is expressed in terms of the momentum fraction variables (in “+” components) in analogy with the parton distributions in the infinite momentum frame. Moreover, hadron spin can also be correctly constructed us-
ing the so-called Melosh rotation. The kinematic subgroup of the LF formalism has the maximum number of interaction-free generators, including the boost operator which describes the center-of-mass motion of the bound state (for a review of LF dynamics, see [4]). The LFQM has been applied in the past to study the heavy-to-heavy and heavy-to-light weak decay form factors [5,6]. However, the weak form factors were calculated only for $q^2 \leq 0$ at the beginning, whereas physical decays occur in the time-like region $0 \leq q^2 \leq (M_i - M_f)^2$, with $M_i, f$ being the initial and final meson masses. Hence extra assumptions are needed to extrapolate the form factors to cover the entire range of momentum transfer [7,8]. Lately, the weak form factors for $P \rightarrow P$ transition were calculated in [9,10] for the first time for the entire range of $q^2$, so additional extrapolation assumptions are no longer required. This is based on the observation [11] that in the frame where the momentum transfer is purely longitudinal, i.e., $q_\perp = 0$, $q^2 = q^+ q^-$ covers the entire range of momentum transfer. The price is that, besides the conventional valence-quark contribution, we must also consider the nonvalence configuration (or the so-called Z graph, see Fig. 1(b)). The nonvalence contribution vanishes if $q^+ = 0$, but is supposed to be important for heavy-to-light transition near zero recoil [5,7,11,12]. Some methods for treating this nonvalence configuration exist: the authors of Ref. [10] considered the effective higher Fock state and calculated the effect in chiral perturbation theory. Ref. [13] follow a Schwinger–Dyson approach and related the nonvalence contribution to an ordinary LF wave function.

In this Letter, we present a new way of handling the nonvalence contribution of $P \rightarrow P$ transition. For comparison, it will be instructive to analyze the known valence contribution in parallel. The main advantage of this way is that relativistic effects of the quark motion and spin are treated consistently in both valence and nonvalence configurations. We assume both normalization conditions of meson and quark states and a single interaction Hamiltonian to obtain both the Melosh transformations of valence and nonvalence contributions. Combining these two contributions, we calculate completely the form factors of the semileptonic decay and the relevant CKM matrix elements.

We are interested in the matrix element which defines the weak form factors by

$$
\langle P' | \bar{Q} \gamma^\mu Q | P \rangle = f_+ (q^2) (P + P')^\mu + f_- (q^2) q^\mu,
$$

where $q = P - P'$ is the momentum transfer. Assuming a vertex function $\Lambda_P$ [5,6] which is related to $Q \bar{q}$ bound state of $P$ meson, the quark–meson diagram depicted in Fig. 1(a) yields

$$
\langle P' | \bar{Q} \gamma^\mu Q | P \rangle = - \int \frac{d^4 p_1}{(2\pi)^4} \Lambda_P \Lambda_{P'} \times \text{Tr} \left[ \gamma_5 \frac{i(p_3 + m_3)}{p_3^2 - m_3^2 + i\epsilon} \gamma_\mu \frac{i(p_2 + m_2)}{p_2^2 - m_2^2 + i\epsilon} \right] \times \gamma^\alpha \frac{i(p_1 + m_1)}{p_1^2 - m_1^2 + i\epsilon}. 
$$

where $p_2 = p_1 - q$ and $p_3 = p_1 - P$. We consider the poles in denominators in terms of the LF coordinates $(p^-, p^+, p_\perp)$ and perform the integration over the LF
\[ \langle P | \bar{Q} \gamma^\mu Q | P \rangle = \int_0^q \left[ d^3 p_1 \right] \frac{A_p}{\mathbf{p}_3 - \mathbf{p}_{3\text{on}}} (I^\mu | p_{1\text{on}}^{-}) \times \frac{A_{p'}}{\mathbf{p}_3' - \mathbf{p}_{3\text{on}}' + \mathbf{p}_2' - \mathbf{p}_{2\text{on}}'} + \int_0^q \left[ d^3 p_1 \right] \frac{A_p}{\mathbf{p}_3 - \mathbf{p}_{3\text{on}}} (I^\mu | p_{1\text{on}}^{-}) \times \frac{A_{p'}}{\mathbf{p}_3' - \mathbf{p}_{3\text{on}}' + \mathbf{p}_2' - \mathbf{p}_{2\text{on}}'}, \tag{3} \]

where \( i = 1, 2, 3, \)

\[ [d^3 p_1] = \frac{d p_1^+ d^2 p_{1\perp}}{16\pi^3 |p_1^+|^3}, \]

\[ I^\mu = \text{Tr} [\gamma_5 (\mathbf{p}_3 + m_3) \gamma_5 (\mathbf{p}_2 + m_2) \gamma^\mu (\mathbf{p}_1 + m_1)], \]

\[ p_{1\text{on}}^{-} = m_1^2 + p_{1\perp}^2 / p_{1\text{on}}^+, \]

\[ p_{1\text{on}}^{-} = p_{3\text{on}}^{-} - p_{3\text{on}}^{-} \text{ in the first and second term of Eq. (3). It is worthwhile to mention every vertex function and its denominator corresponds exactly to the relevant meson bound state. This is clearer if we define } S_j = p_j - p_{j\text{on}}^{-} \text{ and rewrite Eq. (3) in a more symmetrical form:} \]

\[ \langle P | \bar{Q} \gamma^\mu Q | P \rangle = \int_0^q \left[ d^3 p_1 \right] \left[ \frac{A_p}{\mathbf{S}_p + \mathbf{S}_1 + \mathbf{S}_3} I^\mu \right] \times \frac{A_{p'}}{\mathbf{S}_p' + \mathbf{S}_2 + \mathbf{S}_3'} \bigg|_{S_p, p', s=0}, \tag{5} \]

In general, the integrals in Eq. (5) are divergent if we treat the vertices as pointlike. Internal structures for these vertices are therefore necessary. In the LFQM, the internal structure \([10,14,15]\) consists of \( \phi \) which describes the momentum distribution of the constituents in the bound state, and \( R_{S_1, S_2}^{S_3, S} \) which creates a state of definite spin \( (S, S_z) \) out of LF helicity \( (\lambda_1, \lambda_2) \) eigenstates and is related to the Melosh transformation [16]. Here we adopt a convenient approach relating these two parts. The interaction Hamiltonian is assumed to be \( H_I = i \int d^3 x \bar{\Psi} \gamma_5 \psi \Phi \) where \( \Psi \) is quark field and \( \Phi \) is meson field containing \( \phi \) and \( R_{S_1, S_2}^{S_3, S} \). On the one hand, if we normalize the meson state depicted in Fig. 2(a) as \([10]\)

\[ \{M(P', S', S_z') | H_I H_I | M(P, S, S_z)\} = 2(2\pi)^3 p^+ \delta^3 (p' - P) \delta_{S'S} \delta_{S_z S_z'}, \]

and the valence wave function \( \phi^\mu \) as

\[ \int \frac{d^3 p_1}{2(2\pi)^3} \frac{1}{p^+} \langle \phi^\mu \rangle^2 = 1, \tag{7} \]

where \( p_1 \) and \( p_2 \) are the on-mass-shell momenta; the valence configuration of \( R_{S_1, S_2}^{S_3, S} \) is

\[ R_{1,2}^S = \sqrt{\frac{p_1^+ p_2^+}{2p_{3\text{on}}^+ p_{2\text{on}}^+ m_1 m_2}}, \tag{8} \]

On the other hand, if we normalize the quark state depicted in Fig. 2(b) as

\[ \{ Q(p_3', s') | H_I H_I | Q(p_3, s)\} = 2(2\pi)^3 \delta^3 (p_3' - p_3) \delta_{s's}, \tag{9} \]
and the nonvalence wave function \( \phi^n \) as

\[
\int \frac{d^3 p_2}{(2\pi)^3} \frac{1}{p_2^+} |\phi^n|^2 = 1; \tag{10}\]

the nonvalence configuration of \( R_{S,S_z}^{S_1,S_2} \) is

\[
R_{2,3}^n = \frac{\sqrt{p_2^+}}{2\sqrt{p_{20n} \cdot p_{3on} - m_2m_3}} \int d^3 p_3 \phi_{p_3}^n \, . \tag{11}\]

After taking the "good" component \( R \equiv R_{S,S_z}^{S_1,S_2} \), the nonvalence configuration of \( (x,x') \) amounts to having \( p_{13}^+ \leq 0 \) amounts to having \( q'_\perp = 0 \) so that \( q^2 \geq 0 \) and \( k'_\perp = k' \). 

Using the definitions of the LF momentum variables \((x,x',k_\perp,k'_\perp)\) [14] and take a Lorentz frame where \( p_1 = p_{13}^+ = 0 \) amounts to having \( q_\perp = 0 \) and \( k'_\perp = k_\perp \). From Eq. (13) we obtain

\[
\expval{P'}{\bar{Q}^+ Q}{P} = 2P^+ H(r), \tag{15}\]

where

\[
A = m_1x + m_3(1-x), \quad A' = m_2x' + m_3(1-x'), \quad x (x') \text{ is the momentum fraction carried by the spectator antiquark in the initial (final) state in the first term of (14). However, } x' \geq 1 \text{ the second term of (14), which shows that the momentum } p_{23}^+ \text{ of the spectator quark is larger than the } P'^+ \text{ of the final meson.}
\]

As explained above, we shall work in the frame where \( q_\perp = 0 \) so that \( q^2 \geq 0 \). Defining \( r = P'^+ / P^+ \) gives \( q^2 = (1-r)(M_p^2 - M_{p'}^2) / r \). Consequently, for a given \( q^2 \), the two solutions for \( r \) are given by

\[
r_\pm = \frac{1}{2M_p^2} - M_p^2 / M_p^2 - q^2 \pm 2M_pQ(q^2), \tag{16}\]

where

\[
Q(q^2) = \sqrt{(M_p^2 - M_{p'}^2 - q^2)^2 - 4M_p^2M_{p'}^2} / 2M_p.
\]

The ± signs in (16) correspond to the daughter meson recoiling in the ±z-direction relative to the parent meson. The form factors \( f_{\pm}(q^2) \), of course, should be independent of the reference frame chosen for the moving direction of the daughter meson. For a given \( q^2 \), it follows from (1) that

\[
f_{\pm}(q^2) = \pm \frac{1}{r_+ - r_-} H(r_+) - \frac{1}{r_+ - r_-} H(r_-), \tag{17}\]

It is easily seen that \( f_{\pm}(q^2) \) are independent of the choice of reference frames, as it should be. The scalar form factor \( f_0(q^2) \) is related to \( f_{\pm}(q^2) \) by

\[
f_0(q^2) = f_+(q^2) + \frac{q^2}{M_p^2 - M_{p'}^2} f_-(q^2). \tag{18}\]

The differential decay rate for \( P \rightarrow P' \) is given by [13]

\[
d\Gamma / dq^2 = \frac{G_F^2}{24\pi^3} |V_{q\bar{q}_2}|^2 Q(q^2)(1 - 2\delta)^2
\]
\begin{align*}
\times \left\{ \left[ Q(q^2) \right]^2 (1 + \hat{s}) |f_+(q^2)|^2 \\
+ M_P^2 \left( 1 - \frac{M_P^2}{M_N^2} \right)^{2/3} \left| f_0(q^2) \right|^2 \right\}, \quad (19)
\end{align*}

where \( G_F \) is the Fermi constant, \( \hat{s} = m_l^2/2q^2 \), \( m_l \) is the mass of lepton \( l \), and \( V_{q,0} \) is the CKM matrix element.

In principle, the momentum distribution amplitude \( \phi(x, k_{\perp}) \) can be obtained by solving the LF QCD bound state equation [4]. However, before such first-principle solutions are available, we shall have to use phenomenological amplitudes. The simplest conjecture is related to the Melosh transformation effect; for example, \( \phi = N \exp[-(A^2 + k_{\perp}^2)/(2\omega^2)] \), where \( N \) is normalization constant and \( \omega \) is a scale parameter. However, the contributions of the end-point regions \( (x \to 0, 1) \) for this wave function are nonvanishing. Here we make a slight modification to:

\begin{align*}
\phi(x, k_{\perp}) = N \left[ x(1 - x) \right]^{1/n} \left[ \frac{\omega^2}{(A^2 + k_{\perp}^2) + \omega^2} \right]^{\gamma_n}.
\end{align*}

(20)

where \( n \) is an integer. When \( n \) is large (~ 20), the form of this power-law wave function is almost the same as the previous exponential one except at the end-points. In addition, we do not treat \( n \) as a new parameter because the differences between wave functions for different large \( n \)’s are negligible. Thus the three parameters are \( m_1, m_2, \) and \( \omega \) in Eq. (20).

We can use Eqs. (17), (18), (14), and (20) to calculate the form factors of the processes \( K^0 \to \pi^\pm f^\mp v_l(K_{\ell 3}) \) and \( D^0(B^0) \to \pi^- f^+ v_l \) which correspond to the light-to-light and heavy-to-light decay modes, respectively. On the one hand, the parameters appearing in the wave functions \( \phi_{K,\pi}^x \) are fixed by assuming the quark masses \( m_u = m_d \) and fitting to the experimental values of the decay constants \( f_{K,\pi} \) [17] and the charged radii \( \langle r^2 \rangle_{K^+,\pi^+} \) [18,19]. On the other hand, we determine the parameter \( \omega \) in \( \phi_{D,B}^x \) by fitting the data in Ref. [21] and treat it as universal among the other decay modes. As for the \( D \) and \( B \) mesons, the parameters are determined by assuming the quark masses \( m_c = 1.3 \) GeV, \( m_b = 4.5 \) GeV and fitting to the lattice QCD values of the decay constant \( f_{D,B} \) [20]. These parameters are as listed below (in units of GeV):

\begin{align*}
m_u,d = 0.2, \quad m_s = 0.32, \quad m_c = 1.3, \quad m_b = 4.5, \quad \omega_{D}^B = 0.3, \quad \omega_{\pi}^B = 2.34, \quad \omega_{K}^B = 2.66, \quad \omega_{D}^B = 3.19, \quad \omega_{\pi}^B = 4.71. \quad (21)
\end{align*}

The numerical results of the form factor \( f_+ \) for various decay modes are plotted in Figs. 3–5. From these figures, we easily find, for the same final meson, that the nonvalence contributions are smaller when the initial mesons are heavier. In addition, the nonvalence contribution is important for heavy-to-light transition.
Fig. 5. The form factor $f_+$ for $B^0 \to \pi^- l^+ \nu_l$ compared with the lattice QCD data [23].

Table 1

<table>
<thead>
<tr>
<th>Value</th>
<th>$V_{us}$</th>
<th>$V_{cd}$</th>
<th>$V_{ub}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>$0.2179 \pm 0.0016$</td>
<td>$0.247 \pm 0.021$</td>
<td>$0.0037 \pm 0.0007$</td>
</tr>
<tr>
<td>P.D.G. [17]</td>
<td>$0.2196 \pm 0.0023$</td>
<td>$0.224 \pm 0.016$</td>
<td>$0.0036 \pm 0.0012$</td>
</tr>
</tbody>
</table>

near zero recoil ($q^2 \sim q^2_{\text{max}}$). This result is consistent with the prediction in [5,7,11,12].

Finally, we can use the Eqs. (17)–(19) and the experimental data of the relevant decay rates [17] to calculate the three CKM matrix elements $V_{us}$, $V_{cd}$, and $V_{ub}$. These values from this work and Ref. [17] are listed in Table 1. The error bars in this work come from the uncertainties of the decay widths. We find these values are consistent with [17].

In conclusion, a new treatment for the nonvalence configuration have been shown. We emphasize that the vertex functions correspond to LF valence and nonvalence wave functions exactly. The relativistic effects of the quark motion and spin were also treated consistently in both valence and nonvalence configurations. Therefore, we are able to calculate the form factors of the semileptonic decay completely. The numerical results showed the nonvalence contribution of the heavy-to-light transition is smaller than that of the light-to-light one. In addition, the CKM matrix elements evaluated from these form factors were consistent with the data in Particle Data Group.

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References

Final-state interactions and single-spin asymmetries in semi-inclusive deep inelastic scattering

Stanley J. Brodsky, Dae Sung Hwang, Ivan Schmidt

Abstract

Recent measurements from the HERMES and SMC Collaborations show a remarkably large azimuthal single-spin asymmetries $A_{UL}$ and $A_{UT}$ of the proton in semi-inclusive pion leptonproduction $\gamma^*(q)p \to \pi X$. We show that final-state interactions from gluon exchange between the outgoing quark and the target spectator system lead to single-spin asymmetries in deep inelastic lepton–proton scattering at leading twist in perturbative QCD, i.e., the rescattering corrections are not power-law suppressed at large photon virtuality $Q^2$ at fixed $x_{bj}$. The existence of such single-spin asymmetries requires a phase difference between two amplitudes coupling the proton target with $J_{zp} = \pm \frac{1}{2}$ to the same final-state, the same amplitudes which are necessary to produce a nonzero proton anomalous magnetic moment. We show that the exchange of gauge particles between the outgoing quark and the proton spectators produces a Coulomb-like complex phase which depends on the angular momentum $L_z$ of the proton’s constituents and is thus distinct for different proton spin amplitudes. The single-spin asymmetry which arises from such final-state interactions does not factorize into a product of distribution function and fragmentation function, and it is not related to the transversity distribution $\delta q(x, Q)$ which correlates transversely polarized quarks with the spin of the transversely polarized target nucleon. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Single-spin asymmetries in hadronic reactions have been among the most difficult phenomena to understand from basic principles in QCD. The problem has become more acute because of the observations by the HERMES [1] and SMC [2] Collaborations of a strong correlation between the target proton spin $\vec{S}_p$ and the plane of the produced pion and virtual photon in semi-inclusive deep inelastic lepton scattering $\ell p \to \ell' \pi X$ at photon virtuality as large as $Q^2 = 6$ GeV$^2$. Large azimuthal single-spin asymmetries have also been seen in hadronic
reactions such as $pp \rightarrow \pi X$ [3], where the target antiproton is polarized normal to the pion production plane, and in $pp \rightarrow \Lambda^+ X$ [4], where the hyperon is polarized normal to the production plane.

In the target rest frame, single-spin correlations correspond to the $T$-odd triple product $i\vec{S}_p \cdot \vec{p}_\pi \times \vec{q}$, where the phase $i$ is required by time-reversal invariance. The differential cross section thus has an azimuthal asymmetry proportional to $|\vec{p}_\pi|/|\vec{q}|$ $\sin \theta_{q\pi} \sin \phi$ where $\phi$ is the angle between the plane containing the photon and pion and the plane containing the photon and proton polarization vector $\vec{S}_p$. In a general frame, the azimuthal asymmetry has the invariant form $\frac{1}{2\pi} \epsilon_{\mu\nu\sigma\tau} P^\mu S^*_p p^\sigma q^\tau$ where the polarization four-vector of the proton satisfies $S^*_p = -1$ and $S_p \cdot P = 0$.

In order to produce a correlation involving a transversely-polarized proton, there are two necessary conditions: (1) there must be two proton spin amplitudes $M[\gamma^* p(J^p_0) \rightarrow F]$ with $J^p_0 = \pm 1/2$ which couple to the same final-state $|F\rangle$; and (2) the two amplitudes must have different, complex phases. The correlation is proportional to $\text{Im}(M[J^p_+ = +1/2] M[J^p_- = -1/2])$. The analysis of single-spin asymmetries thus requires an understanding of QCD at the amplitude level, well beyond the standard treatment of hard inclusive reactions based on the factorization of distribution functions and fragmentation functions. Since we need the interference of two amplitudes which have different proton spin $J^p_0 = \pm 1/2$ but couple to the same final-state, the orbital angular momentum of the two proton wavefunctions must differ by $\Delta L^z = 1$. The anomalous magnetic moment for the proton is also proportional to the interference of amplitudes $M[\gamma^* p(J^p_0) \rightarrow F]$ with $J^p_0 = \pm 1/2$ which couple to the same final-state $|F\rangle$.

Final-state interactions (FSI) in gauge theory can affect deep inelastic scattering reactions in a profound way, as has been demonstrated recently [5]. The rescattering of the outgoing quark leads to a leading twist contribution to the deep inelastic cross section from diffractive channels as has been demonstrated recently [5]. The rescattering of the outgoing quark leads to a leading twist contribution from diffractive channels. Here we shall show that FSI also provide the required phase needed to produce single-spin asymmetries in deep inelastic scattering.

The dynamics of the constituents in the target can be described by its light-front wavefunctions, $\psi_{n/p}(x_i, \vec{k}_i, \lambda_i)$, the projections of the hadronic eigenstate on the free color-singlet Fock state $|n\rangle$ at a given light-cone time $t = z/c$. The wavefunctions are Lorentz-invariant functions of the relative coordinates $x_i = k^0_i / P^+ = (k^0_i + k^0)/ (P^0 + P^z)$ and $\vec{k}_i$ (with $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \vec{k}_i = \vec{0}_z$), and they are independent of the bound state’s physical momentum $P^+$ and $P_\perp$ [6]. The physical transverse momenta are $\vec{p}_\perp = x_i \vec{P}_\perp + \vec{k}_i$. The $\lambda_i$ label the light-front spin $S^z$ projections of the quarks and gluons along the quantization $z$ direction. If a target is stable, its light-front wavefunction must be real. Thus the only source of a nonzero complex phase in leptoproduction in the light-front frame are final-state interactions. The rescattering corrections from final-state exchange of gauge particles produce Coulomb-like complex phases which, however, depend on the proton spin. Thus $M[\gamma^* p(J^p_0 = \pm 1/2) \rightarrow F] = |M[\gamma^* p(J^p_0 = \pm 1/2) \rightarrow F]| e^{i\Delta_\chi}$. Each of the phases is infrared divergent; however, the difference $\Delta_\chi = \chi_+ - \chi_-$ is infrared finite and nonzero. The resulting single-spin asymmetry is then proportional to $\sin \Delta_\chi$.

2. A model calculation of single-spin asymmetries in gauge theory

We shall calculate the single-spin asymmetry in semi-inclusive electroproduction $\gamma^* p \rightarrow H X$ induced by final-state interactions in a model of a spin-1 proton of mass $M$ with charged spin-$\frac{1}{2}$ and spin-0 constituents of mass $m$ and $\lambda$, respectively, as in the QCD-motivated quark–diquark model of a nucleon. The basic electroproduction reaction is then $\gamma^* p \rightarrow q(qq)_0$, as illustrated in Figs. 1 and 2. We shall take the case where the detected particle $H$ is identical to the quark. One can take the asymmetry for a detected hadron by convoluting the jet asymmetry result with a realistic fragmentation function; e.g., $D_q \rightarrow H(x, Q^2)$.

The amplitude for the $\gamma^* p \rightarrow q(qq)_0$ can be computed from the tree and one-loop graphs illustrated in Fig. 2. A spin asymmetry will arise from the final-state interactions of the outgoing charged lines. The $J^z = +1/2$ two-

Fig. 1. The final-state interaction in the semi-inclusive deep inelastic lepton scattering \( \ell p \uparrow \rightarrow \ell' \pi X \).

Fig. 2. The tree (a) and one-loop (b) graphs for \( \gamma^* p \rightarrow q(\bar{q}q)_{0} \). The interference of the two amplitudes with \( J_z^p = \pm 1/2 \) provides the proton’s single-spin asymmetry.

Particle Fock state is given by [7,8]

\[
|\Psi_{\text{two particle}}(P^+, \bar{P}_\perp = \bar{0}_\perp)\rangle = \int \frac{d^2\vec{k}_\perp dx}{\sqrt{x(1-x)} \, 16\pi^3} \left[ \psi_{\frac{1}{2}^{+}}(x, \vec{k}_\perp)|\frac{1}{2}; x P^+, \vec{k}_\perp\rangle + \psi_{\frac{1}{2}^{-}}(x, \vec{k}_\perp)|\frac{1}{2}; x P^+, \vec{k}_\perp\rangle \right],
\]

(1)

where

\[
\begin{align*}
\psi_{\frac{1}{2}^{+}}(x, \vec{k}_\perp) &= (M + \frac{P^+}{x}) \varphi, \\
\psi_{\frac{1}{2}^{-}}(x, \vec{k}_\perp) &= -(\frac{M + i k^1 + i k^2}{x}) \varphi.
\end{align*}
\]

(2)
The scalar part of the wavefunction $\varphi$ depends on the dynamics. In the perturbative theory it is simply

$$\varphi = \varphi(x, k_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (k_\perp^2 + m^2)/x - (k_\perp^2 + \lambda^2)/1-x}. \quad (3)$$

In general one normalizes the Fock state to unit probability.

Similarly, the $J^z = -1/2$ two-particle Fock states have components

$$\begin{align*}
\psi^+_{1/2}(x, k_\perp) &= \frac{\text{sign}(k_\perp^0)}{\sqrt{2}} \varphi(x, k_\perp), \\
\psi^-_{1/2}(x, k_\perp) &= \frac{\text{sign}(k_\perp^0)}{\sqrt{2}} \varphi(x, k_\perp).
\end{align*} \quad (4)$$

The spin-flip amplitudes in (2) and (4) have orbital angular momentum projection $l^z = +1$ and $-1$, respectively. The numerator structure of the wavefunctions is characteristic of the orbital angular momentum, and holds for both perturbative and nonperturbative couplings.

We require the interference between the tree amplitude of Fig. 2(a) and the one loop graph of Fig. 2(b). The contributing amplitudes for $\gamma^* p \to q(qq)_0$ have the following structure through one loop order:

$$A(\uparrow \to \uparrow) = \left( M + \frac{m}{\Delta} \right) C \left( h + i \frac{e_1 e_2}{8\pi} g_1 \right), \quad (5)$$

$$A(\downarrow \to \uparrow) = \left( -r - i \frac{r^2}{\Delta} \right) C \left( h + i \frac{e_1 e_2}{8\pi} g_2 \right), \quad (6)$$

$$A(\uparrow \to \downarrow) = \left( M + \frac{m}{\Delta} \right) C \left( h + i \frac{e_1 e_2}{8\pi} g_1 \right), \quad (7)$$

$$A(\downarrow \to \downarrow) = \left( M + \frac{m}{\Delta} \right) C \left( h + i \frac{e_1 e_2}{8\pi} g_2 \right). \quad (8)$$

where

$$C = -g e_1 P^+ \sqrt{\Delta 2\Delta(1-\Delta)}, \quad (9)$$

$$h = \frac{1}{P_\perp^2 + \Delta(1-\Delta)(-M^2 + m^2/\Delta + \lambda^2/(1-\Delta))}. \quad (10)$$

The quark light-cone fraction $\Delta = k^+ / P^+$ is equal to the Bjorken variable $x_{bj}$, up to corrections of order $1/Q$. The label $\uparrow / \downarrow$ corresponds to $J^z_{bj} = \pm 1/2$. The second label $\uparrow / \downarrow$ gives the spin projection $J^z_{\uparrow / \downarrow} = \pm 1/2$ of the spin-$1/2$ constituent. Here $e_1$ and $e_2$ are the electric charges of $q$ and $(qq)_0$, respectively, and $g$ is the coupling constant of the proton-$q$-$(qq)_0$ vertex. The first term in (5) to (8) is the Born contribution of the tree graph. The crucial result will be the fact that the contributions $g_1$ and $g_2$ from the one-loop diagram Fig. 2(b) are different, and that their difference is infrared finite. A gauge particle mass $\lambda$ will be used as an infrared regulator in the calculation of $g_1$ and $g_2$.

The calculation will be done using light-cone time-ordered perturbation theory, or equivalently, by integrating Feynman loop diagrams over $dk^-$. The light-cone frame used is $p = (p^+, p^-, \vec{P}_\perp) = (P^+, P_\perp, \vec{P}_\perp)$ and $q = (q^+, q^-, \vec{q}_\perp)$ with $q^+ = 0$ and $q^- = 2q \cdot P/P^+$. The Bjorken variable is $\Delta = Q^2/2q \cdot p = Q^2/2M_v$. Since $q^- = 0$, light-cone time-orderings where the virtual photon produces a $q\bar{q}$ pair do not appear.

The light-cone formalism is invariant under boosts in the $\vec{z}$ direction: $P^+ \to \gamma P^+$. It reduces to a laboratory frame when $P^+ = M$. If we take $\vec{q}$ to lie in the $\vec{z} - \vec{x}$ plane in this frame, $\vec{q} = (q^x, q^y, q^z) = (Q, 0, -v)$; i.e., $\vec{q}$ is oriented at an angle $\theta_{lab} = \tan^{-1} \frac{Q}{v}$, from the negative $\vec{z}$ direction. This is illustrated in Fig. 3. Here $v$ is the laboratory energy of the photon. In the Bjorken scaling limit with $Q^2$ and $v$ large, and $\Delta = x_{bj}$ fixed, the angle $\theta_{lab} \to 0$, so the light-cone laboratory frame and usual laboratory frame with $\vec{q}$ taken in the $-\vec{z}$ direction are identical.
The covariant expression for the four one-loop amplitudes of diagram Fig. 2(b) is:

\[\mathcal{A}^{\text{one-loop}}(I)\]

\[= i\gamma^2 e_2 \int \frac{d^4k}{(2\pi)^4} \frac{N(I)}{\langle k^2 - m^2 + i\epsilon \rangle \langle (k + q)^2 - m^2 + i\epsilon \rangle \langle (k - r)^2 - \lambda_g^2 + i\epsilon \rangle \langle (k - P)^2 - \lambda^2 + i\epsilon \rangle}\]

\[= -i\gamma^2 e_2 \int \frac{d^2k_\perp}{2(2\pi)^2} \int P^+ dx \frac{N(I)}{P^+ + i\epsilon} \frac{1}{x P^+ (x - \Delta) (1 - x)} \]

\[\times \left( \frac{k^- - (m^2 + k_\perp^2) - i\epsilon}{x P^+ (x - \Delta) (1 - x)} \right) \]

\[\times \left( \frac{\lambda_{x+}^2 + \lambda_{x-}^2}{x P^+ (x - \Delta) (1 - x)} \right) \]

\[\times \left( \frac{\lambda_{x+}^2 - \lambda_{x-}^2}{x P^+ (x - \Delta) (1 - x)} \right) \]

\[\times \left( \frac{\lambda_{x+}^2 - \lambda_{x-}^2}{x P^+ (x - \Delta) (1 - x)} \right) \]

\[= -i\gamma^2 e_2 \times (2\pi i) \int \frac{d^2k_\perp}{2(2\pi)^2} \int P^+ dx \frac{N(I)}{P^+ + i\epsilon} \frac{1}{x P^+ (x - \Delta) (1 - x)} \]

where we used \(k^+ = x P^+\). The numerators \(N(I)\) are given by

\[N(\uparrow \rightarrow \uparrow) = 2P^+ \sqrt{\Delta x} \left( M + \frac{m}{x} \right) q^-\]

\[N(\downarrow \rightarrow \uparrow) = 2P^+ \sqrt{\Delta x} \left( x_{-i\epsilon} \right) q^-\]

\[N(\uparrow \rightarrow \downarrow) = 2P^+ \sqrt{\Delta x} \left( x_{+i\epsilon} \right) q^-\]

\[N(\downarrow \rightarrow \downarrow) = 2P^+ \sqrt{\Delta x} \left( M + \frac{m}{x} \right) q^-\]

where \(q^- = Q^2 / \Delta P^+ = 2M v / P^+\). For the [current]–[gauge propagator]–[current] factor, in Feynman gauge only the \(-g^+\) term of the gauge propagator \(-g^+\) contributes in the Bjorken limit, and it provides a factor proportional to \(q^-\) in the numerator which cancels the \(q^-\) in the denominator of the gauge propagator. Therefore, the result scales in the Bjorken limit.

The integration over \(k^-\) in (11) does not give zero only if \(0 < x < 1\). We first consider the region \(\Delta < x < 1\).
The result is identical to that obtained from light-cone time-ordered perturbation theory.

The phases $\chi_i$ needed for single-spin asymmetries come from the imaginary part of (16), which arises from the potentially real intermediate state allowed before the rescattering. The imaginary part of the propagator (light-cone energy denominator) gives

$$\pm i\pi \delta \left( P^+ - \frac{(\lambda^2 + \vec{k}_\perp^2)x}{(1-x)P^+} + q^- \right) = -i\pi \frac{1}{q_\perp^2} \frac{\Delta^2}{q_\perp^2} \delta(x - \Delta - \delta),$$

(17)

where

$$\delta = 2\Delta \frac{\eta \cdot (\vec{k}_\perp - \vec{r}_\perp)}{q_\perp^2}.$$  

(18)

Since the exchanged momentum $\delta P^+$ is small, the light-cone energy denominator corresponding to the gauge propagator is dominated by the $\frac{(\vec{k}_\perp - \vec{r}_\perp)^2 + \lambda^2}{(1-x)\Delta}$ term. This gets multiplied by $(x - \Delta)$, so only $(\vec{k}_\perp - \vec{r}_\perp)^2 + \lambda^2$ appears in the propagator, independent of whether the photon is absorbed or emitted. The contribution from the region $0 < x < \Delta$ thus compliments the contribution from the region $\Delta < x < 1$.

We can integrate (16) over the transverse momentum using a Feynman parametrization to obtain the one-loop terms in (5) to (8).

$$g_1 = \int_0^1 d\alpha \frac{1}{\alpha(1-\alpha)\vec{r}_\perp^2 + \alpha \lambda^2 + (1-\alpha)\Delta(1-\Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta})},$$

(19)

$$g_2 = \int_0^1 d\alpha \frac{\alpha}{\alpha(1-\alpha)\vec{r}_\perp^2 + \alpha \lambda^2 + (1-\alpha)\Delta(1-\Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta})}. $$

(20)

Although not necessary for our analysis, we will assume for convenience that the final-state interactions generate a phase when exponentiated, as in the Coulomb phase analysis of QED. The rescattering phases $e^{i\chi_i}$ $(i = 1, 2)$ with $\chi_i = \tan^{-1} \left( \frac{\vec{r}_{\perp\alpha}}{\vec{r}_{\perp\beta}} \right)$ are thus distinct for the spin-parallel and spin-antiparallel amplitudes. The difference in phase arises from the orbital angular momentum $k_\perp$ factor in the spin-flip amplitude, which after integration gives the extra factor of the Feynman parameter $\alpha$ in the numerator of $g_2$. Notice that the phases $\chi_i$ are each infrared divergent for zero gauge boson mass $\lambda g \rightarrow 0$, as is characteristic of Coulomb phases. However, the difference $\chi_1 - \chi_2$ which contributes to the single-spin asymmetry is infrared finite. We have verified that the Feynman gauge result is also obtained in the light cone gauge using the principal value prescription. The small numerator coupling of the light-cone gauge particle is compensated by the small value for the exchanged $l^+ = \delta P^+$ momentum.

The virtual photon and produced hadron define the production plane which we will take as the $\hat{z} - \hat{x}$ plane.

The azimuthal single-spin asymmetry transverse to the production plane is given by

$$P_y = \frac{e_1e_2}{8\pi} \frac{2(\Delta M + m)r^1}{(\Delta M + m)^2 + \vec{r}_\perp^2} \left[ \frac{\vec{r}_\perp^2 + \Delta(1-\Delta)}{(\Delta M + m)^2 + \vec{r}_\perp^2} \right] \ln \frac{\vec{r}_\perp^2 + \Delta(1-\Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta})}{\Delta(1-\Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta})}. $$

(21)

The linear factor of $r^1 = r^\perp$ reflects the fact that the single spin asymmetry is proportional to $\vec{S}_p \cdot \vec{q} \times \vec{r}$, where $\vec{q} \sim -v^2$ and $\vec{S}_p = \pm \hat{y}$. Here $\Delta = x_{bj}$. 

![](image-url)
Our analysis can be generalized to the corresponding calculation in QCD. The final-state interaction from

gluon exchange has the strength $\frac{e_1 e_2}{4\pi} \rightarrow C_F \alpha_s(\mu^2)$. The scale of $\alpha_s$ in the $\overline{\text{MS}}$ scheme can be identified with

the momentum transfer carried by the gluon $\mu^2 = e^{-5/3}(\vec{k}_\perp - \vec{r}_\perp)^2$ [9]. The matrix elements of the proton to its

constituents will have the same numerator structure as the perturbative model since they are determined by orbital angular momentum constraints. The strengths of the proton matrix elements can be normalized by the anomalous magnetic moment and the total charge. In QCD, $r_\perp$ is the magnitude of the momentum of the current quark jet relative to the virtual photon direction. Notice that for large $r_\perp$, $P_y$ decreases as $\alpha_s(r_\perp^2) x_{bj} M r_\perp \ln r_\perp^2 / r_\perp^2$. The physical proton mass $M$ appears since it is present in the ratio of the $L_z = 1$ and $L_z = 0$ matrix elements. This form is expected to be essentially universal.

3. Model predictions

We show the predictions of our model in Fig. 4 for the asymmetry $P_y = A^{\sin\phi}_{UT}$ of the $i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$ correlation based on Eq. (21). As representative parameters we take $\alpha_s = 0.3$, $M = 0.94$ GeV for the proton mass, $m = 0.3$ GeV for the fermion constituent and $\lambda = 0.8$ GeV for the spin-0 spectator. The single-spin asymmetry $P_y$ is shown as a function of $\Delta$ at $r_\perp = 0.5$ GeV in Fig. 4(a) and as a function of $r_\perp$ at $\Delta = 0.15$ in Fig. 4(b). The

Fig. 4. Model predictions for the single spin asymmetry of the proton in electroproduction resulting from gluon exchange in the final state as a function of $\Delta = x_{bj}$ and quark transverse momentum $r_\perp$. The parameters are given in the text.
HERMES asymmetry $A_{UL}^{\sin \phi}$ contains a kinematic factor

$$K = \frac{Q}{v} \sqrt{1 - y} = \sqrt{\frac{2Mx}{E}} \sqrt{\frac{1 - y}{y}}$$

because the proton is polarized along the direction of the incident electron. The resulting predictions for $K P_y$ are shown in Figs. 4(c) and (d). Note that $\vec{r} = \vec{p}_q - \vec{q}$ is the momentum of the current quark jet relative to the photon direction. The asymmetry as a function of the pion momentum $\vec{p}_\pi$ requires a convolution with the quark fragmentation function.

4. Summary

We have calculated the single-spin asymmetry in semi-inclusive electroproduction induced by final-state interactions. We have shown that the final-state interactions from gluon exchange between the outgoing quark and the target spectator system leads to single-spin asymmetries in deep inelastic lepton–proton scattering at leading twist in perturbative QCD; i.e., the rescattering corrections are not power-law suppressed at large photon virtuality $Q^2$ at fixed $x_{bj}$. The azimuthal single-spin asymmetry $P_y$ transverse to the photon-to-pion production plane decreases as $\alpha_s(r_2^\perp x_{bj} M r_\perp \ln(r_2^\perp) / r_2^\perp)$ for large $r_\perp$, where $r_\perp$ is the magnitude of the momentum of the current quark jet relative to the virtual photon direction. The fall-off in $r_2^\perp$ instead of $Q^2$ compensates for the dimension of the $\bar{q}-q$–gluon correlation. The mass $M$ of the physical proton mass appears here since it determines the ratio of the $L_z = 1$ and $L_z = 0$ matrix elements. We have estimated the scale of $\alpha_s$ as $O(r_2^\perp)$. The nominal size of the spin asymmetry is thus $C_F \alpha_s(r_2^\perp) a_p$ where $a_p$ is the proton anomalous magnetic moment.

It is usually assumed that the cross section for semi-inclusive deep inelastic scattering at large $Q^2$ factorizes as the product of quark distributions times quark fragmentation functions [10,11]. Our analysis shows that the single-spin asymmetry which arises from final-state interactions does not factorize in this way since the result depends on the $\langle p|\bar{\psi}_q A \psi_q|p\rangle$ proton correlator, not the usual quark distribution derived from $\langle p|\bar{\psi}_q(\xi) \psi_q(0)\rangle$ evaluated at equal light-cone time $\xi^+ = 0$. In particular, the spin asymmetry is not related to the transversity distribution $\delta q(x, Q)$ which correlates transversely polarized quarks with the spin of the transversely polarized target nucleon.

Our results are directly applicable to the azimuthal correlation of the proton spin with the virtual photon to current quark jet plane, which can be deduced from jet measures such as the thrust distribution. The sin $\phi$ correlation of the proton spin with the photon-to-pion production plane as measured in the HERMES and SMC experiments can then be obtained using the usual fragmentation function. Detailed comparisons with experiment will be presented elsewhere. Our approach can also be applied to single-spin asymmetries in more general hadronic hard inclusive reactions such as $e^+e^- \rightarrow A \uparrow X$ and $pp \rightarrow A \uparrow X$.

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References

$K^+ \to \pi^+ \nu \bar{\nu}$: a rising star on the stage of flavour physics

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Abstract

Motivated by the new experimental information reported by the BNL-E787 Collaboration, we analyse the present impact and the future prospects opened by the measurement of $B(K^+ \to \pi^+ \nu \bar{\nu})$. Although still affected by a large error, the BNL-E787 result favours values of $B(K^+ \to \pi^+ \nu \bar{\nu})$ substantially larger than what expected within the Standard Model. As a result, this data already provide non-trivial constraints on the unitarity triangle, when interpreted within the Standard Model framework. We stress the importance of the clean relation between $B(K^+ \to \pi^+ \nu \bar{\nu})$, $\sin 2\beta$ and $\Delta M_B / \Delta M_{\bar{B}}$ that in the next few years could provide one of the deepest probes of the Standard Model in the sector of quark-flavour dynamics. A speculative discussion about possible non-standard interpretations of a large $B(K^+ \to \pi^+ \nu \bar{\nu})$ is also presented. Two main scenarios naturally emerge: those with direct new-physics contributions to the $s \to d \nu \bar{\nu}$ amplitude and those with direct new-physics effects only in $B_d \to \bar{B}_d$ mixing. Realistic models originating these two scenarios and possible future strategies to clearly identify them are briefly discussed. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Flavour-changing neutral-current (FCNC) processes provide a powerful tool to investigate the flavour structure of the Standard Model and its possible extensions. Among them, $K \to \pi \nu \bar{\nu}$ decays are certainly a privileged observatory because of their freedom from long-distance uncertainties.

An important step forward in the difficult challenge to measure the $K^+ \to \pi^+ \nu \bar{\nu}$ rate has recently been reported by the BNL-E787 Collaboration [1]. The combined analysis of BNL-E787 data, including previous published results [2], can be summarized as follows:

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = (1.57_{-0.82}^{+1.75}) \times 10^{-10}$$

(1)

The theoretical estimate of $B(K^+ \to \pi^+ \nu \bar{\nu})$ within the Standard Model (SM), as obtained by combining the analysis of Ref. [3] with an updated Gaussian fit of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [4] (discussed below), reads

$$B(K^+ \to \pi^+ \nu \bar{\nu})_{SM} = (0.72 \pm 0.21) \times 10^{-10}.$$  

(2)

Although still compatible within the errors, the difference between the central values in Eqs. (1) and (2) opens interesting perspectives.
The purpose of this Letter is twofold. On the one side, we analyse the impact of Eq. (1) within the SM framework: as we shall show, despite the large error this result already has a non-negligible statistical impact in CKM fits. We also discuss a possible future strategy to take advantage of the theoretically clean nature of $B(K^+ \to \pi^+ \nu \bar{\nu})$, $\sin 2\beta$ and $\Delta M_{B_s}/\Delta M_{B_d}$. These three observables, whose experimental determination will substantially improve in the near future, can be combined to make one of the most significant tests of the Standard Model in the sector of quark-flavour dynamics.

On the other side, we shall discuss possible new-physics scenarios that could accommodate a large value of $B(K^+ \to \pi^+ \nu \bar{\nu})$, assuming that in the future the error in Eq. (1) will decrease, without a substantial reduction of the central value. Interestingly, these scenarios do not necessarily require direct new-physics effects in the $s \to d \nu \bar{\nu}$ amplitude: a $B(K^+ \to \pi^+ \nu \bar{\nu})$ almost twice as big as in Eq. (2) could also arise with direct new-physics effects only in the $B_d$-$\bar{B}_d$ mixing amplitude.

2. $B(K^+ \to \pi^+ \nu \bar{\nu})$ within the SM

Short-distance contributions to the $s \to d \nu \bar{\nu}$ amplitude are efficiently described, within the SM, by the following effective Hamiltonian [3]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\sin^2\theta_W} \times \sum_{l=e,\mu,\tau} \left[ \lambda_l X_{NL}^{l} + \lambda_t X(x_t) \right] 	imes (\bar{s}d)(\bar{v}_l v_l)_{V-A},$$

where $x_t = m_t^2/M_W^2$, $\lambda_q = V_{qs}^* V_{qd}$ and $V_{ij}$ denote CKM matrix elements. The coefficients $X_{NL}^{l}$ and $X(x_t)$, encoding top- and charm-quark loop contributions, are known at the NLO accuracy in QCD [5,6] and can be found explicitly in [3]. The theoretical uncertainty in the dominant top contribution is very small and it is essentially determined by the experimental error on $m_t$. Fixing the $\overline{\text{MS}}$ top-quark mass to $m_t = (166 \pm 5)$ GeV we write

$$X(x_t) = 1.51 \left[ \frac{m_t(m_t)}{166 \text{ GeV}} \right]^{1.15} = 1.51 \pm 0.05. \quad (4)$$

The largest theoretical uncertainty in estimating $B(K^+ \to \pi^+ \nu \bar{\nu})$ originates from the charm sector. Following the analysis of Ref. [3], the perturbative charm contribution is conveniently described in terms of the parameter

$$p_0(X) = \frac{1}{\lambda^2} \left[ \frac{2}{3} X_{NL}^{c} + \frac{1}{3} X_{NL}^{t} \right] = 0.42 \pm 0.06. \quad (5)$$

where $\lambda = |V_{ud}|$ is the expansion parameter in Wolfenstein’s parameterization of the CKM matrix [7]. The numerical error in the r.h.s. of Eq. (5) is obtained from a conservative estimate of NNLO corrections [3]. Recently also non-perturbative effects introduced by the integration over charmed degrees of freedom have been discussed [8]. Despite a precise estimate of these contributions is not possible at present (due to unknown hadronic matrix-elements), these can be considered as included in the uncertainty quoted in Eq. (5). Finally, we recall that genuine long-distance effects associated to light-quark loops are well below the uncertainties from the charm sector [9].

With these definitions the branching fraction of $K^+ \to \pi^+ \nu \bar{\nu}$ can be written as

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{\bar{K}^+}{\lambda^2} \left[ (\text{Im}\lambda_t)^2 X^2(x_t) + (\text{Re}\lambda_t P_0(X) \text{Re}\lambda_t X(x_t)) \right], \quad (6)$$

where [3]

$$\bar{K}^+ = r_{K^+} \frac{3\alpha^2 B(K^+ \to \pi^0 \nu \bar{\nu})}{2\pi^2 \sin^4\theta_W} = 7.50 \times 10^{-6} \quad (7)$$

and $r_{K^+} = 0.901$ takes into account the isospin breaking corrections necessary to extract the matrix element of the $(\bar{s}d)(\bar{v}_l v_l)_{V-A}$ current from $B(K^+ \to \pi^0 \nu \bar{\nu})$ [10]. Employing the improved Wolfenstein decomposition of the CKM matrix [11], Eq. (6) describes in the $\bar{\rho}$--$\bar{\eta}$ an ellipse with small eccentricity, namely

$$(\sigma \bar{\eta})^2 + (\bar{\rho} - \bar{\rho}_0)^2 = \frac{\sigma B(K^+ \to \pi^+ \nu \bar{\nu})}{\kappa^2} |V_{cb}|^2 X^2(x_t). \quad (8)$$

where

$$\bar{\rho}_0 = 1 + \lambda^4 \frac{P_0(X)}{|V_{cb}|^2 X(x_t)} \quad \text{and} \quad \sigma = \left( 1 - \frac{\lambda^2}{2} \right)^{-2}. \quad (9)$$

\[\text{2} \quad \text{The natural order of magnitude of these non-perturbative corrections, relative to the perturbative charm contribution is } m_K^2/(m_t^2 \ln(m_t^2/M_W^2)) \sim 2\%\]
Table 1

<table>
<thead>
<tr>
<th>Input values used in CKM fits</th>
<th>Experimental data</th>
<th>Theoretical inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 0.220 \pm 0.002$</td>
<td>$</td>
<td>V_{cb}</td>
</tr>
<tr>
<td>$\Delta M_{B_s} = 0.487 \pm 0.009 \text{ ps}^{-1}$</td>
<td>$\Delta M_{B_s} &gt; 15 \text{ ps}^{-1}$</td>
<td>$\xi = (F_{Bd}/F_{Bs})/\sqrt{B_{Bs}/B_{Bd}} \sqrt{M_{B_s}/M_{B_d}} = 1.15 \pm 0.06$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>V_{ub}/V_{cb}</td>
</tr>
<tr>
<td></td>
<td>$\sin(2\beta) = 0.79 \pm 0.10$</td>
<td>$P_0(X) = 0.42 \pm 0.06$</td>
</tr>
</tbody>
</table>

The ellipse eventually becomes a doughnut once the uncertainties on the parameters determining $\rho_0$ and on the r.h.s. of (8) are taken into account.

Stringent bounds about the values of $\rho$ and $\eta$ within the SM can be obtained, at present, imposing constraints from $|V_{ub}|$, $\Delta M_{B_s}$, $\Delta M_{B_d}/\Delta M_{B_s}$, $\epsilon_{K}$ and $\sin (2\beta)$ [12]. In Fig. 1 we show the result of a simple Gaussian fit to these quantities, using the input values in Table 1: all errors have been combined in quadrature, whereas the 95% upper limit on $\Delta M_{B_s}$ has been treated as an absolute bound. Up to minor differences [mainly due to the value of $\hat{B}_K$ and the use of $\sin (2\beta)$], the result of this fit are in good agreement with more refined analyses available in the literature [12]. The statistical distribution of $\rho$ and $\eta$ thus obtained has been used to produce the result in Eq. (2). Note that, by construction, the error in Eq. (2) does not define a strict interval; it should be interpreted as the standard deviation of a Gaussian distribution.

The impact of the present experimental information on $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in the $\rho-\eta$ plane is analysed in Figs. 1 and 2. Due to the large central value and the non-Gaussian distribution, the BNL-E787 measurement already provides a non-negligible statistical input. This is hardly visible in a global fit (Fig. 1), but is more clear in Fig. 2, where the 90% C.L. exclusion limit imposed by the lower bound on $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ has been constructed by smooth modification of a Gaussian distribution, fitting the reference figures of 68%, 80%, 90% and 98% C.L. intervals obtained by BNL-E787 [1]. We are grateful to Steve Kettell for providing us the reference figures not reported in [1].

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The statistical distribution of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ has been constructed by smooth modification of a Gaussian distribution, fitting the reference figures of 68%, 80%, 90% and 98% C.L. intervals obtained by BNL-E787 [1]. We are grateful to Steve Kettell for providing us the reference figures not reported in [1].
is shown. Due to the large central value, the overall quality of the global fit decreases once the information on $B(K^+ \to \pi^+ \bar{\nu} \nu)$ is added. However, this is not a significant effect at the moment and, as shown in Fig. 2, the SM is still in good shape.

The prediction in Eq. (2), based on a global CKM fit, suffers to some extent from hadronic uncertainties entering the determination of $|V_{ub}|$ and the extraction of $\bar{\rho} - \bar{\eta}$ constraints from $\epsilon_K$ and $\Delta M_{B_d}$. On the other hand, the vertex of the unitarity triangle can in principle be determined (up to discrete ambiguities) simply by using $\Delta M_{B_s}/\Delta M_{B_d}$ and $\sin(2\beta)$, two quantities with a very small theoretical uncertainty. By definition,

$$\bar{\rho} = 1 - R_t \cos \beta, \quad \bar{\eta} = R_t \sin \beta,$$

where $R_t^2 = (1 - \bar{\rho})^2 + \bar{\eta}^2$. Expressing $R_t$ as a function of $\Delta M_{B_d}/\Delta M_{B_s}$, we can predict with great accuracy the value of $B(K^+ \to \pi^+ \bar{\nu} \nu)$ in terms of theoretically clean observables:

$$B(K^+ \to \pi^+ \bar{\nu} \nu) = \kappa_+ |V_{cb}|^4 X^2(x_t) \left[ \sigma R_t^2 \sin^2 \beta \right.$$ 
$$+ \frac{1}{\sigma} \left( R_t \cos \beta + \frac{\lambda^4 P_0(x)}{|V_{cb}|^2 X(x_t)} \right)^2 \right],$$

$$R_t = \frac{\xi \sqrt{\sigma}}{\lambda} \sqrt{\frac{\Delta M_{B_s}}{\Delta M_{B_d}}} \times \left[ 1 - \lambda \xi \frac{\Delta M_{B_d}}{\Delta M_{B_s}} \cos \beta + O(\lambda^4) \right].$$

In the next few years, when the experimental determination of $\frac{\Delta M_{B_d}}{\Delta M_{B_s}}$, $\sin(2\beta)$ and $B(K^+ \to \pi^+ \bar{\nu} \nu)$ will substantially improve, this relation could provide one of the most significant tests of the Standard Model in the sector of quark-flavour dynamics.

Unfortunately at the moment we cannot fully exploit the potential of Eqs. (11) and (12) in obtaining a precise prediction of $B(K^+ \to \pi^+ \bar{\nu} \nu)$ since $\Delta M_{B_d}$ has not been measured yet. Following Ref. [3], the best we can do at present is to derive a solid upper bound. Saturating simultaneously the following upper limits

$$\sqrt{\frac{\Delta M_{B_s}}{\Delta M_{B_d}}} < 0.180, \quad |V_{cb}| < 0.044,$$

sin(2$\beta$) > 0.5, $\xi < 1.3$, $P_0(X) < 0.50$, $X(x_t) < 1.57$,

that should be regarded as a very conservative assumption, we obtain

$$B(K^+ \to \pi^+ \bar{\nu} \nu)_{\text{SM}} < 1.32 \times 10^{-10}.$$

By construction it is difficult to assign a probabilistic meaning to this result: it should be regarded as an absolute bound under the assumptions in (13). As a consistency check of this statement, we note that Eq. (14) coincides with the $3\sigma$ upper limit derived from the global Gaussian fit. We can thus firmly conclude that the central value in Eq. (1) cannot be accommodated within the SM.

3. New-physics scenarios with a large $B(K^+ \to \pi^+ \bar{\nu} \nu)$

A stimulating coincidence implied by the experimental result in Eq. (1) is the fact that its central value is well in agreement with the constraints imposed by $\epsilon_K$ and $|V_{ub}|$ (see Fig. 1). If in the future the error on $B(K^+ \to \pi^+ \bar{\nu} \nu)$ will decrease, without substantial changes in the central value, we shall have a conflict only between $B(K^+ \to \pi^+ \bar{\nu} \nu)$ and observables sensitive to $B_d - \bar{B}_d$ mixing. In Fig. 3 we show the result of a $\rho - \tilde{\eta}$ fit without the inclusion of $B_d - \bar{B}_d$ data: in this case negative values of $\bar{\rho}$ are clearly more
favoured. Remarkably, a similar qualitative indication is obtained also by the central values of non-leptonic 
$B \to K \pi$ decays and, in particular, by the deviation of the ratio $R_n = B(B_d \to \pi^+ \pi^-)/(2B(B_d \to \pi^0 \pi^0))$ from one (see Ref. [14] and references therein). Also in the $B \to K \pi$ case the statistical significance of the effect is still quite limited, nonetheless there is certainly enough room for speculations about possible new-physics effects in $B_d$ mixing.

As emphasised in the previous section, $\Delta M_{B_s}/\Delta M_{B_d}$ and $\sin2\beta$ on the one side and $B(K^+ \to \pi^+ \nu\bar{\nu})$ on the other are affected by small theoretical uncertainties, thus the potential conflict between $B(K^+ \to \pi^+ \nu\bar{\nu})$ and $\Delta B \approx 2$ amplitudes is mainly an experimental issue: if in the future the discrepancy will become more significant it will unambiguously signal the presence of new physics. Moreover, since the FCNC $s \to d \nu\bar{\nu}$ transition and $B_d$-$\bar{B}_d$ mixing both appear only at the loop level within the SM, on general grounds both amplitudes can equally be considered as a good candidates for possible non-standard effects. In the following we shall analyse separately possible new-physics scenarios affecting one of these two amplitudes.

Scenario I: non-standard contributions to the $s \to d \nu\bar{\nu}$ amplitude

The first question to address about non-standard contributions to the observed transition $K^+ \to \pi^+ \nu\bar{\nu}$ is whether the missing energy is due to a $\nu\bar{\nu}$ pair or not. Since the neutrino pair cannot be detected, all the information about the decay must be deduced by the spectrum of the charged pion and with only two candidate events this is clearly rather poor. Nonetheless, some conclusions can already be drawn. In particular, we can exclude the possibility that these events are generated by a process of the type $K^+ \to \pi^+ X^0$, where $X^0$ is a massless particle that escapes detection [1,2]. On the other hand, since $\pi^+$ momenta of the two events are almost identical, we cannot exclude yet the possibility that these events are due to a two-body decay with a massive particle—sufficiently long lived or with invisible decay products—with mass $\approx 100$ MeV. This rather exotic scenario could easily be discarded in the near future by the observation of candidate events with a different kinematical configuration.

A general discussions about $K^+ \to \pi^+ \nu\bar{\nu}$ beyond the SM can be found in [15]. If we assume purely left-handed neutrinos and we neglect possible lepton-flavour violations, the only dimension-six effective operator relevant to these processes is $\langle \bar{s}d \rangle v_\nu (\bar{v}_\nu)_{\nu-A}$ (as in the SM case) and the measurement of $B(K^+ \to \pi^+ \nu\bar{\nu})$ fixes the magnitude of its Wilson coefficient. At present this is the only available information about this coefficient, thus there is little we can learn from a model-independent analysis. The only outcome of such type of analysis is an update of the upper bound on $B(K_L \to \pi^0 \nu\bar{\nu})$ [15], that in view of Eq. (1) reads

$$B(K_L \to \pi^0 \nu\bar{\nu}) < \frac{\xi_{KL}^2 R_{KL}}{\xi_{K^+}^2 R_{K^+}}B(K^+ \to \pi^+ \nu\bar{\nu}) < 1.7 \times 10^{-9} \text{ (90\% C.L.)}. \quad (15)$$

Among specific new physics models, low-energy supersymmetry is certainly one of the most interesting and well-motivated scenarios. Supersymmetric contributions to the $s \to d \nu\bar{\nu}$ amplitude have been extensively discussed in the recent literature, both within models with minimal flavour violation [16,17] and within models with new sources of quark-flavour mixing [18–20]. As clearly stated in Ref. [17], minimal models, or models without new quark-flavour structures, cannot produce a sizeable enhancement of the $K^+ \to \pi^+ \nu\bar{\nu}$ width and would be immediately ruled out by a large $B(K^+ \to \pi^+ \nu\bar{\nu})$.

Also within models with new sources of quark-flavour mixing is not easy to produce sizeable modifications of the $s \to d \nu\bar{\nu}$ amplitude. Excluding fine-tuned scenarios with large cancellations in $\Delta S = 2$ transitions, sizeable enhancements of $K \to \pi \nu\bar{\nu}$ rates can only be generated by chargino-mediated diagrams with a large (non-standard) $\tilde{u}_L^j - \tilde{u}_R^j$ mixing [18–20]. Moreover, in the limit of large squark masses ($M_{\tilde{W}}^2/M_{\tilde{q}}^2 \ll 1$) box diagrams are systematically suppressed over $Z$-penguin ones and can be safely neglected [20,21]. In this approximation the modification to the SM Hamiltonian in (3) can be obtained by replacing $X(x_t)$ with

$$X' = X(x_t) \left[ 1 + \frac{A_{jl}^t \tilde{H}_{l\ell} \tilde{V}_{ij} \tilde{V}_{j\ell}}{8\delta_{ij} X(x_t)} \right], \quad (16)$$

where [20]

$$A_{jl}^t = \tilde{H}_{l\ell} \tilde{V}_{ij} \tilde{V}_{j\ell} - g_{t\ell} V_{t\ell} \tilde{H}_{l\ell} \bar{V}_{j\ell} \tilde{C}_{2j} \tilde{C}, \quad (17)$$
matrices that diagonalize the chargino mass matrix
\[ \delta U \]
diagonalizes the up-squark mass matrix (written in the
As

ploying a perturbative diagonalization of both squark
conditions:

For

xij

The explicit expressions of

O

and

\[ X(x_t) \]

The expression (16) can be further simplified em-

\[ \approx \frac{1}{\delta X(x_t)} \]


\[ k(x_i, x_j) \]

\[ f(x_t) \]

f

\[ tRqL \]

The smallness of the vector coupling of the Z boson
to charged leptons implies that, to a good accuracy,

\[ X(x_t) \approx 1 + \frac{1}{8X(x_t)} \]

\[ \times \left[ g_t \frac{\delta U}{V_{td}} \delta^x f_1(x_{ij}; \tan \beta) \right. \]

\[ + \left. g_t \frac{\delta U}{V^*_{ts}} \delta f_1(x_{ij}; \tan \beta) \right] \]

\[ + \frac{1}{V_{ts} V_{td}} \delta f_2(x_{ij}) + \mathcal{O}(V_{ij}) \right] \]

where

\[ \delta U_{i_{gq}} = \frac{(M^2_{i g})_{i_{gq}}}{(M^2_{q})}, \quad \delta^x = \frac{M_W}{M_Z} \]

\[ \mathcal{O}(V_{ij}) \]

terms not enhanced by

\[ X(x_t) \approx 1.4, \]

\[ M^2_{i g} \]

\[ f_1 \]

\[ f_2 \]

\[ |f_1| \leq 0.1 \]

\[ |f_2| \leq 0.4 \]

In order to ob-

\[ \delta U_{i_{gq}} \]

\[ \lambda \geq 2 |V_{ts} V_{td}|, \]

\[ \lambda \geq 2 |V_{iq}|, \quad q = s \text{ or } d. \]

These requirements are not in contradiction with the
phenomenological bounds on \( \delta U_{i_{qL}} \)
implies other observables [21] and are consistent with
the constraints imposed by the stability of the superpoten-
tial [22]. However, they necessarily require a rather
non-trivial structure for the A terms.

The possible supersymmetric enhancement of the
\[ K^+ \rightarrow \pi^+ \nu \bar{\nu} \]

\[ K^+ \rightarrow \pi^0 \nu \bar{\nu} \]

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where $C_{10}^{SM} \approx -4.2$ [13]. If at least one of the conditions (21) is satisfied then at least in one of the two cases ($b \rightarrow s$ or $b \rightarrow d$) the axial-current operator receives $O(1)$ non-standard contributions. Since $X(x_{t})/(\sin^{2}\theta_{W}C_{10}^{SM}) \approx 1.5$, on general grounds $C_{10}$ is slightly more sensitive to modifications of the Z-penguin contribution with respect to $X'$. On the other hand, if $\delta_{t}^{U(L)}$ and $\delta_{t}^{U(L)}$ conspire to maximize the effect in Eq. (19), we can expect a smaller relative impact in Eq. (23).

In the most optimistic case, i.e., in presence of a 100% increase of $|C_{10}|$ in the $b \rightarrow s$ transition, the effect could possibly be detected in a short time at $B$-factories, by looking at exclusive $B \rightarrow (K, K^{*})^{\pm} \ell^{\pm} \ell^{-}$ decays. In particular, the lepton-forward-backward asymmetry in $B \rightarrow K^{*}\ell^{+}\ell^{-}$ provides an excellent probe of magnitude and phase of $C_{10}$ [24]. On the other hand, to detect a modification of $C_{10}$ that does not exceed the 30% level in magnitude, either in $b \rightarrow s$ or in $b \rightarrow d$, it is necessary a detailed study of inclusive transitions or pure leptonic decays ($B_{s,d} \rightarrow \ell^{+}\ell^{-}$).

Finally, we note that a non-standard $Z \bar{b}q$ vertex leads to potentially observable effects also in inclusive and exclusive $b \rightarrow sv\bar{v}$ transitions [24]. In particular, Eq. (23) can trivially be extended to the $b \rightarrow sv\bar{v}$ case with the replacement $C_{10}(C_{SM}^{L}) \rightarrow C_{vL}(C_{SM}^{L})$, where $C_{vL}$ is the Wilson coefficient of the only dimension-six operator contributing to these processes within the SM [13], namely, $(\bar{b}s)_{V-A}(\bar{v}v)_{V-A}$.

**Scenario II: non-standard contributions to $B_{d}-\bar{B}_{d}$ mixing**

Contrary to the $s \rightarrow d\bar{v}\bar{v}$ case, the present information about $B-\bar{B}$ mixing is already rich and precise. As a result, the scenario with new physics in $B-\bar{B}$ mixing turns out to be rather constrained also within a model-independent approach.

The first conclusion that can easily be drawn is that this scenario is not flavour blind: we necessarily need to modify the SM relation between $|V_{td}/V_{ts}|$ and $\Delta M_{B_{d}}/\Delta M_{B_{s}}$ in Eq. (12) in order to allow a solution with negative $\rho$ (see Fig. 3). If new physics affects $\Delta M_{B_{d}}$ and $\Delta M_{B_{s}}$ in the same way, with a flavour-blind modification of the loop function, then the ratio $|V_{td}/V_{ts}|$ extracted from $\Delta M_{B_{d}}/\Delta M_{B_{s}}$ would be exactly the same as in the SM. Since the measurement of $\Delta M_{B_{d}}$ alone favours positive values of $\rho$ (within the SM) and $\Delta M_{B_{s}}$ alone is insensitive to $\rho$, the most economical way to implement a non-standard scenario with $\rho < 0$ is to assume sizeable new-physics effects only in $B_{d}-\bar{B}_{d}$ mixing.

In presence of new-physics in $B_{d}-\bar{B}_{d}$ mixing we can write, in full generality,

$$
\mathcal{M}_{12}^{d} = \frac{1}{2M_{B_{d}}} \left[ \langle \bar{B}_{d} | H_{\text{eff}}^{\Delta B=2} | B_{d} \rangle \right] \propto \left[ V_{td}^{2} + Z^{2} \right],
$$

where $Z^{2}$ is a complex quantity encoding the non-standard contribution, normalized to the SM one (except for the CKM factor). The new-physics contribution has been conveniently expressed in terms of the square of $Z = |Z|e^{i\phi}$ since, in most scenarios, contributions to $\Delta B = 2$ amplitudes are proportional to the square of some $\Delta B = 1$ effective coupling. Note, however, that $\phi$ is not necessarily the phase of the new $\Delta B = 1$ effective coupling: it incorporates also a possible $\pm \pi/2$ shift induced by a possible overall minus sign of the new contribution with respect to the SM one. Denoting by $|V_{td}^{0}|$ the modulus of $V_{td}$ determined by $\Delta M_{B_{d}}$ and $\Delta M_{B_{s}}$ assuming only SM contributions and by $-\rho \theta_{0}$ its phase ($V_{td}^{0} = |V_{td}^{0}|e^{-i\rho \theta_{0}}$) determined by $\mathcal{A}_{CP}(B_{d} \rightarrow \Psi K)$, the non-standard contribution $Z$ should satisfy the following equation

$$
|V_{td}^{0}|^{2}e^{-2i\rho \theta_{0}} + |Z|^{2}e^{2i\phi} = |V_{td}^{0}|^{2}e^{-2i\rho \theta_{0}},
$$

where $V_{td} = |V_{td}|e^{-i\theta}$ denotes the true CKM factor. If we require a solution with $\tilde{\rho} < 0$ and $\tilde{\eta} > 0$, such that $\sin(2\tilde{\beta}) < \sin(2\tilde{\beta}_0)$ and $\cos(2\tilde{\beta}) > 0$, we then obtain

$$
1 + \frac{|Z|^{2}}{|V_{td}|^{4}} \geq \frac{2|Z|^{2}}{|V_{td}|^{4}} \cos(2\phi + 2\tilde{\beta}) = \left| \frac{V_{td}^{0}}{V_{td}} \right|^{4} < 1,
$$

(26)

$$
\frac{|Z|^{2}}{|V_{td}|^{2}} \sin(2\phi + 2\tilde{\beta}) = \left| \frac{V_{td}^{0}}{V_{td}} \right|^{2} \sin(2\phi - 2\tilde{\beta}_0) < 0.
$$

(27)

Even without specifying the exact values of $\tilde{\rho}$ and $\tilde{\eta}$, the solution of Eqs. (26) and (27) requires that $(2\phi + 2\tilde{\beta})$ is in the third quadrant, or that $Z$ has large imaginary part. Interestingly, this conclusion is independent
of the discrete ambiguities arising in the determination of $\beta_0$ from $\mathcal{A}_{CP}(B_d \to \Psi K)$. If we further impose that $\delta$ and $\eta$ are within the inner ellipse in Fig. 3, it is easy to check that $0.7 \lesssim \left| \frac{Z}{V_{td}} \right| \lesssim 1.1$ and $|\phi| \gtrsim 75^\circ$. From this general analysis we conclude that in all models where the non-standard $\Delta B = 2$ amplitude is proportional to the square of an effective $\Delta B = 1$ coupling, the latter can be real (i.e., does not imply new CP-violating phases) only if there is a relative minus sign between SM and non-standard $\Delta B = 2$ amplitudes.

Similarly to the $s \to d\nu \bar{\nu}$ case, low-energy supersymmetry is one of the most interesting and well-motivated scenarios to discuss specific predictions. Within the generic framework of the mass-insertion [25] there are several possibilities to implement the proper contribution to Eq. (24). For instance, assuming the dominance of gluino box diagrams, we can simply adjust the coupling $\delta_{bl}^D$ to produce the desired modification of $B_d \to \bar{B}_d$ mixing. In particular, $O(1)$ corrections are obtained for $|\delta_{bl}^D| \sim O(10^{-1})$. Interestingly, in this case SM and supersymmetric loop functions have the same sign [25] (assuming $M_3 \gtrsim M_2$, as suggested by RGE constraints [26]) thus, according to the general argument discussed above, $\delta_{bl}^D$ must be almost purely imaginary.

A more specific framework which justifies the existence of new flavour structures affecting mainly $B$-physics observables, rather than $K$ decays or electric dipole moments, is the so-called effective supersymmetry scenario [27]. Within this model all squarks are rather heavy, with masses of $O(10)$ TeV, with the exception of left-handed bottom and top squarks, whose masses are kept below 1 TeV. By this way supersymmetric contributions to observables not involving the third family are naturally suppressed and, at the same time, the naturalness problem of the Higgs potential is cured by the light squarks of the third family. Integrating out the heavy squarks of the first two generations, the light sbottom mass eigenstate ($\tilde{b}_2$) can be written as $\tilde{b}_2 = Z_{i3} V_{ij} \tilde{d}^j$, where $\tilde{d}^j$ denote flavour eigenstates, $V$ is the CKM matrix and $Z_{ij}$ are coefficients arising by the diagonalization of the $3 \times 3$ left-handed down-squark mass matrix [28]. In practice, the coupling $(Z_{db} Z_{bB}^*)$, where $Z_{db} = Z_{i3} V_{td}$, plays in this context the same role as $\delta_{bl}^D$ in the generic framework of the mass insertion approximation. Indeed gluino-sbottom box diagrams lead to the following effective Hamiltonian [28]

$$\mathcal{H}_{\Delta B=2}^{\text{eff-SUSY}} = \frac{\alpha_s^2}{36 M_B^2} (Z_{db} Z_{bB}^*)^2 f(x_{gB})$$

$$\times b_L \gamma^\mu d_L b_L \gamma^\nu d_L, \quad (28)$$

$$\left| \frac{Z}{V_{td}} \right| e^{i\phi} \approx \frac{Z_{db} Z_{bB}^*}{10^{-2}} \frac{1 \text{ TeV}}{M_B}. \quad (29)$$

The coupling $Z_{db} Z_{bB}^*$ can naturally be of $O(10^{-2})$ [28], inducing the desired $O(1)$ correction; however, similarly to $\delta_{bl}^D$, also $Z_{db} Z_{bB}^*$ needs to be almost purely imaginary in order to produce the correct sign of the effect (i.e. a decrease of $\Delta M_{B_t}$). We further note that a non-trivial flavour structure among the first two generations is necessary to ensure that $Z_{qB} Z_{bB}^*$ is not proportional to $V_{tq}$ and thus the corrections to $\Delta M_{B_t}$ and $\Delta M_{B_s}$ are not correlated.

Both within the generic mass-insertion framework and within the effective supersymmetry scenario it is not easy to point out clear correlations between non-standard contributions to $B_d \to \bar{B}_d$ mixing and those to other observables. On the other hand, a clear model-independent indication about this non-standard scenario could be obtained by a firm experimental evidence (independent from $K^+ \to \pi^+ \nu \bar{\nu}$) of $\tilde{\rho} < 0$. More precise results on non-leptonic $B \to K \pi$ decays would be extremely interesting in this respect [14].

4. Conclusions

In this Letter we have analysed the present impact of the new experimental information on $B(K^+ \to \pi^+ \nu \bar{\nu})$ [1]. Despite an apparent large error, the non-Gaussian tail and the large central value let us to extract from the BNL-E787 result non-trivial constraints on the CKM unitarity triangle. As we have explicitly shown, the theoretically clean information from $B(K^+ \to \pi^+ \nu \bar{\nu})$, combined with $\Delta M_{B_t}/\Delta M_{B_s}$ and

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Note that there is a misprint in the expression of $f(x)$ reported in Ref. [28].
sin 2\(\beta\), already defines a rather narrow region in the \(\rho \sim \eta\) plane. As emphasised, the precise relation linking these three observables will soon provide one of the most interesting consistency tests of the Standard Model in the sector of quark-flavour dynamics.

Stimulated by the large central value of the BNL-E787 result, we have also presented a speculative discussion about possible non-standard interpretations of a large \(B(K^+ \rightarrow \pi^+ \nu \bar{\nu})\). In general, these can be divided into two big categories: models with direct new-physics contributions to the \(s \rightarrow d\nu \bar{\nu}\) amplitude and models with direct new-physics effects only in \(B_d - \overline{B_d}\) mixing. In the latter case a large \(B(K^+ \rightarrow \pi^+ \nu \bar{\nu})\) arises because of a different CKM fit, which allows a solution with \(\rho < 0\). Supersymmetry with non-minimal flavour structures provides a consistent framework to realize both possibilities and, in the case of sizeable non-standard contributions to \(B_d - \overline{B_d}\) mixing, the scenario with heavy masses for the first two families emerges as a natural candidate.

We have outlined the correlations occurring between \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) and rare semileptonic FCNC \(B\) decays in supersymmetry. If the \(s \rightarrow d\nu \bar{\nu}\) amplitudes receive a sizeable supersymmetric enhancement, a substantial deviation from the SM should be observed either in \(b \rightarrow d\ell^+ \ell^-\) or in \(b \rightarrow s\ell^+ \ell^-\) transitions, especially in observables sensitive to the axial-current operator \(Q_{10}\), such as the lepton FB asymmetry. On the other hand, the smoking-gun for the scenario with new-physics in \(B_d - \overline{B_d}\) mixing would be a firm experimental evidence of \(\rho < 0\), independent from \(B(K^+ \rightarrow \pi^+ \nu \bar{\nu})\), obtainable for instance by means of \(B \rightarrow K \pi\) decays.

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References

Orbitally excited $D$ and $B$ mesons in the approach of the QCD string with quarks at the ends

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Abstract

In this Letter we discuss the masses and the splittings of $1^{2S+1}P_J$ states in the spectrum of $D$ and $B$ mesons, as they appear in the approach of the QCD string with quarks at the ends. We find good agreement of our predictions with those of other QCD-motivated models as well as with the lattice and experimental data, including recent experimental results. We discuss the ordering pattern for $P$ levels in $D$- and $B$-mesonic spectrum. © 2002 Published by Elsevier Science B.V.

Data on the spectroscopy of heavy-light mesons coming from various experimental collaborations are challenge for theorists, and these are $D$ and $B$ mesons to play an important role in checks of the validity and accuracy of the models.

In this Letter we address questions concerning the masses of orbitally excited $D$ and $B$ mesons in the method of the QCD string with quarks at the ends, paying special attention to the ordering of the $P$ levels. The choice of $D$ and $B$ mesons is not accidental and is stipulated by the recent data on the masses and decays of the above mentioned heavy-light mesons coming from various experimental collaborations. Despite of the fact that there is no agreement between them and some resonances are not yet confirmed, still we find it interesting to compare these experimental data, as well as those provided by other models and lattice simulations, with the predictions of our approach. First, let us remind the reader the basic ideas of the latter.

Starting from the gauge-invariant wave function of the $q\bar{q}$ meson,

$$\Psi_{q\bar{q}}(x, y|A) = \bar{\Psi}_q(x) \Phi(x, y) \Psi_{\bar{q}}(y),$$

with $\Phi$ being the parallel transporter, we write the Green’s function of the meson,

$$G_{q\bar{q}} = \langle \Psi_{q\bar{q}}^+(\bar{x}, \bar{y}|A) \Psi_{q\bar{q}}(x, y|A) \rangle_{q\bar{q}A},$$

and perform the integration over the quark and the gluonic fields. For the latter case we make use of the minimal area law asymptotic for the Wilson loop bounded by the quark and the antiquark trajectories (see, e.g., [1]),

$$\frac{\text{Tr}}{P} \exp \left( ig \oint_C dz_\mu A_\mu \right) \sim \exp (-\sigma S_{\text{min}}),$$

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where \( \sigma \) is the string tension in the fundamental representation of the SU(3) colour group, and the area \( S_{\text{min}} \) can be approximated by means of the straight-line anzatz [2],

\[
S_{\text{min}} = \int dt \int d\beta \sqrt{(\dot{w} w')^2 - \dot{w}^2 w''^2},
\]

\( w_\mu(t, \beta) = \beta x_{1\mu}(t) + (1 - \beta)x_{2\mu}(t), \quad (4) \)

with \( x_{1,2} \) being the coordinates of the quark and the antiquark. Now, applying the Feynman–Schwinger representation to the single-quark propagators and introducing the einbein fields \( \mu_{1,2} \) to simplify the relativistic kinematics [3], we, finally, arrive at the following expression for the Hamiltonian of the meson [4,5]:

\[
H = H_0 + V_{\text{str}} + V_{\text{sd}},
\]

\[
H_0 = \sum_{i=1}^{2} \left( \frac{\vec{p}_i^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2} + \sigma \right) \frac{1}{r} - C_0, \quad (6)
\]

\[
V_{\text{str}} \approx -\frac{\sigma (\mu_1^2 + \mu_2^2 - \mu_{1\mu_2}) \bar{L}^2}{6\mu_1\mu_2 r} + \frac{\sigma^2 (\mu_1 + \mu_2) (4\mu_1^2 - 7\mu_1\mu_2 + 4\mu_2^2) \bar{L}^2}{72\mu_1^2\mu_2}, \quad (7)
\]

\[
V_{\text{sd}} = \frac{8\pi \kappa}{3\mu_1\mu_2} \bar{\psi}(0) \psi(0) - \frac{\sigma}{2r} \left( \frac{\vec{S}_1 \bar{L}}{\mu_1} + \frac{\vec{S}_2 \bar{L}}{\mu_2} \right)
\]

\[
+ \frac{\kappa}{r^2} \left( \frac{1}{2\mu_1} + \frac{1}{\mu_2} \right) \frac{\vec{S}_1 \bar{L}}{\mu_1}
\]

\[
+ \frac{\kappa}{r^2} \left( \frac{1}{2\mu_2} + \frac{1}{\mu_1} \right) \frac{\vec{S}_2 \bar{L}}{\mu_2}
\]

\[
+ \frac{\kappa}{\mu_1\mu_2 r^2} \left( \frac{3(\vec{S}_1 \bar{n}) (\vec{S}_2 \bar{n})}{2} - (\vec{S}_1 \bar{S}_2) \right)
\]

\[
+ V_{\text{loop}}(\kappa^2), \quad (8)
\]

where in (5) we supply the purely nonperturbative interaction, coming from the string-like picture of confinement, by the perturbative Coulomb interaction \( (\kappa = \frac{e^2}{\alpha_i^3}) \), as well as by the constant negative shift, \( C_0 \), due to the light-quark self-energy [6] strongly needed to bring the Regge trajectory intercepts into their experimental values. The term \( V_{\text{str}} \) deserves special attention, since it is originated from the square root in (4) and describes the contribution of the QCD string into the total inertia of the rotating \( qq \) system.

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td>Parameters of the Hamiltonian (5)–(8)</td>
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<tr>
<td>Meson &amp; ( \sigma ) (GeV(^2)) &amp; ( a_s ) &amp; ( C_0 ) (MeV) &amp; ( m_Q ) (MeV) &amp; ( m_q ) (MeV)</td>
</tr>
<tr>
<td>---</td>
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<tr>
<td>( D ) &amp; 0.17 &amp; 0.4 &amp; 196 &amp; 1400 &amp; 9</td>
</tr>
<tr>
<td>( B ) &amp; 0.17 &amp; 0.39 &amp; 169 &amp; 4800 &amp; 8</td>
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This contribution is important to establish the correct slope of the mesonic Regge trajectories [7]. We keep the first two terms in its expansion in powers of \( \sigma/\mu^2 \). The term \( V_{\text{sd}} \) contains spin-dependent interaction generated by both, perturbative and nonperturbative, potentials. Finally, the last term, \( V_{\text{loop}}(\kappa^2) \), comes from the one-loop corrections to the potential. It is given by Eqs. (3.1) and (3.2) of the paper [8] with the obvious change \( m_{1,2} \rightarrow \mu_{1,2} \), and we choose the renormalization scale to be equal to the reduced effective mass \( \mu \). Finally, to fix the Hamiltonian (5)–(8), we use the values of the parameters listed in Table 1.

Einbein fields \( \mu_{1,2} \) are kept as variational parameters and the spectrum is minimized then with respect to them. The external values of \( \mu \)’s play the role of the constituent masses of the quarks and appear dynamically due to the interaction. This feature of the given approach allows one to start from the current mass of the constituent (gluons also can be described in this formalism) and to arrive at its effective constituent mass self-consistently. However, this simple interpretation should be considered with caution. The first source of error is neglecting the quark Zitterbewegung. Indeed, we neglect the negative-signed solutions for \( \mu_{1,2} \) expecting its small influence on the spectrum [5]. On the other hand, the simple quantum mechanical reduction of the relativistic field-theory problem given by the QCD string approach is not applicable for the description of chiral effects, such as the Bogoliubov-type transformation from bare to dressed quarks and the formation of a nontrivial chiral condensate. Therefore one cannot pretend to describe the pion in this framework. In realistic quantum-field-theory-based models each mesonic state possesses two wave functions which describe the motion forward and backward in time of the \( qq \) pair inside the meson [9]. For the pion, which is expected to be strictly massless in the chiral limit, the two wave functions are of the same order of magnitude, so that none of them can be neglected. Luckily the backward motion is suppressed.
if at least one of the quarks is heavy [9], so that one expects to arrive at reliable predictions in the case of heavy-light mesons.

Since the Hamiltonian $H_0$, which plays the role of the zeroth order approximation for the problem, conserves the angular momentum $\vec{L}$, the total spin $\vec{S}$, and the total momentum $\vec{J} = \vec{L} + \vec{S}$ separately, then its eigenstates can be specified as terms, $n_{2S+1}L_J$, with $n$ being the radial quantum number. In the remainder of this Letter we shall concentrate on the states with $n = L = 1$. Their masses can be represented as

$$M(1^{2S+1}P_J) = \{1^{2S+1}P_J|H|1^{2S+1}P_J\} = M_0 + \Delta E_{\text{str}} + E_{\text{str}}(\vec{S_1}\vec{L})$$

$$+ E_{\text{str}}(\vec{S_2}\vec{L}) + E_\mu(3(\vec{S_1}\vec{n})(\vec{S_2}\vec{n}))$$

$$- (\vec{S_1}\vec{S_2})^2,$$

(9)

where $\Delta E_{\text{str}}$ is the contribution of the string correction and the term with the spin–spin interaction does not contribute since the wave function at the origin vanishes for orbitally excited states, whereas the corresponding one-loop contribution is negligible. The results of numerical calculations, including the values of the coefficients entering Eq. (9), are listed in Table 2 (see [5] for the details of the calculations).

In Table 3 we give the matrix elements of the spin-tensor and spin-orbit operators between $P$-states. Since the spin-orbit interaction mixes states with different total spin, then the masses of the physical states with the total momentum $J = 1$ are subject to a matrix equation,

$$\begin{align*}
\{1^{1}P_1|H|1^{1}P_1\} - E & = \{1^{3}P_1|H|1^{3}P_1\} \\
\{1^{3}P_1|H|1^{1}P_1\} & = \{1^{3}P_1|H|1^{3}P_1\} - E
\end{align*}$$

(10)

In Tables 4, 5 we give our predictions for the masses of the $P$-level $D$ and $B$ mesons and compare them with the predictions of other models as well as with the lattice and experimental data coming from various collaborations.

From Tables 4, 5 one can deduce several conclusions. First, all three mentioned models give good description of the $1P_2$ states, whereas all of them fail to reproduce a very heavy $1P_0$ $B$-mesonic state reported by OPAL [15]. If this experimental value is confirmed, then this will serve as a signal that all theoretical ap-
averaging is performed over the zeroth-order wave
(eigenstate problem for the Hamiltonian (6) in this limit
the light quark,
Hamiltonian. 1 As discussed in [5,20], solution of the
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additional careful study. On the other hand, lattice sim-
ulations give the mass 5.754 GeV for this state [12],
which is about 100 MeV lower than the OP AL value. This stresses once again that the experimental situation strongly needs clarification. A similar state in the spectrum of D mesons is not reported yet by experimental collaborations, though all models and the lattice simulations give a consistent prediction for it to be around 2 430–2 440 MeV.

Another conclusion which one can make from
Tables 4, 5 is that there is no agreement concerning
1P1 states. Different models give different splitting patterns (see also the discussion in [12]). To have a better insight into the nature of this splitting let us study the heavy-quark limit, \( m_Q = m_1 \rightarrow \infty \), analytically, which is possible in our approach. Only the coefficient \( E_{s\alpha \beta} \), in notations of Eq. (9), survives in this limit, and the expression for it reads

\[
E_{s\alpha \beta} = -\frac{\sigma}{2\mu^2} (r^{-1}) + \frac{\kappa}{2\mu^2} (r^{-3}) \\
+ \frac{9\kappa^2}{16\pi^2 \mu^2} \left[ \left( \frac{19}{18} + \gamma_E \right) (r^{-3}) + (r^{-3} \ln(\mu r)) \right],
\]

where \( \gamma_E = 0.5772 \) is the Euler constant and the averaging is performed over the zeroth-order wave function \( \psi_{\alpha \beta}(r) \) corresponding to both states, 1P1/2 and 1P3/2, which are now the true eigenstates of the Hamiltonian.1 As discussed in [5,20], solution of the eigenstate problem for the Hamiltonian (6) in this limit is given by solutions to the Schrödinger equation

\[
\left( \frac{d^2}{d\xi^2} + \frac{\lambda}{|\xi|} \right) \chi_\lambda = a(\lambda) \chi_\lambda,
\]

with the reduced Coulomb-potential strength \( \lambda \) being the solution of the equation (we put the light-quark current mass equal to zero for simplicity)

\[
\lambda^2 = \frac{4}{3} \kappa^2 \left( a + 2\frac{\partial a}{\partial \lambda} \right),
\]

which is \( \lambda_0 = 1.215 \) for \( \alpha_s = 0.39 \) and \( \lambda_0 = 1.250 \) for \( \alpha_s = 0.4 \). The reduced effective mass \( \mu \) takes the value

\[
\mu = \frac{1}{2} \sqrt{\sigma} \left( \frac{\lambda_0}{\kappa} \right)^{3/2} \approx 0.7 \text{ GeV}
\]

and

\[
\{ r^n \} = (2\mu \sigma)^{-N/3} \int_0^\infty x^{N+2} \left| \chi_\lambda(x) \right|^2 dx.
\]

Then the difference of the masses of the two eigenstates corresponding to \( j_2 = 1/2 \) and \( j_2 = 3/2 \) is

\[
M_{1P3/2} - M_{1P1/2} = -\frac{3}{2} E_{s\alpha \beta},
\]

so that the picture of the splitting depends on the sign of the coefficient (11). Numerically this difference equals to +9 MeV for D mesons and +11 MeV for B’s. Of course the considered limit \( m_Q \rightarrow \infty \) is not realistic; it might be reasonably well justified for the b-quark, but not c-quark. In what follows the heavy-
quark mass is kept finite.

From Tables 4, 5 one can see that the predictions of our method for the masses of the 1P1 states are in good agreement with the lattice calculations [12] as well as with the experimental data. Namely, as far as the spectrum of D mesons is concerned, we have good coincidence with the results of CLEO [14] (see Table 4). In the B-mesonic spectrum we identify the state B1 with the mass \( m(B_1) = 5.71 \pm 0.02 \text{ GeV} \), recently claimed by CDF [18], with the lightest member of the \( J = 1 \) doublet, whereas the heaviest one can be associated with the resonance reported by OPAL [15].

---

1 We follow the standard notations using the total momentum of the light quark, \( j_2 = \hat{L} + \hat{S}_2 \), as the subscript.
Fig. 1. Splitting pattern for the $D$- (left plot) and $B$-mesonic (right plot) $P$ levels as a function of the heavy-quark mass. The vertical dashed line corresponds to $m_Q = m_c = 1.4$ GeV for the left plot and $m_Q = m_B = 4.8$ GeV for the right one, respectively.

(see Table 5). Unfortunately, experimental resolution does not enable one to disentangle both $P_1$ states simultaneously, so that at present time one rather has to rely on available lattice simulations. In such a situation other models, as well as improved lattice calculations, are welcome to attack this problem to have well established and clear predictions for experimentalists.

In Fig. 1 we give the splitting pattern for the $1P$ levels for both, $D$ and $B$, mesons for the heavy-quark mass varying from infinity to about 1.3 GeV with the vertical dashed line giving the actual masses of $c$ and $b$ quarks for $D$ and $B$ mesons, respectively. It is also worth mentioning that according to our model the $1P_0$ and $1P_2$ levels change their ordering around the heavy-quark mass $m_Q \approx 7.9$ GeV for the $D$-like meson (left plot in Fig. 1), and around $m_Q \approx 5.5$ GeV for the $B$-like one (right plot in Fig. 1).

In conclusion, let us briefly summarize the results reported in this Letter. We addressed the question on masses and splitting pattern of the $P$-level $D$ and $B$ mesons in the method of the QCD string with quarks at the ends. We took into account the proper dynamics of the QCD string, encoded in the so-called string correction, and supplied the interquark interaction with the one-loop corrections adapted to the case of the self-consistently generated dynamical masses of the quarks. Using the standard values for the string tension, the strong coupling constant and the current quark masses, we calculated the spectrum of $P$-level $D$ and $B$ mesons and found good agreement of our results with the lattice and experimental data, including those reported recently. Finally, we give our predictions for the splittings between the $P$ states.

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Best values for the CP-odd meson–nucleon couplings 
from supersymmetry

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Abstract

In the supersymmetric models, the dominant sources of the hadronic flavor-diagonal CP violation at low energy are the theta term and the chromoelectric dipole moments of quarks. Using QCD sum rules, we estimate the preferred range and the best values for the CP-odd meson–nucleon coupling constants induced by these operators. When the theta term is removed by the axion mechanism, the size of the most important isospin triplet pion–nucleon coupling is estimated to be $\bar{g}_{\pi NN}^{(1)} = 2 \times 10^{-12} (\tilde{d}_u - \tilde{d}_d)$, where chromoelectric dipole moments are given in units of $10^{-26}$ cm. © 2002 Published by Elsevier Science B.V.

The search for CP violation in flavor-conserving processes is of paramount scientific importance. The suppression of CP-violating effects induced by the complex phase of the Kobayashi–Maskawa matrix allows the use of electric dipole moments (EDMs) of neutrons or heavy atoms as well as T-odd asymmetries in the decays and scattering of baryons as powerful tools for probing new physics beyond the Standard Model.

The wide separation between the energy scale of "new physics" (superpartners, technicolor, etc.) and the characteristic momenta of particles in non-accelerator experiments permits consideration of only the first few terms in the effective CP-odd Lagrangian. In the minimal supersymmetric models only the theta term, three-gluon operator, and EDMs and color EDMs of light quarks are important:

$$\delta \mathcal{L} = - \sum_{q=u,d,s} m_q \bar{q} \left( 1 + i \theta_q \gamma_5 \right) q + \theta_G \frac{\alpha_s}{8\pi} G \tilde{G} + w G G \tilde{G} - \frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} F \sigma \gamma q - \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q \bar{q} G \gamma \gamma q,$$

where $G \tilde{G} \equiv G^{\mu \nu} \tilde{G}^{\mu \nu}$, $G \sigma \equiv t^a G^{\mu \nu} \sigma_{\mu \nu}$, and $GG \tilde{G} \equiv f_{abc} G^{\mu \nu} \tilde{G}^{\rho \epsilon} G_{\alpha \beta \gamma \delta}$. The coefficients in (1) are generated by the CP violation in the SUSY breaking sector and evolved down to 1 GeV, which is the border line of viability of the perturbative quark–gluon description.
In this Letter we present a systematic study of the transition from this Lagrangian to the effective T-odd meson–nucleon interactions which determine the magnitude of the CP-violating nuclear moments and the T-odd asymmetries in nucleon scattering. T-odd nuclear forces are the main source for the EDMs of heavy diamagnetic atoms (see, e.g., [1]). The quality of constraints imposed on supersymmetric models from a recently improved measurement of the EDM of the xenon and mercury atoms [2,3] as well as of future experimental efforts with the EDMs of diamagnetic atoms and the T-odd nucleon scattering [4] depends crucially on the treatment of QCD and nuclear effects, i.e., on the extraction of limits on $d_i$ from the experimental bound on the atomic EDM. The implications of this powerful constraint for the CP-violating nuclear moments can be easily computed, leading to the matrix element. For most of the models of CP violation in the effective Lagrangian, the CP-violating nuclear moments at vanishing pion momentum are proportional to the first power of the quark mass, whereas the pion propagator contains $1/m_q$. Thus, the quantum numbers of the $\bar{q}g_\sigma\gamma_q$ operators allow them to produce zero-momentum $\pi^0$'s from the vacuum. The pion–nucleon scattering amplitude at vanishing pion momentum is proportional to the first power of the quark mass, whereas the pion propagator contains $1/m_q$, so that Figs. 1(b) and 1(a) both contribute at the same order of chiral perturbation theory. Indeed, the quantum numbers of the $\bar{q}g_\sigma\gamma_q$ operators allow them to produce zero-momentum $\pi^0$'s from the vacuum.

The first step in the calculation of $\langle N|\bar{q}g_\sigma\gamma_q\sigma_q|N\rangle$ and $\langle N|\bar{q}g_\sigma\gamma_q\sigma_q|N\rangle$ is the reduction of the pion field by means of PCAC [13], Fig. 1(a). The smallness of the $t$-channel pion momentum compared to the characteristic hadronic scale justifies this procedure,

$$\langle N|\bar{q}g_\sigma\gamma_q\sigma_q|N\rangle = \frac{1}{f_\pi}\langle N|\bar{q}g_\sigma\gamma_q\sigma_q|N\rangle.$$  (3)

The commutator of the zero component of the axial current with CP-violating operators $O_{CP} = \bar{q}g_\sigma\gamma_q\sigma_q$ can be easily computed, leading to the matrix elements of the $\bar{q}g_\sigma\gamma_q\sigma_q$ operators over the nucleon state [13]. However, Eq. (3) is an incomplete result. A second class of contributions was pointed out in Ref. [6] and in Refs. [14,15] in the context of the neutron EDM problem. They consist of the pion pole diagrams, Fig. 1(b), which contribute at the same order of chiral perturbation theory. Indeed, the quantum numbers of the $\bar{q}g_\sigma\gamma_q\sigma_q$ operators allow them to produce zero-momentum $\pi^0$'s from the vacuum. The pion–nucleon scattering amplitude at vanishing pion momentum is proportional to the first power of the quark mass, whereas the pion propagator contains $1/m_q$, so that Figs. 1(b) and 1(a) both contribute at the same order of chiral perturbation theory.

Using the low energy theorems that relate the pion–nucleon scattering amplitude with the matrix elements of $m_q\bar{q}q$ over the nucleon state, we arrive at the following intermediate result for the $NN\pi^0$ vertex:

$$\frac{1}{2f_\pi}\langle N|\bar{d}_a(\bar{u}_aG\sigma u - m_u^2\bar{u}u) - \bar{d}_d(\bar{d}_dG\sigma d - m_d^2\bar{d}d)|N\rangle = m_u + m_d \left[ \frac{2\bar{\theta}}{m_\sigma} \right] \times (\langle N|\bar{u}u - \bar{d}d|N\rangle).$$  (4)

In this expression, $m_u = m_u/m_u + m_d$ and $m_d^2 = (0|\bar{q}g_\sigma\gamma_q\sigma_q|0)/\langle \bar{q}q \rangle = -0.8 \pm 0.1$ GeV$^2$ [17] para-
metrizes the strength of the quark–gluon dim = 5 vacuum condensate. In our case, this originates from the \( \langle \pi_0|\bar{q}g_5G\sigma\gamma_5q|0 \rangle \) matrix element and the minus sign is included into the definition of \( m_0^2 \) for convenience. An alternative way of obtaining the amplitude (4) is to chirally rotate quark masses to the basis where pions cannot be produced from the vacuum, \( \langle \pi^0|2\bar{q}m_\eta\gamma_5q+\sum \bar{q}g_5G\sigma\gamma_5q|0 \rangle = 0 \), while keeping the theta term fixed, \( \sum \theta_q = \text{const} \). This eliminates diagrams 1(b), but creates an additional contribution to 1(a), leading to the same result (4).

When the PQ mechanism is activated, removing the theta term, the minimum of the axion vacuum is shifted from \( \theta = 0 \) by the color EDM operators [16]. It turns out that the true minimum is such that the square bracket in Eq. (4) is zero so that only the first line survives. This leads to a relatively simple expression for the couplings

\[
\tilde{g}^{(0)}_{\pi NN} = \frac{\tilde{d}_u + \tilde{d}_d}{2\pi}(p|H_u - H_d|p),
\]

\[
\tilde{g}^{(1)}_{\pi NN} = \frac{\tilde{d}_u - \tilde{d}_d}{2\pi}(N|H_u + H_d|N),
\]

in terms of matrix elements of the \( H_u \) and \( H_d \) operators:

\[
H_u \equiv \bar{u}g_5G\sigma u - m_0^2\bar{u}u,
\]

\[
H_d \equiv \bar{d}g_5G\sigma d - m_0^2\bar{d}d.
\]

Previously, using a combination of QCD sum rules and scaling arguments, Ref. [13] estimated that \( \langle N|qg_5G\sigma q|N \rangle \sim \frac{3m_0^2}{2\pi^2}\langle N|\bar{q}q|N \rangle \). Another analysis [18] finds similar estimate \( \langle N|qg_5G\sigma q|N \rangle \sim m_0^2\langle N|\bar{q}q|N \rangle \). Obviously, these estimates are not sufficient to derive a reliable answer for \( \tilde{g}^{(0)}_{\pi NN} \) and \( \tilde{g}^{(1)}_{\pi NN} \) because of the additional \( -m_0^2\langle N|\bar{q}q|N \rangle \) contribution coming from diagrams 1(b). The danger of mutual cancellation between the two contributions was realized in Ref. [6] where the need for a dedicated analysis of \( \langle N|H_{ud}|N \rangle \) was emphasized. In the rest of this paper we derive the QCD sum rules [19] for the matrix elements of the \( H_{ud} \) operators. The advantage of this approach is that the operator product expansion (OPE) will contain similar vacuum condensates for both sources, \( \bar{q}g_5G\sigma q \) and \( -m_0^2\bar{q}q \), which allows us to trace possible cancellations. Following Refs. [15, 20], we introduce the generalized nucleon interpolator,

\[
\eta_p = (ji + j)q),
\]

which combines the two Lorentz structures, \( j_1 = 2\epsilon_{abc}(u_5^C\gamma_5d_5)\gamma_5c \) and \( j_2 = 2\epsilon_{abc}(u_5^C\gamma_5d_5)\gamma_5c \).

We compute the OPE for the correlator of this current in the presence of \( \bar{d}_u(d)H_{u(d)} \) external sources

\[
\Pi(Q^2) = \int d^4xe^{ip.x}(0)T\{\eta(N)x\bar{\eta}(0)\}|0\rangle_{\bar{d}_u(d)},
\]

where \( Q^2 = -p^2 \), with \( p \) the current momentum.

We limit our calculation to the Lorentz structure proportional to \( p \) because it is less susceptible to direct instanton contributions and excited resonances than the chirally even structure proportional to \( I \). Relevant diagrams for this correlator are shown in Fig. 2. After a straightforward calculation, we find

\[
\rho\rho_{\text{OPE}}(Q^2) = \rho(\bar{q}q)\left[\ln\left(\frac{\Lambda_{\text{UV}}^2}{-p^2}\right)\right]_{\text{LO}} + \frac{1}{p^2}\ln\left(\frac{-p^2}{\Lambda_{\text{IR}}^2}\right)_{\text{NLO}}
\]

\[
+ \frac{1}{p^2}\ln\left(\frac{-p^2}{\Lambda_{\text{NNLO}}^2}\right)
\]

The leading-order term is given by the diagrams 2(a), 2(b),

\[
\pi_{\text{LO}} = \frac{3m_0^2}{64}\left[d_+(5 - 2\beta - 3\beta^2) + d_-(1 - \beta)^2\right]
\]

Here we have introduced the combinations \( d_+ = \tilde{d}_u + \tilde{d}_d \) and \( d_- = \tilde{d}_u - \tilde{d}_d \). It turns out that the diagrams 2(a), where the external source enters through the \( (0)\bar{q}\delta\gamma_5q(0) \) structure, give large and opposite sign contributions for the \( g_5\bar{q}G\sigma q \) and \( -m_0^2\bar{q}q \) sources so that their combined effect in \( H_q \) is nil. Fortunately, this cancellation does not hold for diagrams 2(b) that give (10).

The next-to-leading term corresponds to diagrams 2(c),

\[
\pi_{\text{NLO}} = \frac{3m_0^2}{32}(1 - \beta^2) - \frac{9}{2}\left(7 - 2\beta - 5\beta^2\right)
\]

\[
+ \frac{d_+}{96}g_5^2GG(1 - \beta)^2
\]

which contribute to the OPE (9) with the log of the infrared cutoff \( \Lambda_{\text{IR}} \). \( g_5^2GG \approx 0.4 - 1 \text{ GeV}^4 \) is
the vacuum gluon condensate. The next-to-next-to-leading order
\[ \Pi^{\text{NNLO}} = \frac{d_+}{24} \left[ \chi_s \pi^2 (1 + 2\beta - 3\beta^2) ight. \\
+ \frac{3m_0^4}{16} (13 - 2\beta - 11\beta^2) \\
+ \left. \frac{d_+}{24} \left[ \chi_T \pi^2 (1 - \beta)^2 + \frac{m_0^4}{16} (-11 - 14\beta + \beta^2) \\
- \frac{g_s^2 (GG)}{24} (5\beta^2 + 8\beta - 1) \right] \right] \] (12)
contains the vacuum polarizabilities,
\[ \chi_{S,T} = \int \! d^4x \langle 0| T \left[ \bar{u}u \pm \bar{d}d(x), H_0 \pm H_0(0) \right]|0 \rangle, \] (13)
and the vacuum factorization assumption has been made in (12).

The sum rules prescription involves matching the OPE with the phenomenological part, \( \Pi^{\text{OPE}}(Q^2) = \Pi^{\text{phen}}(Q^2) \), where
\[ \rho \Pi^{\text{phen}} = \rho \left( \frac{2\lambda^2 g_{\pi NN} m_N}{(p^2 - m_N^2)^2} + \frac{A}{p^2 - m_N^2} \ldots \right) \] (14)
contains double and single pole contributions, and the continuum. After Borel transformation of the sum rule
\( \Pi^{\text{OPE}}(Q^2) = \Pi^{\text{phen}}(Q^2) \) we obtain
\[ \frac{\langle \bar{q}q \rangle}{\pi^2 f_{\pi}} \left[ \frac{\pi^{\text{LO}} E_0}{M^2} - \frac{\pi^{\text{NLO}}}{M^2} \left( \ln \frac{M^2}{\Lambda_{\text{IR}}} \right) - 0.58 \right] \\
- \frac{\pi^{\text{NNLO}}}{M^2} \right] = M^{-4} \exp \left[ -\frac{m_N^2}{M^2} \right] (2\lambda^2 m_N g_{\pi NN} + M^2) \]
\[ + M^{-2} B \exp \left[ -\frac{s}{M^2} \right]. \] (15)
Here \( s \) is the continuum threshold and \( E_0 = 1 - e^{-s/M^2} \). \( A \) and \( B \) parametrize the contribution of excited states and are assumed to be independent of \( M \).

It is reasonable to start the numerical treatment from a simple estimate, à la Ioffe [21], which assumes the dominance of the ground state and the LO OPE term, and eliminates \( \lambda \) using the nucleon mass sum rule for \( \bar{p} \). Separating different isospin structures, we find
\[ g_{\pi NN}^{(1)} = (\bar{d}_u - \bar{d}_d) \frac{3}{2} \frac{m_N f_{\pi}}{m_N} \rho_{\bar{q}q}(\bar{q}q) |m_N^2| F_1(\beta), \] (16)
\[ g_{\pi NN}^{(0)} = (\bar{d}_u + \bar{d}_d) \frac{3}{10} \frac{m_N f_{\pi}}{m_N} \rho_{\bar{q}q}(\bar{q}q) |m_N^2| F_0(\beta). \] (17)
Here \( F_1(\beta) = (5 - 2\beta - 3\beta^2)/(5 + 2\beta + 5\beta^2) \) and \( F_0(\beta) = 5(1 - \beta)^2/(5 + 2\beta + 5\beta^2) \). \( F_1(0) = F_0(0) = 1 \). To get numerical estimates, we choose \( \beta = 0 \), extensively used in lattice simulations. It is well known that the \( j_1 \) current has a much better overlap with the nucleon ground state and \( \lambda_1 \gg \lambda_2 \). Substituting \( M = 1 \) GeV, we obtain
\[ g_{\pi NN}^{(1)} = 3 \times 10^{-12} \frac{\bar{d}_u - \bar{d}_d}{10^{-26} \text{cm}} \times \frac{|\langle \bar{q}q \rangle|}{(225 \text{MeV})^3} \frac{|m_N^2|}{0.8 \text{GeV}^2}. \] (18)
\[ g^{(0)}_{\pi NN} = 0.6 \times 10^{-12} \frac{\tilde{d}_u + \tilde{d}_d}{10^{-26} \text{ cm}} \times \frac{|\tilde{\eta}|}{(225 \text{ MeV})^3} \times 0.8 \text{ GeV}^2. \]  

(19)

In most SUSY models, \( d_{u(d)} = \) loop factor \( \times M^{-2}_{\text{SUSY}} \times \) a linear combination of \( m_u \) and \( m_d \). When combined with \( \tilde{\eta} \eta \) from Eqs. (18), (19), this forms \( m_\pi^2 f_{\pi}^2 \) times a function of \( m_u/m_d \), thus eliminating a major source of uncertainty in EDM calculations due to the poor knowledge of \( m_u + m_d \) [14,15,20]. The estimate (18) is twice smaller than the value of \( g^{(0)}_{\pi NN} \) used in [6].

Also in agreement with [6], Eqs. (18), (19) suggest that \( g^{(0)}_{\pi NN}/g^{(1)}_{\pi NN} \sim 0.2d_+/d_- \).

For a more systematic analysis, one has to include NLO and NNLO terms in the OPE. Here, we immediately face the problem of the unknown vacuum condensates \( \chi_{S,T} \). Even though the vacuum correlators \( \langle \tilde{\eta} \eta, \tilde{\eta} \eta \rangle \) can be determined using chiral perturbation theory [22], there is no direct information on \( \langle \tilde{\eta} \eta, \tilde{\eta} \eta, G \sigma \eta \rangle \) other than that it is likely to be comparable with \( m_\pi^2 \langle \tilde{\eta} \eta, \tilde{\eta} \eta \rangle \). At this point we would like to take advantage of the possibility to choose \( \beta \) in such a way as to minimize higher order terms in the OPE. We note that \( \chi_S \) in (12) is multiplied by \( 1 + 2\beta - 3\beta^2 \) which becomes 0 at \( \beta = -1/3 \) and 1. The choice of \( \beta = 1 \) also suppresses the leading order, while \( \beta = -1/3 \) maximizes it. For the expected size of \( \chi_S \) [22], \( \chi_S \sim 0.16 \times m_\pi^2 |\tilde{\eta}| f_{\pi}^{-4} \), we can choose \( \beta \) in a range such that the whole square bracket in front of \( d_- \) in Eq. (12) is zero. This gives a range of interpolating currents around \( \beta = -1/3, -0.5 < \beta < 0 \),

(20)

where we can tune the NNLO terms to zero in the \( g^{(0)}_{\pi NN} \) channel. Variation of \( \beta \) in this range contributes to an estimate of the uncertainty in our analysis. In the \( g^{(1)}_{\pi NN} \) channel there is no obvious choice of \( \beta \) that would remove \( \chi_T \) and leave the leading order term unsuppressed, so we will choose the same \( \beta \) as for \( g^{(0)}_{\pi NN} \). We note that this range is close to \( \beta = 0 \) as used for (18), (19). One should also worry about the dependence on \( \Lambda_R \) in NLO. Remarkably, in the range (20), this dependence is softened by cancellation of the \( m_d^4 \) and \( g^{2}_{\pi} (GG) \) terms.

The preferred range for \( g^{(1)}_{\pi NN} \) and \( g^{(0)}_{\pi NN} \) is determined according to the following procedure. We take

the OPE side of (15) at the lower point of the usual Borel window, \( M^2 = 0.8 \text{ GeV}^2 \), and vary it through the range of parameters \(-0.5 \leq \beta \leq 0, 300 \text{ MeV} \leq \Lambda_R \leq 500 \text{ MeV}, 0.7 \text{ GeV}^2 \leq |m_0^2| \leq 0.9 \text{ GeV}^2, 0.4 \text{ GeV}^4 \leq g_{\pi}^4 \leq 1 \text{ GeV}^4, \) and \( 2 \text{ GeV}^2 \leq s \leq 3 \text{ GeV}^2, 0.8 \text{ GeV}^6 \leq (2\pi)^2 \lambda_+^2 \leq 0.9 \text{ GeV}^6 \) and finally \(-6 \text{ GeV}^{-1} \leq \chi_T/|\tilde{\eta}| \leq 6 \text{ GeV}^{-1}. \) On the r.h.s of (15) we assume the dominance of the double pole contributions for \( M^2 = 0.8 \text{ GeV}^2 \) and allow for a 50% correction due to the presence of the unknown parameters \( A \) and \( B \), thus effectively widening the allowed range for \( g^{(0)}_{\pi NN}. \) As expected, the couplings are most sensitive to the value of \( m_d^2 \). The final results are presented in Table 1. Our “best” value for \( g^{(1)}_{\pi NN} \) is determined by averaging over \( \beta \) and choosing the central values for condensates, which also suppresses the logarithmic term. In order to separate the contribution of \( g^{(1)}_{\pi NN} \) from the \( A \) and \( B \) terms, we impose a relation among \( A, B \) and \( \pi^{10} \) obtained by requiring the same large \( M^2 \) asymptotic behavior for both sides of (15). The resulting sum rule is fitted numerically and produces a result 1.5 times smaller than the naive estimate (18). For \( g^{(0)}_{\pi NN} \) the best value cannot be determined as the OPE side changes sign depending on the value of \( \chi_T \).

Also included in this table are the preferred ranges for the CP-odd couplings of nucleons with \( \eta, \rho \) and \( \omega \) mesons. The couplings with \( \rho \) and \( \omega \) have the EDM-like structures \( -\frac{1}{2} N (\partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}) \sigma_{\mu\nu} \gamma_5 N \) with properly arranged isospin indices. They can be

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Preferred range</th>
<th>Best value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-10) \rightarrow ) ( d_{-26} )</td>
<td>2 ( d_{-26} )</td>
</tr>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-0.5) \rightarrow ) ( d_{+26} )</td>
<td>-</td>
</tr>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-1) \rightarrow ) ( d_{+26} )</td>
<td>-</td>
</tr>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-0.3) \rightarrow ) ( d_{-26} )</td>
<td>-</td>
</tr>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-0.5) \rightarrow ) ( d_{+2} )</td>
<td>1.4 ( d_+ )</td>
</tr>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-0.9) \rightarrow ) ( d_{-} )</td>
<td>0.9 ( d_- )</td>
</tr>
<tr>
<td>( g^{(0)}_{\pi NN} \times 10^{12} )</td>
<td>((-2) \rightarrow ) ( d_{-} )</td>
<td>-1.4 ( d_- )</td>
</tr>
</tbody>
</table>
easily extracted from the calculation of the neutron EDM $d_n$, induced by $\tilde{d}_{u(d)}$ [15] after a simple reassignment of charges for the external vector currents. Best values for $\bar{g}_{\rho NN}$ and $\bar{g}_{\omega NN}$ follow from the central values of $d_n(\tilde{d}_u, \tilde{d}_d)$ given in [15]. Finally, the coupling to the $\eta$ meson is dominated by the strange quark chromoelectric dipole moment $\tilde{d}_s$ in the isospin-singlet and by $\tilde{d}_u - \tilde{d}_d$ in the isospin-triplet channels, and in both cases only the expected range can be quoted.

In conclusion, we have shown that the size of the CP-odd pion–nucleon constant generated by quark chromoelectric dipole moments is given by the matrix element of $\bar{q}g_{\sigma\rho}G^\sigma q - m_0^2 \bar{q}q$ over the nucleon state. We have constructed a QCD sum rule for this matrix element and determined the preferred range and the best value for the $\bar{g}_{\pi NN}^{(1)}$ coupling. The upper part of the preferred range agrees with previous estimates. However, in the interpretation of the experimental limit on the EDM of the mercury atom [3] in terms of limits imposed on new CP-violating physics, a more conservative value $\bar{g}_{\pi NN}^{(1)} = (\tilde{d}_u - \tilde{d}_d)/10^{-14}$ cm should be used. This translates the result of Ref. [3] (see [1,6] for details) into the bound $|\tilde{d}_u - \tilde{d}_d| < 2 \times 10^{-26}$ cm. This constraint provide a sensitive probe of CP violation in the supersymmetric spectrum up to $M_{\text{SUSY}}$ of few TeV.

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References

Field renormalization constant for unstable particles

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Abstract

A recently proposed normalization condition for the imaginary part of the self-energy of an unstable particle is shown to lead to closed, exact expressions for the renormalized self-energy and propagator, which are consequently free of ultraviolet divergences to all orders in perturbation theory. In turn, the corresponding closed expressions for the mass and field renormalization counterterms are necessary in some important cases to avoid power-like infrared divergences in high orders of perturbation theory. In the same examples, the width plays the rôle of an infrared cutoff, and, consequently, the field renormalization counterterm is not an analytic function of the coupling constant. © 2002 Elsevier Science B.V. All rights reserved.

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The unrenormalized transverse propagator of a gauge boson is of the form:

\[ D^{(u)}_{\mu\nu}(s) = -iQ_{\mu\nu} \frac{s}{s - M_0^2 - A(s)}, \]  

where \( Q_{\mu\nu} = g_{\mu\nu} - q_{\mu}q_{\nu}/s, \) \( q_{\mu} \) is the four-momentum, \( s = q^2, \) \( M_0 \) is the bare mass, and \( A(s) \) is the unrenormalized self-energy. An analogous expression holds for a scalar boson, with \( -iQ_{\mu\nu} \rightarrow i. \) The complex position of the propagator’s pole is given by

\[ \bar{s} = M_0^2 + A(\bar{s}). \]  

Combining Eqs. (1) and (2), we have

\[ D^{(u)}_{\mu\nu}(s) = \frac{-iQ_{\mu\nu}}{s - \bar{s} - [A(s) - A(\bar{s})]}, \]  

Parameterizing \( \bar{s} = m_2^2 - im_2\Gamma_2, \) where we employ the notation of Ref. [1], and considering the real and imaginary parts of Eq. (2), we see that

\[ m_2^2 = M_0^2 + \text{Re} A(\bar{s}), \]  

\[ m_2\Gamma_2 = -\text{Im} A(\bar{s}). \]  

If \( m_2 \) is identified with the renormalized mass, Eq. (4) tells us that the mass counterterm is given by \( \delta m_2^2 = \text{Re} A(\bar{s}). \) This is to be contrasted with the conventional mass renormalization

\[ M^2 = M_0^2 + \text{Re} A(M^2). \]  

where \( M \) is the on-shell mass. The great theoretical advantage of using \( m_2 \) and \( \Gamma_2 \) as the basis to define mass and width is that they are intrinsically gauge-independent quantities, while \( M \) is known to be gauge dependent in next-to-next-to-leading order [1,2].

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The renormalized propagator $D_{\mu\nu}^{(r)}(s)$ is obtained by dividing Eq. (1) by the field renormalization constant $Z = 1 - \delta Z$. Recalling Eq. (4), one readily obtains
\begin{equation}
D_{\mu\nu}^{(r)}(s) = \frac{-i Q_{\mu\nu}}{s - m_f^2 - S(s) + \Re \delta S(s) - \delta Z(s - m_f^2)},
\end{equation}
where
\begin{equation}
S(s) = Z A(s).
\end{equation}
Thus, the renormalized self-energy is given by
\begin{equation}
S^{(r)}(s) = S(s) - \Re \delta S(s) + \delta Z(s - m_f^2),
\end{equation}
where the second and third terms are identified with the mass renormalization parameter,
\begin{equation}
\delta m_f^2 = Z \delta m^2 = \Re \delta S(s),
\end{equation}
and the field renormalization counterterm, respectively. Since the hermiticity of the Lagrangian density requires these counterterms to be real, $\delta Z$ must be chosen in such a way that, for real $s$, $\Re S^{(r)}(s)$ is ultraviolet convergent to all orders of perturbation theory. Once this is done, $\Im S^{(r)}(s) = \Im S(s) = Z \Im A(s)$ must also be ultraviolet convergent to all orders, since there are no further counterterms available. This means that $\delta Z$ can also be determined by choosing a suitable normalization condition on $\Im S(s) = Z \Im A(s)$.

Recently, a novel normalization condition for
\begin{equation}
\Im S(m_f^2) = Z \Im A(m_f^2) = -m_f^2 \Gamma_2,
\end{equation}
was proposed independently in Eqs. (22) and (23) of Ref. [3] and in Ref. [4]. The above relation between the width and the self-energy is known to be valid at the one-loop order. Eq. (11) extends its validity as an exact normalization condition, valid to all orders. While the objective of Ref. [3] was to solve the notorious problem of threshold singularities in the conventional definition of $Z$ [5], that of Ref. [4] was to provide a second normalization condition for the systematic order-by-order removal of ultraviolet divergences in $S^{(r)}(s)$.

An interesting feature of the analysis of Ref. [3] is that it leads to closed, exact expressions for $Z$. This may be understood immediately by combining Eq. (11) with Eqs. (5) and (8)
\begin{equation}
\Im S(m_f^2) = \Im A(s) = \frac{\Im \delta S(s)}{Z}.
\end{equation}
Thus,
\begin{equation}
Z = \frac{\Im S(s)}{\Im S(m_f^2)} = 1 - \frac{\Im \delta S(s) - \Im S(m_f^2)}{m_f^2 \Gamma_2},
\end{equation}
where we again employed Eq. (11). Eq. (13) tells us that, once the normalization condition of Eq. (11) is adopted, the field renormalization counterterm is given by the closed expression
\begin{equation}
\delta Z = \frac{\Im \delta S(s) - \Im S(m_f^2)}{m_f^2 \Gamma_2}.
\end{equation}
In particular, the renormalized self-energy may be written as
\begin{equation}
S^{(r)}(s) = S(s) - \Re \delta S(s) + \frac{\Im \delta S(s) - \Im S(m_f^2)}{m_f^2 \Gamma_2} (s - m_f^2).
\end{equation}
Combining Eq. (13) with Eq. (8), $Z$ may be also expressed in terms of the unrenormalized self-energy, as
\begin{equation}
Z = \frac{1}{1 + \Im A(s) - \Im A(m_f^2)}/(m_f^2 \Gamma_2),
\end{equation}
the expression given in Eq. (23) of Ref. [3]. As explained in that paper, Eq. (16), in conjunction with Eq. (5), leads to $Z \Im A(m_f^2) = -m_f^2 \Gamma_2$ as a mathematical identity, a result that coincides with the normalization condition of Eq. (11).

In general, $\Gamma_2 = O(g^2)$, where $g$ is a generic gauge coupling. If $\delta M^2 = \Re \delta S(s)$ and $\delta Z = \Im \delta S(s) - \Im S(m_f^2)/(m_f^2 \Gamma_2)$ admit expansions in powers of $\Gamma_2$, one readily obtains the expressions for the counterterms to all orders. For instance,
\begin{equation}
\delta M^2 = R - R' = \frac{I_2^3}{2} R'' + \frac{I_3}{6} I^{(3)} + \frac{I_4}{24} R^{(4)} + \cdots,
\end{equation}

\begin{equation}
\delta Z = -R' + \frac{I_2}{2} I'' + \frac{I_3^2}{6} R^{(3)} - \frac{I_4^3}{24} R^{(4)} + \cdots,
\end{equation}

where
\begin{equation}
I_n = (-1)^n \frac{1}{(2n)!} \left[ \frac{2n}{2n} R^{(2n)} - \frac{I^{2n+1}}{2n+1} I^{(2n+1)} \right].
\end{equation}
where $R \equiv \text{Re} S(s)$, $I \equiv \text{Im} S(s)$, the primes and superscripts $(n)$ indicate derivatives with respect to $s$, and all the functions are evaluated at $s = m_2^2$. Separating out the contributions of the photon $R$ associated with the photon \[6\]. This leads to a logarithmic analysis valid in the narrow-width approximation, in terms of the form $I(n)$ and $R(n)$ for $n = 3, 5, \ldots$, and $n = 2, 4, \ldots$, respectively.

These catastrophic problems can be neatly avoided by employing the closed, exact expressions of Eqs. (10) and (14) to treat such contributions. For instance, $c(s - \bar{s}) \ln(\bar{s} - s)/\bar{s}$ contributes zero to $\delta M^2$ [cf. Eq. (10)] and $-\ln a$, where $a = \Gamma_2/\sqrt{m_2^2 + \Gamma_2^2}$, to $\delta Z$ [cf. Eq. (14)]. Thus, in this contribution, the width plays the role of an infrared cutoff. Since generally $\Gamma_2 = O(g^2)$, we also see that $\delta Z$ is not an analytic function of $g^2$ in the neighborhood of $g^2 = 0$.

An expression for the renormalized propagator, alternative to Eq. (7), is obtained by dividing Eq. (3) by $Z$:

$$D^{(R)}_{\mu\nu}(s) = \frac{-iQ_{\mu\nu}}{s - \bar{s} - [\bar{S}(s) - S(\bar{s})] - \delta Z(s - \bar{s})}.$$  \hspace{1cm} (19)

Writing \[6\]

$$\frac{s - \bar{s}}{\bar{s}} = \rho(s)e^{\theta(s)},$$  \hspace{1cm} (20)

where

$$\rho(s) = \frac{1}{m_2} \sqrt{\frac{(s - m_2^2)^2 + m_2^2 \Gamma_2^2}{m_2^2 + \Gamma_2^2}}$$

$$\rho(s) \sin \theta(s) = \frac{s \Gamma_2}{m_2(m_2^2 + \Gamma_2^2)} (-\pi \leq \theta \leq \pi),$$  \hspace{1cm} (21)

and using Eq. (14), the contribution of $c(s - \bar{s}) \ln(\bar{s} - s)/\bar{s}$ to Eq. (19) becomes

$$-iQ_{\mu\nu} \frac{1}{(s - \bar{s})(1 - c[\ln \rho(s) - \ln a + i\theta(s)])}.$$  \hspace{1cm} (22)

At $s = m_2^2$, we have $\rho(m_2^2) = a$ and $\sin \theta(m_2^2) = -m_2/\sqrt{m_2^2 + \Gamma_2^2}$, so that the expression between curly brackets in Eq. (22) is purely imaginary. Furthermore, $\theta(m_2^2) \approx -\pi/2$ in the case $\Gamma_2 \ll m_2$. Thus, with the choice of Eq. (14), the role of $\delta Z$ is to cancel the
contribution $\ln \rho(s)$ at $s = m_2^2$, which depends logarithmically on $F_2$. These results may be understood directly by setting $s = m_2^2$ in Eqs. (7) or (19) and recalling Eqs. (11) and (20). On the other hand, far away from the resonance region, i.e., for $|s - m_2^2| \gg m_2^2 F_2$, $\ln \rho(s)$ does not depend logarithmically on $F_2$, while $\ln[\rho(s)/a]$ does. The latter feature is not surprising, since away from the resonance region $S(s)$ has a regular behaviour, while $\delta Z$ is logarithmically divergent in the limit $F_2 \to 0$.

As mentioned before, the original motivation that led to Eq. (16) or its equivalent, Eq. (13), was to solve the problem of threshold singularities in the evaluation of $Z$, which occurs when the mass of the unstable particle is degenerate with the sum of masses of a pair of contributing virtual particles [3]. Since $m_\gamma = m_\gamma = 0$, the contributions of the $(W, \gamma)$ and $(q, g)$ virtual pairs to the $Z$ factors of the $W$ boson and the unstable quark $q$ are particular, albeit very important, examples of threshold singularities. It is for this reason that the definition of the $Z$ factor, embodied in the closed, exact expressions of Eqs. (13) and (16), provides a consistent formulation to treat the associated “infrared contributions”.

In summary, we have shown that the normalization condition of Eq. (11) leads to closed, exact formulae for the self-energy [Eq. (15)] and the renormalized propagator [Eqs. (7) and (14)] of an unstable particle, which are consequently free of ultraviolet divergences to all orders. Furthermore, in some important cases, the corresponding closed expressions for the mass counterterm [Eq. (10)] and the field renormalization counterterm [Eq. (14)] are necessary in order to treat contributions that would lead to catastrophic, power-like infrared divergences if expanded perturbatively beyond the narrow-width approximation.

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References

On the $B \to X_s l^+ l^-$ decays in general supersymmetric models

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Abstract

We analyze the inclusive semileptonic decays $B \to X_s l^+ l^-$ in the framework of the supersymmetric standard model with non-universal soft-breaking terms at GUT scale. We show that the general trend of universal and non-universal models is a decreasing of branching ratio (BR) and increasing of energy asymmetry (AS). However, only non-universal models can have chances to get very large enhancements in BR and AS, corresponding to large (negative) SUSY contributions to the $b \to s \gamma$ amplitude.

Flavor changing neutral current (FCNC) and CP violating phenomena can be considered as one of the best indirect probe for physics beyond the standard model (SM). Due to the absence of tree level FCNCs in the SM and the suppression of the Glashow–Iliopoulos–Maiani mechanism, they are particularly sensitive to any non standard physics contribution.

In the framework of low energy supersymmetric (SUSY) models, FCNC processes play an important role in severely constraining the soft breaking sector of supersymmetry [1]. As known, these constraints require an high degree of degeneracy in the squark mass matrices, suggesting that the mechanism which transmits the SUSY breaking to the observable sector should be flavour blind. For instance, minimal supergravity (mSUGRA) and gauge-mediation mechanisms successfully explain this degeneracy. In particular, in mSUGRA scenario all the tests on FCNCs can be satisfied due to the assumption of universality for the soft breaking terms at GUT scale. However, recently there has been a growing interest concerning supersymmetric models with non-universal SUSY soft-breaking terms [2]. This is motivated by the fact that superstring inspired models, where supergravity theories are derived, naturally favour non-universality in the soft-breaking terms [3]. This is mainly due to the fact that superstring theories live in extra dimensions and after compactification, non-flat Kähler metrics and flavour-dependent SUSY soft-breaking terms can arise.

Particularly interesting among this class of models are the ones with non-universal trilinear soft-breaking terms in the scalar sector, the so-called $A$-terms. These models can have interesting phenomenological consequences. They could solve in principle the SUSY CP

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are several studies about inclusive ones. For this reason we will restrict our analysis to the dependent calculations of hadronic matrix elements. Uncertainties than the inclusive ones due to model exclusive processes are affected by larger theoretical are very close the SM expectations \[8,9\]. However, these decay channels is quite exciting. The BELLE problem, satisfy all the FCNC constraints, and provide at the same time new significant contributions to the direct CP violating parameter \(\varepsilon'/\varepsilon\) as suggested by the recent experimental results on \(\varepsilon'/\varepsilon\) \[4\]. Moreover, it has been argued that this class of models should also pass the strong constraint on \(B \rightarrow X_s\gamma\) decay \[5\] and gives rise to a large contribution to the CP asymmetry, of order 10–15% which can be accessible at \(B\) factories \[6\].

In this Letter we analyze the impact of a large class of supersymmetric models with non-universal soft SUSY breaking terms (which is motivated by the string inspired scenarios) in the semileptonic (inclusive) \(B \rightarrow X_l l^+ l^-\) decays (with \(l = e,\mu\)). As for the \(B \rightarrow X_s\gamma\) decay, these processes are also very interesting for several reasons: first they are very sensitive to large \(\tan\beta\) since they involve the magnetic dipole operator \(Q_7\) (see Eq. (3)) which allows the quark \(b \rightarrow s\gamma\) transition. Second, they involve other operators as well, the semileptonic operators \(Q_9\) and \(Q_{10}\) (see Eqs. (5), (6)), and so can serve as complementary tests of the model. Third, they provide several measurable quantities, such as branching ratios and asymmetries. At present these decays are known in QCD at the next-to-leading (NLO) order logarithmic accuracy for the SM \[7\], and also \(1/m_b\) nonperturbative contributions are small and well under control.

From the experimental side, the situation about these decay channels is quite exciting. The BELLE experiment has recently announced the first evidence for the exclusive process \(B \rightarrow K^* l^+ l^-\) \[8\], and upper bounds for the three body decays \(B \rightarrow (K, K^*) + (e^+ e^-, \mu^+ \mu^-)\), reported by BABAR and BELLE, are very close the SM expectations \[8,9\]. However, exclusive processes are affected by larger theoretical uncertainties than the inclusive ones due to model dependent calculations of hadronic matrix elements. For this reason we will restrict our analysis to the inclusive ones.

In the framework of supersymmetric models, there are several studies about \(B \rightarrow X_s l^+ l^-\) decays in the literature \[10–15\]. However, a detailed analysis about SUSY models with non-universal soft breaking terms at GUT scale has not been considered. As suggested by a recent study \[10\], based on the low energy approach to supersymmetric models, the non-universality in the soft-breaking sector could generate significant departures from the SM in the semileptonic \(B \rightarrow X_s l^+ l^-\) decays. In this analysis the mass insertion method has been used, where the pattern of flavour change is parametrized by the ratios

\[
(\delta_{ij})_{AB} = \frac{(m_{ij}^2)_{AB}}{M_{sq}^2},
\]

where \((m_{ij}^2)_{AB}\) are the off-diagonal elements of the \(f = \bar{u}, d\) scalar mass squared matrix which mixes flavour \(i, j\) for both left- and right-handed scalars \((A, B = \text{left, right})\), and \(M_{sq}\) is the average squark mass. The main conclusion of this work is that FCNCs constraints and vacuum stability bounds, which strongly constrain these \(\delta s\), could not prevent large effects in \(B \rightarrow X_s l^+ l^-\) decays. In particular, large SUSY contributions to the Wilson coefficients \(C_9\) and \(C_{10}\) at EW scale, corresponding, respectively, to the semileptonic operators \(Q_9\) and \(Q_{10}\), are possible. Therefore, generic SUSY models (with non-universalities in the scalar sector and \(A\)-terms implemented at GUT scale) seem, indeed, an ideal scenario where these large effects could be found. However, it should be stressed that in the analysis of Ref. \[10\], the enhancement of \(C_9\) and \(C_{10}\) is obtained by taking all the \(\delta s\) and other SUSY parameters at low energy as free parameters, in particular the gluino, the lightest stop mass and the bilinear Higgs couplings (the \(\mu\) term). In the class of models analyzed here, we will see that these sizable effects to \(C_9\) and \(C_{10}\) will not show up, leaving to potential large deviations only in the Wilson coefficient \((C_7)\) of the magnetic-dipole operator \(Q_7\). The main reason is due to the fact that the relevant (low energy) SUSY parameters for enhancing \(C_9\) and \(C_{10}\) are strongly correlated, leaving the \(B \rightarrow X_s\gamma\) and the experimental bounds on SUSY mass spectrum very effective in preventing such enhancements.

Furthermore, we will consider the effect of the SUSY models with non-abelian flavour symmetry on these semileptonic decays. The main effect of this symmetry is to prevent excessive FCNC effects in case that the mechanism of SUSY breaking should not be flavour blind. As a specific example, we will analyze here the model proposed in Ref. \[16\], in which the pattern of flavour change is implemented by the breaking of an \(U(2)\) (horizontal) flavour symmetry. For the same reason given above, also these models have large potentialities to give sizable deviations in
$B \to X_s l^+ l^-$ decays, since they contain a new flavour structure in addition to Yukawa matrices. However, we will see that the same conclusions about sizable contributions to $C_9$ and $C_{10}$ will hold for these models as well.

Now we start with the SM results for the inclusive $B \to X_s l^+ l^-$ decays. Inclusive hadronic rates in $B$ meson decays can be precisely calculated by using perturbative QCD and $1/m_b$ quark expansion. The effective Hamiltonian for the $b$ quark semileptonic decay $b \to s l^+ l^-$ is given by

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum_{i=1}^{10} C_i(\mu_b) Q_i(\mu_b),$$

(2)

where $Q_i(\mu)$ are the $\Delta B = 1$ transition operators evaluated at the renomalization scale $\mu \simeq \mathcal{O}(m_b)$.

A complete list of operators involved in this decay are given in Refs. [7,17]. The relevant operators that can be affected by the SUSY contributions are given by

$$Q_7 = \frac{e}{16\pi^2} m_b s_L \sigma^{\mu \nu} b_R F_{\mu \nu},$$

(3)

$$Q_8 = \frac{g}{16\pi^2} m_b s_L T^a \sigma^{\mu \nu} b_R G_{\mu \nu},$$

(4)

$$Q_9 = (\bar{s} L_y \gamma_\mu b_L) \tilde{t} \gamma^\mu t,$$

(5)

$$Q_{10} = (\bar{s} L_y \gamma_\mu b_L) \tilde{b} \gamma^\mu b.$$

(6)

At one-loop, the SUSY contributions to these operators are given by $Z$ and $\gamma$ super-penguin and box diagrams, where inside the loop can run charged Higgs, charginos, gluinos, neutralinos, squarks, and sleptons [13,14].

The general SUSY Hamiltonian also contains the operators $\tilde{Q}_i$ which have opposite chirality with respect to the $Q_i$ ones. In the SM and minimal flavor SUSY models, these contributions are suppressed by $\mathcal{O}(m_{\tilde{g}}/m_b)$ However, in generic SUSY models, and in particular, in case of non-degenerate $A$-terms this argument is no longer true. For instance, the gluino contribution to these operators depend on $(\delta_{23}^d)_{LR}$ and $(\delta_{31}^d)_{RL}$. Here both of these mass insertions are linear combinations of the down type quark masses rather than $m_b$ or $m_s$ exclusively. Therefore, to be consistent, one has to include the contributions of these operators. Indeed, the effects of the operators $\tilde{Q}_{7,8}$ have been found to be very significant for the branching ratio of the $B \to X_s \gamma$ decay [5] and for the CP asymmetry of this decay as well [6]. The Wilson coefficients $C_i(\mu)$ can be decomposed as

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \mathcal{O}(\alpha_s^2),$$

(7)

where $C_i^{(0)}$ and $C_i^{(1)}$ refer to the LO and NLO results, respectively. For our purpose, the SUSY corrections from including the NLO and NNLO are unimportant. The new physics effects in $b \to s l^+ l^-$ can be parametrized by $R_i$ and $\tilde{R}_i$, $i = 7, 8, 9, 10$ defined at the EW as

$$R_i = \frac{C_i^{(0)} - C_i^{(0,\text{SM})}}{C_i^{(0,\text{SM})}}, \quad \tilde{R}_i = \frac{\tilde{C}_i^{(0)} - C_i^{(0,\text{SM})}}{C_i^{(0,\text{SM})}}.$$

(8)

Note that there is no SM contribution to $\tilde{C}_i$. In the minimal supersymmetric standard model (MSSM), the expressions for $R_i$ and $\tilde{R}_i$ are given in Refs. [5,13,14]. However, we anticipate that in the class of models analyzed here, the SUSY contribution to $R_9$ and $R_{10}$ is very small in comparison to $R_7$, the same conclusion hold for $\tilde{R}_9$ and $\tilde{R}_{10}$ as well. Therefore, in order to simplify our analysis, we will use the approximation in which the SUSY dependence in $b \to s l^+ l^-$ decay enters only through the ratios of Wilson coefficients $R_7$, $R_9$, $R_{10}$, and $\tilde{R}_7$. Note that the dependence on $R_S$ and $R_S$ is modest in $b \to s l^+ l^-$, due to the fact that the operator $Q_8$ mixes with $Q_7$ at the NLO. For this reason we will neglect their contribution. We have explicitly checked that this approximation does not significantly affect our results.

Using this parametrization, the non-resonant branching ratios (BR)\(^1\) are expressed in terms of the new physics contribution as [14,15].

$$\text{BR}(B \to X_s e^+ e^-) = 7.29 \times 10^{-6} (1 + 0.714 R_{10} + 0.357 \tilde{R}_{10}^2$$

$$+ 0.35 R_7 + 0.0947 (\tilde{R}_7^2 + \tilde{R}_{10}^2) + 0.179 R_9$$

$$- 0.0313 R_7 R_9 + 0.045 \tilde{R}_7 R_{10}).$$

(9)

\(^1\)In order to reduce the large non-perturbative contributions to the $B \to X_s l^+ l^-$ decays, the resonant regions in the final invariant mass of the dilepton system $l^+ l^-$ should be avoided. This can be easily implemented by excluding some special areas from the integration regions in the dilepton invariant mass. The resulting BR where these regions have not been included, is usually called the non-resonant BR.
BR\(B \to X_s \mu^+\mu^-\)
\[
= 4.89 \times 10^{-6} \left(1 + 1.07 R_{10} + 0.535 R_{10}^2 + 0.0982 R_7 + 0.0491 (R_7^2 + \tilde{R}_7^2) + 0.264 R_9 - 0.0467 R_7 R_9 + 0.0671 R_9^2\right). \tag{10}
\]

The SM values \(7.29 \times 10^{-6}, 4.89 \times 10^{-6}, \) which correspond to \(BR(B \to X_s e^+ e^-)\) and \(BR(B \to X_s \mu^+\mu^-)\), respectively, are recovered by setting \(R_i = \tilde{R}_i = 0\) in these formula. An important observation from Eqs. (9), (10) is that the decay \(b \to s l^+l^-\) is quite sensitive to \(R_{10}\) rather than the other variables. Therefore, any enhancement for \(R_{10}\) could lead to significant changing in the prediction of BR of this decay without any consequences on \(b \to s \gamma\) decay, which mainly depends on \(R_7\). It is worth noticing that the different sensitivity in \(R_7\) in Eqs. (9), (10) is due to the fact that the coefficients proportional to \(R_7\) come from integrating the \(1/q^2\) pole (with \(q^2\) the momentum square of the virtual photon) of the magnetic operator \(Q_7\). Therefore, being the minimum value of \(q^2\) proportional to the mass square of final leptons, the sensitivity to \(R_7\) becomes larger in the electron channel.

We will also consider the lepton–anti-lepton energy asymmetry (AS) in the decay \(b \to s l^+l^-\) which is defined as

\[
A = \frac{N(E_l^- > E_{l^+}) - N(E_{l^+} > E_l^-)}{N(E_{l^-} > E_{l^+}) + N(E_{l^+} > E_{l^-})}, \tag{11}
\]

where, for instance, \(N(E_{l^-} > E_{l^+})\) is the number of the lepton pairs whose negative charged member is more energetic in the \(B\) meson rest frame than its positive partner. As for the BRs we will consider the AS in Eq. (11) integrated over non-resonant regions. With the above parametrization we find [14,15]

\[
A_{ll} = 0.48 \times 10^{-6} \left(1 + 0.911 R_{10} - 0.00882 R_{10}^2 - 0.625 R_7 (R_{10} + 1) + 0.884 R_9 (R_{10} + 1)\right). \tag{12}
\]

where \(R_{BR} = BR/BR^{SM}\).

Finally, regarding the \(B \to X_s \gamma\) decay, we have used the following parametrization [5,15]

\[
BR(B \to X_s \gamma) = (3.29 \pm 0.33) \times 10^{-4} \times (1 + 0.622 R_7 + 0.090 (R_7^2 + \tilde{R}_7^2) + 0.066 R_8 + 0.019 (R_7 R_8 + \tilde{R}_7 R_8) + 0.002 (R_7^2 + \tilde{R}_7^2)), \tag{13}
\]

where the overall factor corresponds to the SM value and its theoretical uncertainty.

We start our analysis by revisiting the predictions for the rate of these decays in the supersymmetric models with minimal flavor violation (such as the minimal SUGRA inspired model). In particular, we will show that the new bound on the Higgs mass [18] and the CLEO measurement for the BR of \(B \to X_s \gamma\) decay [19]

\[2.0 \times 10^{-4} < BR(B \to X_s \gamma) < 4.5 \times 10^{-4}\]

impose severe constraints on the parameter space of this class of models and it is no longer possible to have deviations on the non-resonant BR of \(B \to X_s e^+ e^-\) and \(B \to X_s \mu^+ \mu^-\) decays by more than 25% and 10%, respectively, relative to their SM expectations.

The main reason for that is the following. As emphasized above, the main contributions to these processes are due to the operators \(Q_7, Q_9,\) and \(Q_{10}\) where their Wilson coefficients are proportional to the mass insertions \((\delta_{23}^{u,d})_{LR}\) and \((\delta_{23}^{d})_{LL}\) However. In the minimal SUGRA scenario, and due to the universality assumption upon the soft SUSY breaking parameters at GUT scale, the flavor transitions are suppressed by the smallness of the CKM angles and/or the smallness of the Yukawa couplings. Moreover in this scenario, requiring the lightest Higgs mass to be \(m_h > 110\) GeV implies that the universal gaugino masses \(m_{1/2}\) has to be larger 250 GeV. This leads to a heavy stop mass and hence a further suppression for the SUSY contribution to \(B \to X_s l^+ l^-\) decays is found.

In our analysis we present our results for a specific choice of the sign(\(\mu\)). This choice corresponds to the one which gives positive contributions to the 
\(g - 2\) of the muon, as it is favoured by the new experimental results on 
\(g - 2\) [20]. Incidentally, this specific choice of sign(\(\mu\)) is the one for which the \(B \to X_s \gamma\) constraints are less effective. In Fig. 1, we present the scatter plots of the BR and the AS for the decay \(B \to X_s e^+ e^-\) (which is the most sensitive semileptonic decay) and \(B \to X_s \mu^+ \mu^-\) as a function of the lightest stop mass. In obtaining these figures, we var-
Fig. 1. Branching ratio (BR) and energy asymmetry (AS) of $B \rightarrow X_s e^+e^−$ and $B \rightarrow X_s \mu^+\mu^−$ (normalized to the corresponding SM ones) versus the lightest stop mass in minimal SUGRA model.

ied the universal soft scalar mass $m_0$ and gaugino mass $m_{1/2}$ from 50 GeV up to 1 TeV. The trilinear $A$-term is fixed to be $A_0 = m_0$ and $\tan \beta$ vary in the range [3, 40]. In our numerical analysis we assume the radiative electroweak symmetry breaking and impose the current experimental bounds on the SUSY spectra. We have also imposed the constraints which come from requiring vacuum stability (necessary to ensure that the potential is bounded from below) and from avoiding charge and color breaking minima deeper than the real one. It turns out that the present experimental limit on the lightest Higgs mass sets the most important constraint in minimal SUGRA models. In particular, it excludes the parameter space that leads to stop masses lower than 400 GeV. It is clear that with such heavy stop masses the dominant contribution to $b \rightarrow s l^+l^-$, which comes from chargino exchanges, is quite suppressed.

As can be seen from Figs. 1–3, the general trend of this class of models, for this particular choice of $\text{sign}(\mu)$, is in a decreasing of BR and increasing of AS with respect to the SM expectations, in both universal and non-universal models. The origin of this behaviour can be explained as follows. As discussed above, the variations of BR and AS are mainly due to $R_7$. For this choice of $\text{sign}(\mu)$ the $B \rightarrow X_s \gamma$ constraints are less restrictive and mostly allow for negative values of $R_7$. Negative values of $R_7$ (in the range of $[-1, 0]$) will produce destructive and constructive interferences in BR and AS, respectively, as can be understood from the parametrizations in Eqs. (9)–(12). However, we have also checked that for the other choice of $\text{sign}(\mu)$ the behaviour is opposite, giving an enhancement of BR and decreasing of AS, but with more moderate effects due to a stronger action of $B \rightarrow X_s \gamma$ constraints.

In the large $\tan \beta$ region, where chargino and Higgs contributions to $R_7$ are enhanced ($R_9$ and $R_{10}$ are moderately affected by $\tan \beta$), a sizeable changing in the BR and AS of $B \rightarrow X_s l^+l^-$ decays might arise. Nevertheless, $R_7$ gives also the major contribution to the BR of $B \rightarrow X_s \gamma$, and by imposing the CLEO limits we dismiss such large effects for $B \rightarrow X_s l^+l^-$ decay. Therefore, we can conclude that in SUSY models with universal soft breaking terms, it is not possible to get any significant enhancement for BR in $B \rightarrow X_s l^+l^-$ decays, while a decreasing up to 25% can be obtained in the electron channel. As explained

\footnote{We stress that these last conditions may be automatically satisfied in minimal SUGRA, while in generic SUSY models, like those we will consider below, these conditions have to be explicitly checked.}
above, the decreasing of BR is reflected in a large enhancement of AS, in particular up to 75% and 50% for the electron and muon channels, respectively. However, we will see that in general SUSY models, mainly due to the non-universality in the scalar sector, the Higgs bounds can be relaxed and larger deviations on BR and AS for $B \rightarrow X_i \tau^+ \tau^-$ decays can be achieved, deviations which correspond to the allowed region of large negative values of $R_7$ (namely in the range of $[-6, -4]$).

Now we turn to the most general supersymmetric extension of the SM. In particular, we will consider SUSY models with non-degenerate $A$-terms and non-universal soft scalar and gaugino masses. Such models are naturally obtained from string inspired models [2] and some aspect of their phenomenological implications have been recently studied. Note that the squark mass matrices are often diagonal in string inspired models and this is what we will adopt here. Generic SUSY models might also have non-universality in the off-diagonal terms of squark mass matrices. Nevertheless, these off-diagonal terms are severely constrained by $\Delta M_K$, $\Delta M_B$, and $\varepsilon_K$. Models with flavor symmetries naturally avoid such constraints. We will consider later a model with $U(2)$ flavor symmetry as an example for this class models.

In order to parametrize the non-universality of a large class of string inspired models (with diagonal soft-breaking terms in the sfermion sector), we assume here the following soft scalar masses, gaugino masses $M_a$ and trilinear couplings:

$$M_a = \delta_a m_{1/2}, \quad a = 1, 2, 3,$$

$$m^2_0 = m^2_Z = m^2_0 \text{diag}[1, 1, \delta_4],$$

$$m^2_{ij} = m^2_0 \text{diag}[1, 1, \delta_5],$$

$$m^2_{ij} = m^2_0 \text{diag}[1, 1, \delta_6],$$

$$m^2_{H_1} = m^2_0 \delta_7, \quad m^2_{H_2} = m^2_0 \delta_8,$$

$$A^\alpha = A^{\alpha'} = A^\prime = m_0 \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

where the parameters $\delta_i$ and $a_{ij}$ can vary in the $[0, 1]$ and $[-3, 3]$ ranges, respectively. It has recently been emphasized that these models could be free from the EDMs constraints and also have testable implications for the CP violation experiments [4]. In Ref. [5], the prediction for the BR of $B \rightarrow X_i \gamma$ decay has been considered in two representative examples for this class of models and it was found that $B \rightarrow X_i \gamma$ does not essentially constrain the non-universality of $A$-terms.

In our convention for the trilinear couplings, the $A$ terms are defined such that $A_{ij} = A_{ij} Y_{ij}$ (indices not summed) and $Y_{ij}$ are the corresponding Yukawa couplings. We assume that the Yukawa matrices at EW scale are given by

$$y^d = \frac{1}{v_1} V^*_{\text{CKM}} \text{diag}(m_d, m_s, m_b),$$

$$y^u = \frac{1}{v_2} \text{diag}(m_u, m_c, m_t) V^T_{\text{CKM}}.$$  (21)

For any value of the parameters $m_0$, $\delta_i$, $m_{1/2}$, $a_{ij}$ at GUT scale, and $\tan \beta$ (we determine the $\mu$ and $B$ parameters from the electroweak breaking conditions) we compute the relevant SUSY spectrum and interaction vertices at low energy needed for the calculation of the $b \rightarrow s l^+ l^-$ decay amplitudes. In order to connect the high energy SUSY parameters, gauge and Yukawa couplings with the corresponding low energy ones, we have used the most general renormalization group equations in MSSM at 1-loop level. As stated above, we impose the current experimental bounds on SUSY spectrum, in particular lightest Higgs mass $m_h > 110$ GeV, and $B \rightarrow X_i \gamma$ constraints in Eq. (14).

In Fig. 2 we present scatter plots for the BR and AS for the $B \rightarrow X_i e^+ e^-$ and $B \rightarrow X_i \mu^+ \mu^-$ decays versus the lightest stop mass. As for the universal models, we varied the fundamental mass parameters $m_0$, $m_{1/2}$ from 50 GeV up 1 TeV, and $\tan \beta$ in the range $[3, 40]$. The parameters $\delta_i$ and $a_{ij}$ have been also randomly selected in the ranges $[0, 1]$ and $[-3, 3]$, respectively. It is worth mentioning that the gluino contributions are negligible in the universal limit and the non-universality in the $A$-terms is essential for enhancing such contributions. Moreover, with non-universality in the gaugino masses we can have light chargino and stop masses close to their experimental limit and the Higgs mass bound is satisfied. In this region of parameter space indeed the chargino contributions to $R_7$, $R_9$ and $R_{10}$ are enhanced. However, it is noted that in all the parameter space, $R_9, R_{10}$ are much smaller than $R_7$ and still the main contributions to these processes are due to $R_7$ which also gives the main contribution to the $B \rightarrow X_i \gamma$ decay.
As can be seen from Fig. 2 there is a disconnected region of points, for stop masses lighter than 300 GeV, where very large enhancements in both BR and AS are reached. In particular, a factor 3 and 2.5 of enhancements in both BR and AS are obtained for electron and muon channels, respectively. This region corresponds to the large (negative) SUSY contributions to $R_7$, roughly in the range of $[-6, -4]$, obtained for $\tan \beta > 30$. Nevertheless, these huge enhancements belong to the less populated areas of scatter plots which means that a larger amount of fine tuning between the SUSY parameters is needed in this case.

The more populated areas in Fig. 2 correspond to the other (disconnected) range of allowed values for $R_7$, namely, $-1 < R_7 < 1$. In this region, the $B \to X_s e^+ e^-$ constraints reduce the enhancements (with respect to the SM one) on the BR of $B \to X_s e^+ e^-$ to be less than 20% and the decreases up to 25%. More moderate effects are obtained for the muon channel, since it is less sensitive to $R_7$. In correspondence to these variations on the BR, larger effects are obtained for the AS. In particular up to 75% and 50% enhancements in the AS for electron and muon channel, respectively, while a more moderate increasing (about 40% and 25%, respectively) are expected.

Now we compare our results with the model independent analysis of Ref. [10], based on a low energy approach. Using the mass insertion approximations and general MSSM at low energy it was shown in Ref. [10] that the SUSY contributions to BR and AS of the semileptonic decays can get maximum enhancement (up to $4 \times 10^{-5}$ for the BR $B \to X_s e^+ e^-$) i.e., 4 times the SM value. However, this needs the following values for the mass insertions $(\delta_{23})$:

$$ $(\delta_{23}^{u,d})_{LL} \simeq -0.5, \quad (\delta_{23}^u)_{LR} \simeq 0.9.$$

Such values can be obtained only in a very small region of the parameter space of the SUSY models with non-universal soft terms, specially after imposing the electroweak breaking conditions, the new bounds on the Higgs mass, and $B \to X_s \gamma$ constraints. However, we found that in general the typical values of these mass insertions are $|(\delta_{23}^{u,d})_{LL}| \simeq 10^{-2}, \quad (\delta_{23}^u)_{LR} \simeq 10^{-3}$. This, indeed, leads to a BR for the $B \to X_s e^+ e^-$ decay of order $10^{-6}$ with at most 20% enhancement than the SM value.

Finally, we proceed to consider SUSY models with non-abelian flavour symmetry. This class of models has a flavour structure in the soft scalar masses, and hence, the LL sector contains larger mixing than what is found in the previous models with diagonal squark masses. As mentioned, the $\Delta M_K$ and $\varepsilon_K$ impose severe constraints on the squark mixing, namely $\sqrt{|\text{Re}(\delta_{12})^2_{LL}|} \lesssim 10^{-2}$ and $\sqrt{|\text{Im}(\delta_{12})^2_{LL}|} \lesssim 10^{-3}$, respectively [1].
Here as an illustrative example, we consider a model based on a $U(2)$ symmetry acting on the two light families [16] where the above mentioned constraints are satisfied. In this case, the Yukawa textures, at GUT scale, are given by [16]

$$Y_u = \frac{m_t}{v \sin \beta} \begin{pmatrix} 0 & c_{ee'} & 0 \\ -c_{ee'} & 0 & a_{ee} \\ 1 & b_{ee} & 1 \end{pmatrix},$$

$$Y_d = \frac{m_b}{v \cos \beta} \begin{pmatrix} 0 & c_{ee'} & 0 \\ -c_{ee'} & 0 & a_{ee} \\ 1 & \rho & 1 \end{pmatrix}$$

and the squark mass matrices take the form

$$M_Q^2 = m_{3/2} \begin{pmatrix} 1 & 0 & \alpha\rho_{ee'} \\ \alpha^{*\rho_{ee'}} & 0 & r_3 \end{pmatrix},$$

$$M_D^2 = m_{3/2} \begin{pmatrix} 1 & 0 & \alpha'\rho_{ee'} \\ \alpha'^{\rho_{ee'}} & 1 + \lambda|\rho|^2 & \beta^{*\rho} \\ 0 & \beta^{*\rho} & r_3' \end{pmatrix},$$

$$M_U^2 = m_{3/2} \begin{pmatrix} 1 & 0 & \alpha''\rho_{ee'} \\ \alpha''^{\rho_{ee'}} & 0 & r_3'' \end{pmatrix}.$$  \hspace{1cm} (24)

The definition of the parameters appearing in these matrices can be found in Ref. [16]. The important feature of the flavor structure of this model is the presence of a large mixing between the second and the third generation which would have significant effect on enhancing the BR and AS of $b \to s l^+ l^-$ decays. However, this mixing essentially enhances $R_7$ which means enhancing for the BR of $B \to X_s \gamma$ decay as well. Therefore imposing the $B \to X_s \gamma$ constraints leads to a similar prediction to that we obtained with the previous model.

In Fig. 3 we display the predictions of this model for BR and AS of $B \to X_s e^+ e^-$ and $B \to X_s \mu^+ \mu^-$ decays versus the lightest stop mass. As in the previous models we have considered, most of the parameter space (favored by the $B \to X_s \gamma$ and other constraints) leads to decreasing in the BR and increasing of AS. This is due to the fact that even for these models the major effect in the variation is due to $R_7$. The large enhancements in BR and AS, obtained in the other scenario with large and negative contributions to $R_7$, are not very likely to show up. This is mainly due to the constraints on the Higgs mass, which prevent stop masses to be lighter than 300 GeV.

**Conclusions**

We have analyzed the predictions for the inclusive semileptonic decays $B \to X_s l^+ l^-$ in different SUSY models. In particular, we have considered SUSY models with minimal flavor violation, non-degenerate
A-terms and non-universal soft scalar and gaugino masses, and finally SUSY models with non-abelian symmetry that leads to a flavor structure for the soft scalar masses. We showed that in all these models the major effect on the variations of $B \to X_s l^+ l^-$ decays, with respect to their SM expectations, is due to the SUSY contributions to the magnetic dipole operator parametrized by $R_7$ (which also give the major contribution to the inclusive $B \to X_s \gamma$ decay). The SUSY contributions to the semileptonic operators is almost negligible.

We found that the general trend of our results, favoured by the CLEO $B \to X_s \gamma$ constraints and Higgs mass bound, is in decreasing the non-resonant BR and increasing the AS. Nevertheless, only non-universal models can have chances to get very large enhancements in BR and AS. In particular, in this case up to 3 and 2.5 time enhancements of BR and AS with respect to the SM expectations can be obtained in the electron and muon channel, respectively.

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References

A generating functional for ultrasoft amplitudes in hot QCD

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Abstract

The effective amplitudes for gluon momenta \( p \ll gT \) in hot QCD exhibit damping as a result of collisions. The whole set of \( n \)-point amplitudes is shown to be generated from one functional \( K_{\mu\nu}(X, Y; A) \), in addition to the induced current \( j_{\mu}(X; A) \).

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The ultrasoft amplitudes are the effective amplitudes that are obtained in pure gauge theory at high temperature, when one integrates out the scales \( (T)^{-1} \) and \( (gT)^{-1} \) [1–3]. The colour collective excitations, which describe the physics at such a large wavelength, may be created by a weak external disturbance. The ultrasoft fields obey the Maxwell equations [2]

\[ D^\nu F_{\mu\nu}(X) = j_{\mu}^{\text{ext}}(X) + j_{\mu}^{\text{ind}}(X), \]

where the induced current is the response of the plasma to the initial perturbation \( j_{\mu}^{\text{ext}}(X) \).

\[ j_{\mu}^{\text{ind}}(X) = m_D^2 \int v_{\mu} W^a(X, v), \]

\[ (v.D_X + \hat{C}^{ab} W^b(X, v) = v.E^a(X) \]

with \( v_{\mu}(v_0 = 1, v), (v^2 = 1) \) and \( \int v = \int d\Omega_v/4\pi; m_D^2 = g^2 N T^2/3 \) and \( \hat{C} \) is a collision term [1–3]

\[ \hat{C} W^b(X, v) = \int C(v', v') W^b(X, v'). \]

\[ \hat{C} \] is a symmetric operator in \( v \) space, local in \( X \) and blind to colour, with positive eigenvalues except for a zero mode

\[ \int C(v, v') = 0. \]

The gluons \( p \sim T \) take part in the collective motion, the gluons \( p \sim gT \) are exchanged in the collision process.

The solution to Eqs. (2), (3) may be written in terms of the retarded Green function

\[ i(v.D_x + \hat{C}) G_{\text{ret}}(X, Y; A; v, v') = \delta^{(4)}(X - Y) \delta_S(v - v') \]

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with $G_{\text{ret}}(X, Y) = 0$ for $X_0 < Y_0$. One writes
\begin{equation}
\nu^\text{\prime}.E(Y) = \partial_{\nu^\prime}\left(v^{\mu}A_{\mu}\right) - \nu^\prime.D\gamma A_{\gamma}(Y)
\end{equation}
and, with Eq. (5), one obtains [4,5]
\begin{equation}
j^{\mu \text{ind}}(X; A) = \frac{m_D^2}{\alpha_0} \gamma_0 A_0(X)
+ i \int_{\nu, \nu^\prime} d^4Y \nu_{\mu} G_{\text{ret}}^{ab}(X, Y; \nu, \nu^\prime) \partial_{\nu^\prime}\left(v^\prime A^b(Y)\right).
\end{equation}
The (one-particle-irreducible) $n$-point retarded ultrasoft amplitudes are derived from the induced current [2,4]
\begin{equation}
g^{n-2}G_{\text{ret}}(X; X_1 \cdots X_{n-1}) = \frac{1}{\partial(A(X_{n-1}) \cdots \partial A(X_1))^{\mu \text{ind}}(\nu, \nu^\prime)}(X; A)\bigg|_{A=0}.
\end{equation}
The identity [4,5]
\begin{equation}
\partial_{\nu^\prime} G_{\text{ret}}(X, Y; A; \nu, \nu^\prime) = \int G_{\text{ret}}(X, X_1; A; \nu, \nu^\prime) gT^\nu \nu^\prime g^{-1}
\times G_{\text{ret}}(X_1, Y; A; \nu^\prime, \nu^\prime)
\end{equation}
shows that the effective ultrasoft amplitudes have a tree-like structure, as the addition of a gluon leg amounts to the insertion of the leg along the propagator (in Eq. (10), the time arguments satisfy $X^0 \geq X_1^0 \geq Y^0$ and $T^\nu$ is in the adjoint representation). After differentiation, the Green function that enters the amplitudes is $G_{\text{ret}}(X; Y; A = 0; \nu, \nu^\prime)$, i.e., in momentum space
\begin{equation}
(v.p + i\tilde{C})G_{\text{ret}}(p; \nu, \nu^\prime) = \mathcal{I},
G_{\text{ret}}(p; \nu, \nu^\prime) = (v.p + i\tilde{C})^{-1}_{\nu^\prime}. 
\end{equation}
The amplitudes exhibit damping (i.e., colour relaxation), as a result of collisions. $\tilde{C}$ introduces the scale $g^2 T \ln 1/g$ [1–3]. This damping is compatible with gauge symmetry, because the operator $\tilde{C}$ is local in $X$, blind to color, and has the property (5). These amplitudes obey tree-like Ward identities [5].

The retarded amplitudes (9) are only a subset of the $n$-point amplitudes, as $X^0$ is the later time. The purpose of this Letter is to show that the whole set of $n$-point amplitudes may be generated if one introduces the functional
\begin{equation}
k^{ab}_{\mu \nu}(X, Y; A) = m_D^2 T \int_{\nu, \nu^\prime} [v_{\mu} G_{\text{ret}}^{ab}(X, Y; A; \nu, \nu^\prime) v_{\nu^\prime}^\prime + v_{\nu} G_{\text{ret}}^{ba}(Y, X; A, \nu, \nu^\prime) v_{\mu}^\prime].
\end{equation}
Because of damping, the effective theory has a time arrow. Among the possible real-time formalisms for field theory in thermal equilibrium [6–8], this feature selects the Retarded-Advanced basis ($R/A$), where, because of causality, the propagator keeps its two-component structure $G_{\text{ret}}, G_{\text{adv}}$ with $G_{\text{ret}}(p; \nu, \nu^\prime)$ as in Eq. (11) and $G_{\text{adv}}(p; \nu, \nu^\prime) = G_{\text{ret}}^{\nu^\prime}(p; \nu, \nu^\prime)$. The retarded $n$-point functions are, in fact, common to two bases, the $R/A$ basis and the Keldysh basis, where they correspond to one Keldysh index “2” and the other indices “1”. There exists a Bogoliubov transformation between the two bases [9].

The ultrasoft amplitudes are such that $K_{\mu \nu}(X, Y; A)$, defined in Eq. (12), is the generating functional for the amplitude with two indices “2”. Moreover, the amplitudes with more than two indices “2” vanish. In the following, it is shown that this prescription generates a consistent set of $n$-point amplitudes in the $R/A$ basis, and these obey the expected Ward identities.

Before addressing the case of the amplitudes with $p ≪ gT$, one needs to recall some general properties of the bases and their connection. In both bases, all momenta are incoming the $n$-point amplitudes, i.e., $\sum_{i=1}^{n} p_i = 0$. The Keldysh basis is a rotated representation of the Closed-Time-Path representation [8], the Keldysh indices are $k_1 = 1, 2$. In the $R/A$ basis [7], an incoming momentum may be of type $R$ or of type $A$, its incoming energy is $p_i^0 + i\epsilon_i, \epsilon_i > 0$ type $R$, $\epsilon_i < 0$ type $A$ with $\sum_{i=1}^{n} p_i^0 = 0$ and $\sum_{i=1}^{n} \epsilon_i = 0$.

The relations between the (amputated) $n$-point functions in the Keldysh basis and in the $R/A$ basis were obtained in perturbation theory in Ref. [9]
\begin{equation}
G_{\alpha_1 \cdots \alpha_n}(p_1 \cdots p_n) = T_{\alpha_1 k_1}(p_1) \cdots T_{\alpha_n k_n}(p_n) K_{k_1 \cdots k_n}(p_1 \cdots p_n),
\end{equation}
where $k_1 = 1, 2$ and $\alpha_i = R, A$. There exist different convenient choices in both bases. We follow Ref. [9].
and choose

\[
\begin{bmatrix}
T_{R1}(p) & T_{R2}(p) \\
T_{A1}(p) & T_{A2}(p)
\end{bmatrix} = \begin{bmatrix}
N(p) & 1 \\
1 & 0
\end{bmatrix},
\]

(14)

\[
N(p) = \frac{1}{2} \coth \frac{\beta}{2} p^0 = n(p^0) + \frac{1}{2}.
\]

(15)

Then, the relations are

\[
\Gamma_{AA}(p, p^0) = \frac{1}{2} \delta(p - p^0) \frac{\partial}{\partial p^0} n(p^0) + \frac{1}{2}.
\]

(16)

\[
\Gamma_{RA}(p, p^0) = \frac{1}{2} \delta(p - p^0) \frac{\partial}{\partial p^0} n(p^0) - \frac{1}{2}.
\]

(17)

\[
\Gamma_{RR}(p, p^0) = \frac{1}{2} \delta(p - p^0) \frac{\partial}{\partial p^0} n(p^0) - \frac{1}{2}.
\]

(18)

\[
\Gamma_{RR}(p, p^0) = \frac{1}{2} \delta(p - p^0) \frac{\partial}{\partial p^0} n(p^0) - \frac{1}{2}.
\]

(19)

and so on. Moreover, the R/A basis possesses an important complex conjugate relation [9–11]. With the choice of Eq. (14) and our conventions, it is

\[
N(p_1 \cdots p_m) \Gamma(p_{1A} \cdots p_{mA}, p_{(i+1)R} \cdots p_{nR}) = (-1)^m N(p_{1A} \cdots p_{mA}) \Gamma^*(p_{1R} \cdots p_{nR}, p_{(i+1)A} \cdots p_{mA}),
\]

(20)

where

\[
N(p_1 \cdots p_m) = \prod_{i=1}^{n_m} \frac{n(p_i)}{n_{(i+1)}}.
\]

(21)

For example,

\[
\Gamma_{RR}(p_1, p_2 \cdots p_n) = (-1)^n N(p_2 \cdots p_n) \Gamma_{RA}(p_1, p_2 \cdots p_n),
\]

(22)

i.e., the thermal weight are attached to the legs of type R, and the functions \( \Gamma_{RA} \) are the retarded amplitudes of the response approach.

Concentrating now on the case of the ultrasoft n-point amplitudes, a consistent approximation is obtained if one writes in Eqs. (18) to (22)

\[
N(p_i) \approx \frac{T}{p_i^0}.
\]

(23)

From Eqs. (17) and (9), the amplitudes with one Keldysh index “2” are functional derivatives of the induced current \( j_\mu(X; A) \). The amplitudes with two Keldysh indices “2” are functional derivatives of the function \( K_{\mu\nu}(X, Y; A) \) defined in Eq. (12). \( K_{22} \) is \( K_{\mu\nu}(X, Y; A = 0) \), one differentiation gives \( K_{221} \), and so forth. An amplitude is a sum of terms made of a string of operators, and each term is linked to a specific time ordering of the gluon legs. The properties of the functional derivation are such that: (i) the amplitudes are symmetric in all gluon legs of the same Keldysh index; (ii) the order in \( v \) space and in color space follow the time order [from Eq. (10)], with a corresponding property in momentum space.

The relation (18) has an important interpretation. The retarded amplitudes \( K_{221-1} \) is such that \( p_1 \) is always associated with the latest time. The amplitude \( K_{221-1} \) is such that \( X \) and \( Y \) (i.e., \( p_1 \) and \( p_2 \)) are latest and earliest times, or vice versa. In relation (18), the terms that are present in \( K_{221-1} \) cancel the terms from \( K_{211-1}(T/p_1^0) \) such that \( p_2 \) is associated with the earliest time, and the terms from \( K_{121-1}(T/p_1^0) \) where \( p_1 \) is associated with the earliest time. The resulting terms in \( \Gamma_{RR} \) are such that either \( p_1 \) or \( p_2 \) is associated with the latest time, while neither \( p_1 \) nor \( p_2 \) corresponds to the earliest time, a feature expected from causality.

The constraint \( \Gamma_{RR} = 0 \) gives the Keldysh component of the propagator

\[
K_{22}(p_1, -p_1) = \frac{T}{p_1^0} (K_{21} - K_{12})
\]

\[
= \frac{T}{p_1^0} [\Pi(p_{1R}) - \Pi(p_{1A})].
\]

(24)
Explicit examples are
\[
\Pi_{\mu\nu}(p_{1R}) = m_D^2 g_{\alpha\beta} \left[ \int \nu_\mu(v.p_1 + i \tilde{C})^{-1} v'_\nu(p_0) - g_{\mu\alpha} g_{\nu\beta} \right].
\] (25)

\[
\Gamma_{RAA} = \Gamma_{\mu\nu\rho(p_1R, p_2A, p_3A)} = m_D^2 \int \left[ i f^{acb} v_\mu(v.p_1 + i \tilde{C})^{-1} v'_\rho \times (-v.p_2 + i \tilde{C})^{-1} v''_\nu(-p_0^0) \right. \\
\left. + (2 \leftrightarrow 3) \right].
\] (26)

\[
K_{221} = K_{\mu\nu\rho(p_1, p_2, p_3)} = m_D^2 T \int \left[ i f^{acb} v_\mu(v.p_1 + i \tilde{C})^{-1} v'_\rho \times (-v.p_2 + i \tilde{C})^{-1} v''_\nu \right. \\
\left. + (2 \leftrightarrow 1) \right]
\] (27)

where \((i \leftrightarrow j)\) means a term obtained by the exchange of all indices, i.e., momentum, Lorentz, colour.

The complex conjugate relation (20) imposes multiple consistency conditions which are now examined. The complex conjugation amounts to a time reversal of each string of operators, up to a sign \((-1)^{n-1}\). The constraint
\[
\Gamma_{RRA} = -\left( \frac{T}{p_1^0} + \frac{T}{p_2^0} \right) \Gamma_{AAAR}^\ast
\] (28)
is satisfied very simply by the operators’ strings. In \(\Gamma_{RRA}\), \(p_3\) is associated with the earliest time, as \(p_1\) and \(p_2\) cannot be, and this is also the case for the complex conjugate of \(\Gamma_{AAAR}\) (where \(p_3\) is the latest time). In a similar way
\[
\Gamma_{RAA} \left( \frac{T}{p_3^0} + \frac{T}{p_4^0} \right) = \Gamma_{AAAR}^\ast \left( \frac{T}{p_1^0} + \frac{T}{p_2^0} \right).
\] (29)

In \(\Gamma_{RAA}\), \(p_1\) or \(p_2\) is the latest time and neither of them is the earliest time. This time ordering is equivalent to the one of \(\Gamma_{AAAR}^\ast\), where \(p_3\) or \(p_4\) is the earliest time and neither of them is the latest time.

Turning to relation (19), the constraint \(\Gamma_{RRR} = 0\) leads to the result \(K_{222} = 0\), and the constraint
\[
\Gamma_{RRR} = \Gamma_{AAAR}^\ast T^2 \left( \frac{1}{p_1^0 p_2^0} + \frac{1}{p_1^0 p_3^0} + \frac{1}{p_2^0 p_3^0} \right)
\] (30)
is obeyed with \(K_{2221} = 0\). Then, from \(\Gamma_{RRRR} = 0\) one deduces that \(K_{2222} = 0\). All the constraints on time ordering are satisfied with the vanishing of the amplitudes with three or more Keldysh indices “2”. One can go to the 5-point functions and check that all the constraints from Eq. (20) are obeyed.

The set of vertices in the \(R/A\) basis may be used in a perturbative expansion. For example, one-loop self-energy diagrams with loop momentum \(g^2 T \ln 1/g\) are made either with a pair \(\Gamma_{RRA}, \Gamma_{AAR}\), or with \(\Gamma_{RAAA}\) [5] (a retarded propagator joins an \(A\) leg to an \(R\) leg, as it joins an outgoing \(p_R\) (i.e., incoming \((- p_A))\) to an incoming \(p_R\)).

One now turns to the Ward identities which are satisfied by the Keldysh amplitudes. They are a consequence of the gauge covariance of the Green function (6) and of the conservation laws resulting from Eq. (5), i.e.,
\[
D_{\mu} J_{\mu}^{\text{ind}} = 0 = D_{\nu}^\ast K_{\mu\nu} = D_{\nu}^\ast K_{\mu\nu}.
\] (31)
The \(n\)-point retarded amplitudes are related to \((n-1)\) retarded ones [5]. Another set of identities takes place within the subspace with two indices “2”. For a contraction of a leg with Keldysh index “2”, one uses relations such as
\[
\int \nu_\mu(v.p_1 + i \tilde{C})^{-1} v'.v = \int (v.p_1 + i \tilde{C})(v.p_1 + i \tilde{C})^{-1} = 1
\] (32)
with the help of Eq. (5). For a contraction of a leg with index “1”, one uses relations such as
\[
v.p_3 = (v.(p_3 + p_1) + i \tilde{C}) - (v.p_1 + i \tilde{C}).
\] (33)
Those identities allow one to cancel one propagator of each operators’ string. For example, the resulting Ward identities for \(K_{2211}\) are
\[
-i p^\mu_1 K_{\mu\nu\rho\delta}^{abcd}(p_{1(2)}, p_{2(2)}, p_{3(1)}, p_{4(1)}) = f_{acm} K_{\nu\rho\delta}^{bnd}(p_{2(2)}, (p_1 + p_3)_{(2)}, p_{4(1)}) + (3 \leftrightarrow 4),
\] (34)
\[
-i p^\mu_4 K_{\mu\nu\rho\delta}^{abcd}(p_{1(2)}, p_{2(2)}, p_{3(1)}, p_{4(1)}) = f_{acm} K_{\nu\rho\delta}^{bnd}(p_{1(2)}, p_{2(2)}, (p_3 + p_4)_{(1)}) + f_{acm} K_{\mu\nu\rho\delta}^{bnd}(p_{1(2)}, p_{2(2)}, p_{3(1)}) + (1 \leftrightarrow 2).
\] (35)
One can check that these identities lead to the Ward identities which are expected in the $R/A$ basis [5]. For example, for the $RRAA$ amplitude, there enter the identities, factors such as

$$N(p_1, p_2 + p_4) \approx \frac{T}{p_1^0} + \frac{T}{p_2^0 + p_4^0},$$

$$N(p_2, p_1 + p_3) \approx \frac{T}{p_2^0} + \frac{T}{p_1^0 + p_3^0}. \quad (36)$$

To conclude, we summarize the argument. The induced current $j_\mu$ is the generating function of the retarded ultrasoft amplitudes, and, in particular, of full set of two- and three-point functions. With the approximation $N(p_0) \approx T/p_0$ (the usual companion to the soft amplitudes), one deduces, in the Keldysh basis, the expressions for $K_{22}$, $K_{221}$ and the relation $K_{222} = 0$. Then, from $K_{22}$, $K_{221}$, gauge symmetry suggests to step across to the generating function $K_{\mu\nu}$ of all amplitudes with two Keldysh indices 2, i.e., the Green function (6) enters $K_{\mu\nu}$. Moreover, general constraints exist on the set of $n$-point amplitudes. They are stated in the $R/A$ basis, and they allow to advance towards the full set of $n$-point amplitudes.

One obtains the result that all amplitudes with more than two indices 2 vanish.

References


A lattice calculation of thermal dilepton rates

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Abstract

Using clover improved Wilson fermions we calculate thermal vector meson correlation functions above the deconfinement phase transition of quenched QCD. At temperatures $1.5 T_c$ and $3T_c$ they are found to differ by less than 15% from that of a freely propagating quark–anti-quark pair. This puts severe constraints on the dilepton production rate and in particular rules out a strong divergence of the dilepton rate at low energies. The vector spectral function, which has been reconstructed using the Maximum Entropy Method, yields an enhancement of the dilepton rate over the Born rate of at most a factor two in the energy interval $4 \lesssim \omega/T \lesssim 8$ and suggests that the spectrum is cut-off at low energies by a thermal mass threshold of about $(2–3)T$.

1. Introduction

The dilepton spectrum is one of the key observables in the study of thermal properties of the hot and dense medium created in heavy ion collisions [1–3]. Thermal modifications of the dilepton spectrum have been observed at large invariant masses where they reflect the suppression of heavy quark resonances [4]. At low energies it is expected that non-perturbative in-medium modifications of the quark–anti-quark interactions as well as quark–anti-quark annihilation in the quark–gluon plasma influence the thermal dilepton rate. Perturbative 2-loop [5] calculations as well as calculations based on hard thermal loop (HTL) resummed perturbation theory [6] lead to an enhancement of the spectrum at low energies, which dominates over the suppression arising from massive quasi-particle poles [1,6] or non-perturbative gluon condensates [7]. Indeed, an enhancement of low mass dilepton production has been observed in heavy ion experiments [8].

The thermal dilepton rate describing the production of lepton pairs with energy $\omega$ and total momentum $\vec{p}$ is related to the Euclidean correlation function of the vector current, $J_\mu^V(\tau, \vec{x}) = \bar{\psi}(\tau, \vec{x}) \gamma_\mu \psi(\tau, \vec{x})$, which can be calculated numerically in the framework of lattice regularized QCD. This relation is established through the vector spectral function, $\sigma_V(\omega, \vec{p}, T)$, which directly gives the differential dilepton rate in two-flavor QCD,

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2 \omega^2(e^{\omega/T} - 1)} \sigma_V(\omega, \vec{p}, T),$$

and at the same time is related to the Euclidean vector meson correlation function, $G_V(\tau, \vec{p}, T) =$...
\[ \int d^3x \exp(i \vec{p} \cdot \vec{x})(J_{V}^\mu(\tau, \vec{x})J_{V}^\dagger_{\mu}(0, \vec{0})) \text{, through an integral equation,} \\
\]
\[ G_V(\tau, \vec{p}, T) = \int_0^\infty d\omega \sigma_V(\omega, \vec{p}, T) \frac{\text{ch}(\omega(1/2T))}{\text{sh}(\omega/2T)} \text{,} \]

where the Euclidean time \( \tau \) is restricted to the interval \([0, 1/T]\). A direct calculation of the differential dilepton rate thus becomes possible, if the above integral equation can be inverted to determine \( \sigma_V(\omega, \vec{p}, T) \). Although finite temperature lattice calculations, which are performed on lattices with finite temporal extent \( N_\tau \), will generally provide information on \( G_V(\tau, \vec{p}, T) \) only for a discrete and finite set of Euclidean times \( \tau = k/N_\tau, k = 1, \ldots, N_\tau \), this may be achieved through an application of the maximum entropy method (MEM) [9,10]. We will present first results on such a calculation in this Letter. However, even without invoking statistical tools like MEM accurate numerical results on \( G_V(\tau, \vec{p}, T) \) will themselves provide stringent constraints on spectral functions calculated in other perturbative or non-perturbative approaches. This in turn will put constraints on the thermal dilepton rates.

We present here results of a calculation of vector current correlation functions and reconstructed spectral functions at two temperatures above \( T_c \), i.e., \( T = 1.5T_c \) and \( 3T_c \). We will restrict our discussion to the zero momentum limit \( (\vec{p} = 0) \) of these quantities and we will therefore suppress any explicit momentum dependence in the arguments of \( G_V \) and \( \sigma_V \) in the following.

2. Thermal correlation functions

The correlation functions \( G_V(\tau, T) \) have been calculated within the quenched approximation of QCD using non-perturbatively improved clover fermions [12,13]. This eliminates the \( \mathcal{O}(a) \) discretization errors in the fermion sector. Moreover, in order to explicitly control the cut-off dependence of the numerical results we have used lattices of size \( N_\lambda^3 \times N_\tau \) with fixed ratio of spatial to temporal lattice extent, \( N_\lambda/N_\tau = 4 \), and two different temporal lattice sizes, \( N_\tau = 12 \) and \( 16 \), respectively. Due to the large temporal extent of the lattice, the lattice spacing is rather small already at the deconfinement transition and becomes even smaller for larger temperatures. Using \( T_c = 265 \text{ MeV} \) [14] to set a scale for the lattice spacing one finds, \( a = 1/N_\lambda T \approx (0.75T_c/N_\lambda) \) fm. For the two different temperatures and lattice sizes used here it ranges from \( a = 0.015 \text{ fm} \) to \( a = 0.04 \text{ fm} \). The comparison of results obtained on lattices with different temporal extent and fixed \( T_c \) indeed, confirms that the results for \( G_V(\tau, T) \) are not significantly influenced by finite cut-off effects.

The thermal correlation functions have been constructed from quark propagators obtained from the inversion of the clover improved Wilson fermion matrix. As we have performed calculations in the high temperature, chirally symmetric phase of QCD there are no massless Goldstone modes which could interfere with the convergence of the iterative solvers for this matrix inversion. We thus could perform calculations directly in the limit of vanishing quark masses, i.e., directly at the critical values of the hopping parameter \( \kappa_c \) given in Table 1. The values for \( \kappa_c \) as well as critical \( \kappa \) values were obtained from [13] and interpolation. Note that a finite temperature critical \( \kappa \) defined by a vanishing quark mass might differ from the \( T = 0 \) value \( \kappa_c \) by finite lattice spacing and finite volume effects. Indeed, the values for the quark mass obtained from the PCAC relation [15] are very small but not exactly zero, see Table 1. However, since the correlation functions above \( T_c \) are almost quark mass independent close to the chiral limit [16] it is not crucial to hit \( \kappa_c \) \((T \neq 0)\) precisely.

Some care has to be taken over the proper renormalization of the vector current \( J_{V\mu} \) used in the lattice calculations. In order to be able to compare re-

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### Table 1

| \( T/T_c \) |

\( N_\tau \)  | \( N_\lambda \)  | \( \beta \)  | \( \kappa_c \)  | \( m_q/T \)  | \( Z_V \)  | \# conf. |

| 1.5 | 12 | 6.640 | 1.4579 | 0.13536 | 0.0027(4) | 0.8184 | 25 |

| 16 | 6.872 | 1.4125 | 0.13495 | 0.0019(1) | 0.8292 | 40 |

| 3.0 | 12 | 7.192 | 1.3550 | 0.13440 | 0.0034(1) | 0.8421 | 40 |

| 16 | 7.457 | 1.3389 | 0.13390 | 0.0032(1) | 0.8512 | 40 |
sults obtained with different $N_c$ and $T$ as well as with calculations performed in different continuum regularization schemes we use the renormalized local current $J_{V\mu}^{\text{renorm}} \equiv (2k \cdot Z_V) J_{\mu}$. Here $Z_V(q^2)$ is the current renormalization constant for which we use the non-perturbative values determined in [13]. The overall error on the normalization [13,17], including the effect of the quark mass not being exactly zero has been estimated to be less than 1%. In the high temperature limit the ratio $G_V/T^3$ can be calculated perturbatively. The leading order result is obtained by using free, massless quark propagators in the calculation of the current–current correlation function. For vanishing momentum of the quark–anti-quark pair ($\vec{p} = 0$) the corresponding spectral function is given by $\sigma_V(\omega, T) = 0.75 \pi^{-2} \omega^2 \tanh(\omega/4T)$. In this limit the differential dilepton rate reduces to the Born rate,

$$\frac{d\Omega^{\text{Born}}}{d\omega d^3 p} (\vec{p} = 0) = \frac{5\alpha^2}{36\pi^4} \frac{1}{(\omega^2 + 1)^2}.$$  \hspace{1cm} (3)

In Fig. 1(a) we show our numerical results for $G_V/T^3$ calculated at $T/T_c = 1.5$ and 3. These results are compared to the lattice version of the free vector correlation function, $G_{V}^{\text{free.lat}} / T^3$ [18]. It is evident that the correlators obtained at two different temperatures agree with each other quite well and stay close to the leading order perturbative (free) vector correlation function already quite close to $T_c$. The deviations from $G_{V}^{\text{free.lat}} / T^3$ are better visible in the ratio $G_V^{\text{lat}} / G_{V}^{\text{free.lat}}$ which is shown in Fig. 1(b). It is noteworthy that $G_V(\tau)/T^3$ is definitely larger than the corresponding free correlator, $G_{V}^{\text{free.lat}} / T^3$ for both temperatures and all Euclidean times $0 < \tau T < 1$. This is furthermore supported by a calculation of these ratios for different values of the cut-off, e.g., for $N_\ell = 12$ and 16 at the same value of $T/T_c$. Results agree within statistical errors.

As is obvious from Eq. (2), the correlation function $G_V(\tau, T)$ receives contributions from all energies $\omega$. Due to the presence of the integration kernel $K(\tau, \omega) = \text{ch}(\omega(\tau - 1/2T))/\text{sh}(\omega/2T)$ large energies are, however, exponentially suppressed. This suppression becomes more efficient with increasing Euclidean time $\tau T$. For $\tau T \simeq 1/2$ the correlation function thus is most sensitive to the low energy part of $\sigma_V(\omega, T)$. In fact, using the free spectral function in Eq. (2) we estimate that energies larger than $\omega/T \simeq 15$ contribute only 2% to the central value $G_V(1/2T, T)/T^3$ whereas in the low energy regime, $\omega/T \lesssim 3$, the corresponding contribution still amounts to 12%. $G_V(1/2T, T)/T^3$ thus is sensitive to modifications of the vector spectral functions up to $\omega/T \simeq 15$ and provides a stringent test of models and approximate calculations of the low energy part of the vector spectral function. We find

$$G_V(1/2T, T) \frac{T}{T^3} = \begin{cases} 2.23 \pm 0.05, & T/T_c = 1.5, \\ 2.21 \pm 0.05, & T/T_c = 3, \end{cases}$$  \hspace{1cm} (4)

which is about 10% larger than the free, infinite volume value $G_V^{\text{free}}(1/2T, T) \equiv 2$ as well as the corresponding free value on a finite $64^3 \times 16$ lattice, $G_{V}^{\text{free.lat}}(1/2T, T) \equiv 2.0368$. Our result thus suggests

Fig. 1. Thermal vector current correlation functions, $G_V$, versus Euclidean time $\tau$ (a) and the ratio of $G_V$ and the free correlator $G_{V}^{\text{free.lat}}$ (b). Shown are results for $T/T_c = 1.5$ and 3. The solid line in (a) shows the correlation function for a freely propagating quark–anti-quark pair, $G_{V}^{\text{free.lat}}$, calculated with Wilson fermions on a finite lattice with spatial extent $N_\ell = 64$ and temporal extent $N_\tau = 16$. 

that at least for a significant range of energies the vector spectral function is enhanced over the free case. Such an increase cannot easily be incorporated in simple quasi-particle pictures, which only include massive quasi-particle poles in the quark spectral functions. They lead to a reduction of \(G_V(\tau, T)\) relative to \(G_V^{\text{free}}(\tau, T)\) and in turn to a reduced differential dilepton rate \([1,7,19]\). On the other hand, HTL-resummed perturbative calculations, which incorporates not only poles at non-zero quasi-particle masses but also additional contributions from cuts in the HTL-resummed quark spectral functions, show that both non-perturbative features of quarks propagating in a hot plasma play an important role for the low energy part of the meson spectral functions \([6]\). The cut contributions in the HTL approach \([6]\) as well as in 2-loop perturbative calculations \([5]\) even lead to divergent vector spectral functions at small energies which also makes \(G_V(\tau, T)\) to diverge for all Euclidean times \(\tau\). Assuming that at our aspect ratio of \(T/V^{1/3} = 4\) finite volume effects do not play an important role, the mere fact that we do find a finite result for the vector correlator via Eq. (2) implies that \(\sigma_V(\omega, T)\) vanishes in the limit \(\omega \to 0\).

3. Reconstructed spectral functions

The correlation functions \(G_V\) shown in Fig. 1 clearly stay close to the leading order perturbative result. This suggests that also the spectral functions \(\sigma_V\) are close to that of the free case. We have reconstructed \(\sigma_V\) from \(G_V\) using MEM, which has successfully been applied at \(T = 0\) \([9,11]\) and also been tested at finite temperature \([10]\). In order to take into account finite lattice cut-off effects in the reconstruction of the spectral function we introduce a lattice version, \(K_L(\tau, \omega, N_T)\), of the continuum kernel \(K(\tau, \omega)\) which appears in Eq. (2). With the lattice kernel the spectral function \(\sigma_V\) for vanishing momentum is defined through

\[
G_V(\tau, T) = \int_0^\infty d\omega \sigma_V(\omega, T) K_L(\tau, \omega, N_T),
\]

where \(K_L(\tau, \omega, N_T)\) is the finite lattice approximation of a kernel appropriate to describe the correlation function of a free boson at finite temperature,

\[
K_L(\tau, \omega, N_T) = \frac{2\omega}{T} \sum_{n=0}^{N_T-1} \frac{\exp(-i2\pi n\tau T)}{(2N_T \sin(n\pi/N_T))^2 + (\omega/T)^2}.
\]

This lattice kernel differs from the \(T = 0\) version used in \([9]\) in so far as we have explicitly taken into account the finite temporal extent of the lattice. Of course, \(K_L\) approaches \(K\) in the limit of large \(N_T\). Moreover, for the energy range important for the description of the correlation functions close to \(\tau T = 1/2\), i.e., for \(\omega/T < 15\), they differ by less than 2% already on lattices with temporal extent \(N_T = 16\). Using this lattice kernel we have checked that the MEM analysis of correlation functions constructed from given input spectral functions successfully reproduces these input functions. In particular, we find that the free vector spectral function for massless quarks can be reproduced already by using only 16 equally spaced points in the time interval \([0, 1/T]\).

In Fig. 2(a) we show spectral functions reconstructed from the vector current correlation functions at \(T/T_c = 1.5\) and 3 on the \(64^3 \times 16\) lattices. In the MEM analysis we have taken into account energies up to \(\omega/T = 4N_T\) and we used the lowest order zero temperature perturbative result \(m(\omega) = 0.75\pi^{-2}\omega^2\) as a default model. In order to test the statistical significance of our results we have split our total data sets in 8 jackknife blocks and performed a MEM analysis on each of these blocks. The resulting jackknife error, which is given as a band in the insertion of Fig. 2(a), shows that our data sample is large enough to yield statistically significant results. We also have checked that a variation of the default model by 20% only leads to minor changes in the low energy part of \(\sigma_V\), which stay within the error band shown in Fig. 2(a). Another question is to what extent the spectral function obtained from the MEM analysis is unique. As the MEM analysis is based on a finite data set the reconstruction of the spectral function itself is not unique. This uncertainty is incorporated in the MEM-error, which is calculated from the covariance matrix of the spectral function \([20]\) for four energy bins in the interval \(0 \leq \omega/T \leq 16\). The resulting error on the average value of \(\sigma_V(\omega, T)/\omega^2\) in these bins is also shown in Fig. 2(a).
Fig. 2. Reconstructed vector spectral function $\sigma_V$ in units of $\omega^2$ at zero momentum (a) and the resulting zero momentum differential dilepton rate (b) at $T/T_c = 1.5$ (dotted line) and 3 (dashed line). The solid lines give the free spectral function (a) and the resulting Born rate (b). The insertion in (a) shows the error band on the spectral function at $3T_c$ obtained from a jackknife analysis and errors on the average value of $\sigma_V(\omega,T)/\omega$ in four energy bins (see text).

The reconstructed spectral functions show a broad enhancement over the free spectral function for $\omega/T \gtrsim 16$, i.e., $\omega a \gtrsim 1$. This regime clearly is influenced by lattice cut-off effects. In fact, a similar enhancement is observed in $T = 0$ spectral functions and has been attributed to unphysical contributions of the heavy fermion doublers present in the Wilson (and clover) lattice fermion action [11]. As discussed above this regime of large energies does not contribute to the central values of the vector correlator for $tT \simeq 1/2$. The relevant energy regime is given in the insertion of Fig. 2(a). The enhancement observed for $G_V(1/2T, T)$ thus is due to the peak in $\sigma_V(\omega,T)/\omega^2$ found for $\omega/T \simeq (5–6)$. We furthermore note that for both temperatures the spectral functions have a sharp cut-off at small energies. They drop rapidly below $\omega/T \simeq 5$ and vanish below $\omega/T \simeq 3$. This is in contrast to perturbative calculations of $\sigma_V(\omega, T)$, which lead to a divergent spectral function for small energies.

In Fig. 2(b) we show the differential dilepton rate calculated from Eq. (1) and the low energy part of $\sigma_V$ shown in Fig. 2(a). The comparison with the Born rate shows that for all energies $\omega/T \gtrsim 4$ the difference is less than a factor 2. For energies $\omega/T \lesssim 3$ the dilepton rate drops rapidly and reflects the sharp cut-off found in the reconstructed spectral functions. Of course, with our present analysis, which is based on rather small statistics, we cannot rule out the existence of sharp resonances like the van Hove singularities present in the HTL-resummed calculations. It also may well be that the broad peak found by us will sharpen with increasing statistics as it has been observed in related MEM studies of hadron correlation functions at zero temperature [9]. Nonetheless, any further sharpening of resonance like peaks in a certain energy interval of the spectral function has to be compensated by an even closer agreement with the free spectral function in other energy intervals due to the constraint given by Eq. (4).

4. Conclusions

To conclude, we find that already close to the QCD phase transition temperature, i.e., for $T = 1.5T_c$ and $3T_c$, the vector correlation function deviates only by less than 15% from the leading order perturbative result. This also is reflected in the reconstructed spectral function which deviates by less than a factor two from the leading order perturbative form for energies $\omega \gtrsim 4T$. The most pronounced feature of the spectral function and the resulting dilepton rate is the presence of a sharp cut-off at low energies, $\omega \sim (2–3)T$. If a threshold of similar magnitude persist also closer to $T_c$, there will be no thermal contribution to the dilepton rate at energies $\omega \lesssim 2T_c$ during the entire expansion of the hot medium created in a heavy ion collision. This is consistent with present findings at SPS energies [8] and may become visible as a threshold in the dilepton rates for long-lived plasma states expected to be created at RHIC or LHC energies.
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References

Finite temperature behavior of the 3D Polyakov model with massless quarks

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Abstract

The $(2 + 1)$D Georgi–Glashow (or Polyakov) model with the additional fundamental massless quarks is explored at finite temperature. In the case of vanishing Yukawa coupling, it is demonstrated that the interaction of a monopole and an antimonopole in the molecule via quark zero modes leads to the decrease of the Berezinsky–Kosterlitz–Thouless critical temperature when the number of quark flavors is equal to one. If the number of flavors becomes larger, monopoles are shown to exist only in the molecular phase at any temperatures exceeding a certain exponentially small one. This means that for such a number of flavors and at such temperatures, no fundamental matter can be confined by means of the monopole mechanism.

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3D Georgi–Glashow model (else called the Polyakov model) is known to be one of the eldest and the most famous examples of theories allowing for an analytical description of confinement [1]. However, the phase structure of this model at finite temperature has been addressed only recently. Namely, first in Ref. [2] it has been shown that at the temperature $T_c = g^2/2\pi$ the weakly coupled monopole plasma in this model undergoes the Berezinsky–Kosterlitz–Thouless (BKT) [3] phase transition into the molecular phase. Then, in Ref. [4], it has been shown that approximately at the twice smaller temperature, there occurs another phase transition associated to the deconfinement of $W$-bosons.

In this Letter, we shall be interested in the finite-temperature properties of the monopole ensemble, rather than the ensemble of $W$-bosons. Because of that, let us first discuss in some more details the nature of the above-mentioned BKT phase transition. At high enough temperature, one can apply the idea of dimensional reduction. The dimensionally-reduced theory is then the 2D $XY$-model, but with the temperature-depending strength of the monopole–antimonopole ($M\bar{M}$) interaction. Due to this fact, the phase structure of the model becomes reversed with respect to that of the usual 2D $XY$-model. Namely, at the temperatures below $T_c$, monopoles exist in the plasma phase, that leads to the confinement of fundamental matter [1,5]. At $T > T_c$, the vacuum state is the molecular gas...
of bound $M\bar{M}$-pairs, and consequently fundamental quarks are deconfined [2]. The analogy with the 2D XYZ-model established in Ref. [6] is that spin waves of the 2D XYZ-model correspond to the free photons of the Polyakov model, while vortices correspond to magnetic monopoles.

Let us briefly discuss the BKT phase transition, occurring at $T = T_c$, in the language of the 2D XYZ-model. At $T < T_c$, the spectrum of the model is dominated by massless spin waves, and the periodicity of the angular variable is unimportant in this phase. The spin waves are unable to disorder the spin–spin correlation functions, and those decrease at large distances by a certain power law. On the contrary, at $T > T_c$, the periodicity of the angular variable becomes important. This leads to the appearance of topological singularities (vortices) of the angular variable, which, contrary to spin waves, have nonvanishing winding numbers. Such vortices condense and disorder the spin–spin correlation functions, so that those start decreasing exponentially with the distance. Thus, the nature of the BKT phase transition is the condensation of vortices at $T > T_c$. In another words, at $T > T_c$, there exist free vortices, which mix in the ground state (vortex condensate) of indefinite global vorticity. Contrary to that, at $T < T_c$, free vortices cannot exist, but they rather mutually couple into bound states of vortex–antivortex pairs. Such vortex–antivortex molecules are small-sized short-living (virtual) objects. Their dipole-type fields are short-ranged and therefore cannot disorder significantly the spin–spin correlation functions. However, when the temperature starts rising, the sizes of these molecules increase, until at $T = T_c$ they diverge, that corresponds to the dissociation of the molecules into pairs. Therefore, coming back to the Polyakov model, one of the methods (which will be employed below) to determine there the critical temperature of the BKT phase transition is to evaluate the mean squared separation in the $M\bar{M}$-molecule and find the temperature at which it starts diverging.

In this Letter, we shall consider the extension of the Polyakov model by the fundamental dynamical quarks, which are supposed to be massless. As it will be demonstrated, quark zero modes in the monopole field lead to the additional attraction between a monopole and an antimonopole in the molecule at high temperatures. In particular, when the number of these modes (equal to the number of massless flavors) is sufficiently large, the molecule shrinks so that its size becomes of the order of the inverse W-boson mass. Another factor which governs the size of the molecule is the characteristic range of localization of zero modes. Namely, it can be shown that the stronger zero modes are localized in the vicinity of the monopole center, the smaller molecular size is. In this Letter, we shall consider the case when the Yukawa coupling vanishes, and originally massless quarks do not acquire any mass. This means that zero modes are maximally delocalized. Such a weakness of the quark-mediated interaction of monopoles opens a possibility for molecules to undergo eventually the phase transition into the plasma phase. However, this will be shown to occur only provided that the number of flavors is equal to one, whereas at any larger number of flavors, the respective critical temperature becomes exponentially small. This means that the interaction mediated by such a number of zero modes is already strong enough to maintain the molecular phase at any temperature larger than that one.
Next, in 3D, the electric coupling $g$, the Yukawa coupling $h$, and the vacuum expectation value of the Higgs field $\eta$ have the dimensionality [mass]$^{1/2}$. The Higgs coupling $\lambda$ has the dimensionality [mass]. The masses of the $W$- and Higgs bosons are large compared to $g^2$ in the standard perturbative (else called weak-coupling) regime $g \ll \eta$ and read: $m_W = g \eta$, $m_H = \eta \sqrt{2} \lambda$. The inequality $g \ll \eta$ is necessary to ensure the spontaneous symmetry breaking from SU(2) to U(1). Note also that for the sake of simplicity, we omit the summation over the flavor indices, but consider the general case with an arbitrary number of flavors.

One can further see that the Dirac equation in the field of the third isotopic component of the 't Hooft–Polyakov monopole [7] decomposes into two equations for the components of the $SU(2)$-doublet $\psi$. The masses of these components stemming from such equations are equal to each other and read $m_q = \eta$. Next, the Dirac equation in the full monopole potential has been shown [8] to possess the zero mode, whose asymptotic behavior at $r \equiv |x| \gg m_q^{-1}$ has the following form:

$$\chi_{vn}^+ = N \frac{e^{-mr}}{r} \left( s^+ v_n - s^- v^+_n \right), \quad \chi_{vn}^- = 0. \quad (2)$$

Here, $\chi_{vn}^+$ are the upper and the lower components of the mode, i.e., $\psi = (\chi_{vn})^T$, next $n = 1, 2$ is the isotopic index, $v = 1, 2$ is the Dirac index, $s^+ = (1, 0)$, $s^- = (0, 1)$, and $N$ is the normalization constant.

It is a well known fact that in 3D, the 't Hooft–Polyakov monopole is actually an instanton [1,5]. Therefore, we can use the results of Ref. [9] on the quark contribution to the effective action of the instanton–antiinstanton molecule in QCD. Let us thus recapitulate the analysis of Ref. [9] adapting it to our model. To this end, we fix the gauge $\Phi^a = \eta \delta^a_3$ and define the analogue of the free propagator $S_0$ by the relation $S_0^{-1} = -i(\gamma \partial + m_q \gamma^5)$. Next, we define the propagator $S_M$ in the field of a monopole located at the origin, $\tilde{A}^aM \rightarrow e^{im_q x^j/(gr^2)}$ at $r \gg m_q$, by the formula $S_M^{-1} = S_0^{-1} - g^2 \tilde{A}^aM \tilde{A}^aM$. Obviously, the propagator $S_M$ in the field of an antimonopole located at a certain point $\vec{R}$, $\tilde{A}^aM(\vec{x}) = -\tilde{A}^aM(\vec{x} - \vec{R})$, is defined by the equation for $S_M^{-1}$ with the replacement $\tilde{A}^aM \rightarrow \tilde{A}^aM$. Finally, one can consider the molecule made out of these monopole and antimonopole and define the total propagator $S$ in the field of such a molecule, $\tilde{A}^a = \tilde{A}^aM + \tilde{A}^a\bar{M}$, by means of the equation for $S_M^{-1}$ with $\tilde{A}^aM$ replaced by $\tilde{A}^a$.

One can further introduce the notation $|\psi_n\rangle$, $n = 0, 1, 2, \ldots$, for the eigenfunctions of the operator $-i \gamma \cdot \vec{D}$ defined at the field of the molecule, namely $-i \gamma \cdot \vec{D} |\psi_n\rangle = \lambda_n |\psi_n\rangle$, where $\lambda_n = 0$. This yields the following formal spectral representation for the total propagator $S$:

$$S(\vec{x}, \vec{y}) = \sum_{n=0}^{\infty} \frac{|\psi_n(\vec{x})\rangle \langle \psi_n(\vec{y})|}{\lambda - i m_q \gamma^5}. \quad (3)$$

Next, it is convenient to employ the mean-field approximation, according to which zero modes dominate in the quark propagator, i.e.,

$$S(\vec{x}, \vec{y}) \simeq \frac{|\psi_0(\vec{x})\rangle \langle \psi_0(\vec{y})|}{-i m_q \gamma^5} + S_0(\vec{x}, \vec{y}).$$

Indeed, this approximation is valid, since in the weak-coupling regime the monopole sizes, equal to $m_W^{-1}$, are much smaller than the average distance in the $M\bar{M}$–plasma. This average distance has an order of magnitude $\zeta^{-1/3}$ (see, e.g., Ref. [10] for a discussion). Here, $\zeta \propto e^{-4\pi m_W / g^2}$ stands for the so-called monopole fugacity, which has the dimensionality [mass]$^{4/3}$, and $\epsilon \sim 1$ is a certain dimensionless function of $(m_H / m_W)$. Obviously, $\zeta$ is exponentially small in the weak-coupling regime under study. The approximation (3) remains valid for the molecular phase near the phase transition (i.e., when the temperature approaches the critical one from above), we shall be interested in. That is merely because in this regime, molecules become very much inflated being about to dissociate.
Within the notations adapted, one now has $S = (S_0^{-1} + S_M^{-1} - S_0^{-1})^{-1} = S_M^{-1} S_0^{-1} S_M$, where

$$S = S_0 - (S_M - S_0)S_0^{-1}(S_M - S_0) = S_0 - \frac{\langle \psi_0^M \rangle \langle \psi_0^M \rangle}{-im_\tau} S_0^{-1} \frac{\langle \psi_0^M \rangle \langle \psi_0^M \rangle}{-im_\tau},$$

and $|\psi_0^M\rangle$, $|\psi_0^M\rangle$ are the zero modes of the operator $-i \vec{D}$ defined at the field of a monopole and an antimonopole, respectively. Denoting further $a = \langle \psi_0^M \rangle \langle \psi_0^M \rangle S_0^{-1} |\psi_0^M\rangle$, it is straightforward to see by the definition of the zero mode that $a = \langle \psi_0^M \rangle \langle -i \vec{D} \psi_0^M \rangle = \langle \psi_0^M \rangle S_0^{-1} |\psi_0^M\rangle$. This yields $S = S_0 + (a^*/m_q^2)|\psi_0^M\rangle |\psi_0^M\rangle$, where the star stands for the complex conjugation, and, therefore, det$S = |1 + (a/m_q^2)|$ det$S_0$. Finally, defining the desired effective action as $\Gamma = \ln(\text{det}S^{-1}/\text{det}S_0^{-1})$, we obtain for it in the general case with $N_f$ flavors the following expression: $\Gamma = \text{const} + N_f \ln(m_q^2 + |a|^2)$. The constant in this formula, standing for the sum of effective actions defined at the monopole and the antimonopole, cancels out in the normalized expression for the mean squared separation in the $M\bar{M}$-molecule.

Let us further set $h$ equal to zero, and so $m_q$ is equal to zero as well. Notice first of all that although in this case the direct Yukawa interaction of the Higgs bosons with quarks is absent, they keep interacting with each other via the gauge field. Owing to this fact, the problem of finding a quark zero mode in the monopole field is still valid.\(^2\) The dependence of the absolute value of the matrix element $a$ on the distance $R$ between a monopole and an antimonopole can now be straightforwardly found. Indeed, we have $|a| \propto \int d^3 r/(r^2 |\vec{r} - \vec{R}|) = -4\pi \ln(\mu R)$, where $\mu$ stands for the IR cutoff.

Now we switch on the temperature $T = \beta^{-1}$, so that all the bosonic (fermionic) fields should be supplied with the periodic (antiperiodic) boundary conditions in the temporal direction, with the period equal to $\beta$. The magnetic-field lines of a single monopole thus cannot cross the boundary of the one-period region and should go parallel to this boundary at the distances larger than $\beta$. Therefore, monopoles separated by such distances interact via the 2D Coulomb law, rather than the 3D one. Recalling that the average distance between monopoles is of the order of $\zeta^{-1/3}$, we conclude that at $T \geq \zeta^{1/3}$, the monopole ensemble becomes two-dimensional (see, e.g., Ref. [2] for a detailed discussion of the dimensional reduction in the Polyakov model). However, at the temperatures below the exponentially small one, $\zeta^{1/3}$, monopoles keep interacting by the usual 3D Coulomb law, and the monopole confinement mechanism for the fundamental matter works at such temperatures under any circumstances.

We are now in the position to explore a possible modification of the standard BKT critical temperature [2] $T_c = g^2/2\pi$ due to the zero-mode mediated interaction. As it was discussed above, this can be done upon the evaluation of the mean squared separation in the $M\bar{M}$-molecule and further finding the temperature below which it starts diverging. In this way we should take into account that in the dimensionally-reduced theory, the usual Coulomb interaction of monopoles\(^3\) $R^{-1} = \sum_{n=-\infty}^{n=\infty} (R^2 + (\beta n)^2)^{-1/2}$ goes over into $-2T \ln(\mu R)$, where $R$ denotes the absolute value of the 2D vector $\vec{R}$. This statement can be checked, e.g., by virtue of the Euler–Mac Laurin formula. As far as the novel logarithmic interaction, $\ln(\mu R) = \sum_{n=-\infty}^{n=\infty} \ln(\mu (R^2 + (\beta n)^2))^{1/2}$, is concerned, it transforms into

$$\pi T \ln R + \ln[1 - \exp(-2\pi T \ln R)] - \ln 2.$$  

\(^2\) Note that according to Eq. (2) this mode will be non-normalizable in the sense of a discrete spectrum. However, in the gapless case $m_q = 0$ under discussion, the zero mode, which lies exactly on the border of the two contiguous Dirac seas, should clearly be treated not as an isolated state of a discrete spectrum, but rather as a state of a continuum spectrum. (A similar treatment of the zero mode of a massless left-handed neutrino on electroweak Z-strings has been discussed in Ref. [11].) This means that it should be understood as follows: $|\psi_0(x)\rangle = \lim_{p \to 0} (e^{ip \tau} / \sqrt{p})$, where $p = |\vec{p}|$. Once being considered in this way, zero modes are normalizable by the standard condition of normalization of the radial parts of spherical waves, $R_{pl}$, which reads [12] $\int_0^\infty dr r^2 R_{pl}^2 R_{pl} = 2\pi \delta(p^2 - p)$.

\(^3\) Without the loss of generality, we consider the molecule with the temporal component of $\vec{R}$ equal to zero.
Let us prove this statement. To this end, we employ the following formula [13]:

\[
\sum_{n=1}^{\infty} \frac{1}{n^2 + x^2} = \frac{1}{2x} \left[ \pi \coth(\pi x) - \frac{1}{x} \right].
\]

This yields

\[
x \sum_{n=-\infty}^{+\infty} \frac{1}{x^2 + (2\pi n/a)^2} = \frac{x}{x} + \frac{x a^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2 + (x a/2\pi)^2} = \frac{a}{2} \coth \left( \frac{ax}{2} \right).
\]

On the other hand, the L.H.S. of this expression can be written as

\[
\frac{1}{2} \frac{d}{dx} \sum_{n=-\infty}^{+\infty} \ln \left( x^2 + \left( \frac{2\pi n}{a} \right)^2 \right).
\]

Integrating over \( x \) with the constant of integration set to zero, we get

\[
\sum_{n=-\infty}^{+\infty} \ln \left( x^2 + \left( \frac{2\pi n}{a} \right)^2 \right) = a \int dx \coth \left( \frac{ax}{2} \right) = 2 \ln \sinh \left( \frac{ax}{2} \right) = ax + 2 \ln(1 - e^{-ax}) - 2 \ln 2.
\]

Setting \( 2\pi /a = \mu \beta \) and \( x = \mu R \) we arrive at Eq. (4).

Thus, the statistical weight of the quark-mediated interaction in the molecule at high temperatures has the form

\[
\exp(-2N_f \ln |a|) \propto [\pi T R + \frac{1}{2} - \exp(-2\pi T R)] - \ln 2)^{-2N_f}.
\]

Accounting for both (former) logarithmic and Coulomb interactions, we eventually arrive at the following expression for the mean squared separation \( \langle L^2 \rangle \) in the molecule as a function of \( T, g, \) and \( N_f \):

\[
\langle L^2 \rangle = \frac{\int_{m_W}^{\infty} dR R^{1 - \frac{2N_f}{\pi}} [\pi T R + \ln |1 - \exp(-2\pi T R)| - \ln 2]^{-2N_f}}{\int_{m_W}^{\infty} dR R^{1 - \frac{2N_f}{\pi}} [\pi T R + \ln |1 - \exp(-2\pi T R)| - \ln 2]^{-2N_f}}.
\]

In this equation, we have put the lower limit of integration equal to the inverse mass of the W-boson, which acts as an UV cutoff.

At large \( R, \ln 2 \ll \pi T R \) and \( |\ln |1 - \exp(-2\pi T R)|| \ll \exp(-2\pi T R) \ll \pi T R \). Consequently, we see that \( \langle L^2 \rangle \) is finite at \( T > T_c = (2 - N_f) g^2 /4\pi \), that reproduces the standard result [2] at \( N_f = 0 \). For \( N_f = 1 \), the plasma phase is still present at \( T < g^2 /4\pi \), whereas for \( N_f \geq 2 \) the monopole ensemble may exist only in the molecular phase at any temperature larger than \( \zeta^{1/3} \). Clearly, at \( N_f \gg \max(1, 4\pi T /g^2) \), \( \sqrt{\langle L^2 \rangle} \rightarrow m_W \), which means that such a large number of zero modes shrinks the molecule to the minimal admissible size. Note finally that both the obtained critical temperature \( g^2 /4\pi \) and the standard one (in the absence of quarks), \( g^2 /2\pi \), are obviously much larger than \( \zeta^{1/3} \), that fully validates the idea of dimensional reduction.

In conclusion of this Letter, we have found the critical temperature of the monopole BKT phase transition in the weak-coupling regime of the Polyakov model extended by the massless dynamical quarks, which interact with the Higgs boson only via the gauge field. It has been shown that for \( N_f = 1 \), this temperature becomes twice smaller than the one in the absence of quarks, whereas for \( N_f \geq 2 \) it becomes exponentially small, namely of the order of \( \zeta^{1/3} \). The latter effect means that this number of quark zero modes, which strengthen the attraction of a monopole and an antimonopole in the molecule, becomes enough for the support of the molecular phase at any temperature exceeding that exponentially small one. Therefore, for \( N_f \geq 2 \), no fundamental matter (including dynamical quarks themselves) can be confined at such temperatures by means of the monopole mechanism.
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Texture zeros and majorana phases of the neutrino mass matrix

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Abstract

We present the generic formulas to calculate the ratios of neutrino masses and the Majorana phases of CP violation from the neutrino mass matrix with two independent vanishing entries in the flavor basis where the charged lepton mass matrix is diagonal. An order-of-magnitude illustration is given for seven experimentally acceptable textures of the neutrino mass matrix, and some analytical approximations are made for their phenomenological consequences at low energy scales. © 2002 Elsevier Science B.V. All rights reserved.

1. The atmospheric and solar neutrino oscillations observed in the Super-Kamiokande experiment [1] have provided robust evidence that neutrinos are massive and lepton flavors are mixed. A full description of the mass spectrum and flavor mixing in the framework of three lepton families requires twelve real parameters: three charged lepton masses ($m_e, m_\mu, m_\tau$), three neutrino masses ($m_1, m_2, m_3$), three flavor mixing angles ($\theta_x, \theta_y, \theta_z$), one Dirac-type CP-violating phase ($\delta$) and two Majorana-type CP-violating phases ($\rho$ and $\sigma$). So far only the masses of charged leptons have been accurately measured [2]. Although we have achieved some preliminary knowledge on two neutrino mass-squared differences and three flavor mixing angles from current neutrino oscillation experiments, much more effort is needed to determine these parameters precisely. The more challenging task is to pin down the absolute neutrino mass scale and the CP-violating phases. Towards reaching these goals, a number of new neutrino experiments have been proposed [3].

After sufficient information on neutrino masses and lepton flavor mixing parameters is experimentally accumulated, a determination of the textures of lepton mass matrices should become possible. On the other hand, the textures of charged lepton and neutrino mass matrices may finally be derived from a fundamental theory of lepton mass generation, which is unfortunately unknown for the time being. It is therefore important in phenomenology to investigate how the textures of lepton mass matrices can link up with the observables of lepton flavor mixing.

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Recently Frampton, Glashow and Marfatia [4] have examined the possibility that a restricted class of lepton mass matrices may suffice to describe current experimental data. They find seven acceptable textures of the neutrino mass matrix with two independent vanishing entries in the flavor basis where the charged lepton mass matrix is diagonal.

In this Letter we carry out a further study of two-zero textures of the neutrino mass matrix. Our work is different from Ref. [4] in several aspects: (a) we write out the generic constraint equations for the neutrino mass matrix with two independent vanishing entries, from which the analytically exact expressions of neutrino mass ratios can be derived; (b) the formulas to calculate the Majorana-type CP-violating phases are presented; (c) the relative magnitudes of neutrino masses, the Majorana phases, the ratio of two neutrino mass-squared differences, and the effective mass term of the neutrinoless double beta decay are estimated by taking typical inputs of the flavor mixing angles and the Dirac-type CP-violating phase; and (d) an order-of-magnitude illustration is given for seven two-zero textures of the neutrino mass matrix.

2. In the flavor basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix can be written as

\[ M = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T, \]  

where \( m_i \) (for \( i = 1, 2, 3 \)) denote the real and positive neutrino masses, and \( V \) is the lepton flavor mixing matrix linking the neutrino mass eigenstates \((\nu_1, \nu_2, \nu_3)\) to the neutrino flavor eigenstates \((\nu_e, \nu_\mu, \nu_\tau)\) in the chosen basis. A full description of \( V \) needs six real parameters: three mixing angles and three CP-violating phases. Note that \( V \) can always be expressed as a product of the Dirac-type flavor mixing matrix \( U \) (consisting of three mixing angles and one CP-violating phase) and a diagonal phase matrix \( P \) (consisting of two nontrivial Majorana phases): \( V = UP \). Then we may rewrite \( M \) in Eq. (1) as

\[ M = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T, \]  

where two Majorana-type CP-violating phases are included into the complex neutrino mass eigenvalues \( \lambda_i \), and the relation \(|\lambda_i| = m_i\) holds. Without loss of generality, we take

\[ \lambda_1 = m_1 e^{2i\rho}, \quad \lambda_2 = m_2 e^{2i\sigma}, \quad \lambda_3 = m_3. \]  

In the following we shall show that both the neutrino mass ratios \((m_1/m_3\) and \(m_2/m_3\)) and the Majorana phases \((\rho \text{ and } \sigma)\) can be determined, if two independent entries of \( M \) vanish.

As \( M \) is symmetric, it totally has six independent complex entries. If two of them vanish, i.e., \( M_{ab} = M_{a'b'} = 0 \), we obtain the following constraint relations:

\[ \sum_{i=1}^{3} (U_{ai}U_{bi}\lambda_i) = 0, \quad \sum_{i=1}^{3} (U_{ai}U_{bi}\lambda_i) = 0, \]  

where each of the four subscripts runs over \( e, \mu \) and \( \tau \), but \((a, b) \neq (a, b)\). Solving Eq. (4), we find

\[ \frac{\lambda_1}{\lambda_3} = \frac{U_{a3}U_{b3}U_{a2}U_{b2} - U_{a2}U_{b2}U_{a3}U_{b3}}{U_{a2}U_{b2}U_{a1}U_{b1} - U_{a1}U_{b1}U_{a2}U_{b2}}, \]  

and

\[ \frac{\lambda_2}{\lambda_3} = \frac{U_{a1}U_{b1}U_{a3}U_{b3} - U_{a3}U_{b3}U_{a1}U_{b1}}{U_{a2}U_{b2}U_{a1}U_{b1} - U_{a1}U_{b1}U_{a2}U_{b2}}. \]
One can observe that the left-hand sides of Eqs. (5) and (6) are associated with the Majorana-type CP-violating phases, while the right-hand sides of Eqs. (5) and (6) are associated with the Dirac-type CP-violating phase hidden in the elements of $U$. Therefore, two Majorana phases must depend upon the Dirac-type CP-violating phase. This dependence results simply from the texture zeros of $M$ that we have taken.

Comparing Eq. (5) or Eq. (6) with Eq. (3), we arrive at the expressions of two neutrino mass ratios as follows:

$$ \frac{m_1}{m_3} = \frac{|U_{a3}U_{b3}U_{c3} - U_{a2}U_{b2}U_{c2} + U_{a1}U_{b1}U_{c2}|}{|U_{a2}U_{b2}U_{c1} - U_{a1}U_{b1}U_{c2}|}, \quad \frac{m_2}{m_3} = \frac{|U_{a1}U_{b1}U_{c3} - U_{a3}U_{b3}U_{c1}|}{|U_{a2}U_{b2}U_{c1} - U_{a1}U_{b1}U_{c2}|}. $$

(7)

Furthermore, the expressions of two Majorana phases are found to be

$$ \rho = \frac{1}{2} \text{arg} \left[ \frac{U_{a3}U_{b3}U_{c2} - U_{a2}U_{b2}U_{c3}}{U_{a2}U_{b2}U_{c1} - U_{a1}U_{b1}U_{c2}} \right], \quad \sigma = \frac{1}{2} \text{arg} \left[ \frac{U_{a1}U_{b1}U_{c3} - U_{a3}U_{b3}U_{c1}}{U_{a2}U_{b2}U_{c1} - U_{a1}U_{b1}U_{c2}} \right]. $$

(8)

With the inputs of three flavor mixing angles and the Dirac-type CP-violating phase, we are able to predict the relative magnitudes of three neutrino masses and the values of two Majorana phases. This predictability allows us to examine whether the chosen texture of $M$ with two independent vanishing entries is empirically acceptable or not.

Indeed, the prediction for $m_1/m_2$ and $m_2/m_3$ in a given pattern of $M$ is required to be compatible with the hierarchy of solar and atmospheric neutrino mass-squared differences:

$$ R_\odot \equiv \left| \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} \right| \approx \frac{\Delta m^2_{\odot}}{\Delta m^2_{\text{atm}}} \ll 1. $$

(9)

The magnitude of $R_\odot$ depends upon the explicit solution to the solar neutrino problem. For the large-angle Mikheyev–Smirnov–Wolfenstein (MSW) oscillation of solar neutrinos [5], which is most favored by the present Super-Kamiokande [1] and SNO [6] data, we have $R_\odot \sim \mathcal{O}(10^{-2})$. Because of $|V_{e3}|^2 = |U_{e3}|^2 \ll 1$ [7], the atmospheric neutrino oscillation is approximately decoupled from the solar neutrino oscillation.

With the help of Eqs. (7) and (8), one can calculate the effective mass term of the neutrinoless double beta decay, whose magnitude amounts to $|M_{ee}|$. The explicit expression of $|M_{ee}|$ reads as follows:

$$ |M_{ee}| = m_3 \left| \frac{m_1}{m_3} \right|^2 e^{2i\rho} + m_2 \left| \frac{m_2}{m_3} \right|^2 e^{2i\sigma} + U_{ee}^2. $$

(10)

The Heidelberg–Moscow Collaboration has reported $|M_{ee}| < 0.34$ eV at the 90% confidence level [8]. Useful information on the absolute mass scale of neutrinos could in principle be extracted from a more accurate measurement of $|M_{ee}|$ in the future.

3. As already pointed out in Ref. [4], there are totally fifteen logical possibilities for the texture of $M$ with two independent vanishing entries, but only seven of them are in accord with current experimental data and empirical hypotheses. The seven acceptable patterns of $M$ are listed in Table 1, where all the non-vanishing entries are symbolized by ‘×’s. To work out the explicit expressions of $\lambda_1/\lambda_3$ and $\lambda_2/\lambda_3$ in each case, we adopt the following parametrization for the Dirac-type flavor mixing matrix $U$:

$$ U = \begin{pmatrix} c_x c_z & s_x c_z & s_c \\ -c_s s_x c_z & -s_x c_z e^{-i\delta} & -s_x s_c e^{-i\delta} \\ -c_s c_y s_z + s_x s_y e^{-i\delta} & -s_x c_y s_z + c_x c_y e^{-i\delta} & s_x c_z \end{pmatrix}, $$

(11)

where $s_x \equiv \sin \theta_x, \ c_x \equiv \cos \delta_x, \ s_y \equiv \sin \theta_y, \ c_y \equiv \cos \delta_y, \ s_z \equiv \sin \theta_z, \ c_z \equiv \cos \delta_z$, and so on. The advantage of this phase choice is that the Dirac-type CP-violating phase $\delta$ does not appear in the effective mass term of the neutrinoless double beta decay [9]. In other words, the
An order-of-magnitude illustration of $M$ is given by using typical inputs of $\theta_x$, $\theta_y$, $\theta_z$ and $\delta$, as explained in the text. An instructive result for $m_1/m_3$, $m_2/m_3$, $\rho$, $\sigma$, $R_\nu$ and $|M_{ee}|$ may then be obtained.

**Pattern A$_1$.** $M_{ee} = M_{e\mu} = 0$ (i.e., $a = b = e$; $\alpha = e$ and $\beta = \mu$). We obtain

$$
\frac{\lambda_1}{\lambda_3} = \frac{s_x}{c_x} \left( s_x s_y e^{i\delta} - c_x \right), \quad \frac{\lambda_2}{\lambda_3} = \frac{s_z}{c_z} \left( c_x s_y e^{i\delta} + c_z \right).
$$

As current experimental data favor $\sin^2 2\theta_x \sim O(1)$, $\sin^2 2\theta_y \approx 1$ and $\sin^2 2\theta_z \leq 0.1$ [1,6,7], one may make an analytical approximation for the exact result obtained above. By use of Eqs. (7)–(10), we arrive explicitly at

$$
\frac{m_1}{m_3} \approx \frac{t_x t_y s_z}{s_x}, \quad \frac{m_2}{m_3} \approx \frac{t_y}{t_x^2}, \quad \rho \approx \frac{\delta}{2}, \quad \sigma \approx \frac{\delta}{2} \pm \frac{\pi}{2},
$$

$$
R_\nu \approx \frac{t_\nu^2}{t_x^2} \left| 1 - t_\nu^2 \right| s_z^2, \quad |M_{ee}| = 0
$$

to lowest order, where $t_x \equiv \tan \theta_x$ and so on. Taking the typical inputs $\theta_x = 30^\circ$, $\theta_y = 40^\circ$, $\theta_z = 5^\circ$ and $\delta = 90^\circ$, we obtain $m_1/m_3 \approx 0.04$, $m_2/m_3 \approx 0.13$, $\rho \approx 45^\circ$, and $\sigma \approx 135^\circ$ (or $-45^\circ$). In addition, we get $R_\nu \approx 0.014$, consistent with our empirical hypothesis that the solar neutrino deficit is attributed to the large-angle MSW oscillation. The vanishing of $|M_{ee}|$ implies that it is in practice impossible to detect the neutrinoless double beta decay.
Pattern A. $M_{ee} = M_{e	au} = 0$ (i.e., $a = b = e$; $\alpha = e$ and $\beta = \tau$). We obtain

$$\frac{\lambda_1}{\lambda_3} = -\frac{s_z}{c_x} \left( \frac{c_x c_y e^{i \delta} + s_z}{c_z s_y} \right), \quad \frac{\lambda_2}{\lambda_3} = \frac{s_z}{c_x} \left( \frac{c_x c_y e^{i \delta} - s_z}{c_z s_y} \right).$$

(14)

In the lowest-order approximation, we explicitly obtain

$$\frac{m_1}{m_3} \approx t_{y} s_z, \quad \frac{m_2}{m_3} \approx \frac{1}{t_{y}} s_z, \quad \rho \approx \frac{\delta}{2} \pm \frac{\pi}{2}, \quad \sigma \approx \frac{\delta}{2},$$

$$R_{e} \approx \frac{1}{t_{y}^2} |1 - t_{y}^4 | s_z, \quad |M_{ee}| \approx 0.$$

(15)

Using the same inputs as above, we get $m_1/m_3 \approx 0.06$, $m_2/m_3 \approx 0.18$, $\rho \approx 135^\circ$ (or $-45^\circ$), $\sigma \approx 45^\circ$, and $R_{e} \approx 0.03$. We see that the phenomenological consequences of Patterns A1 and A2 are nearly the same [4]. However, Pattern A2 seems to be more interesting for model building [10], in particular when the spirit of lepton–quark similarity is taken into account.

Pattern B. $M_{\mu\mu} = M_{e\tau} = 0$ (i.e., $a = b = \mu$; $\alpha = e$ and $\beta = \tau$). We obtain

$$\frac{\lambda_1}{\lambda_3} = \frac{s_z}{c_x} \left( 2 c_x^2 s_y^2 - s_x^2 c_y^2 - s_y s_z (c_x^2 s_y^2 e^{i \delta} + c_y^2 s_x^2 e^{-i \delta}) \right),$$

$$\frac{\lambda_2}{\lambda_3} = \frac{s_z}{c_x} \left( s_x^2 c_y^2 + (s_x^2 - c_x^2) c_y^2 e^{i \delta} + s_y s_z (1 + c_x^2) e^{-i \delta} \right).$$

(16)

The smallness of $s_x^2$ allows us to make a similar analytical approximation as before. To lowest order, we find

$$\frac{m_1}{m_3} \approx t_{y}^2, \quad \rho \approx \sigma \approx \frac{\delta}{2}, \quad R_{e} \approx \frac{1 + t_{y}^2}{t_{y}} |s_z| c_x, \quad |M_{ee}| \approx m_{3} t_{y}^2,$$

(17)

where $t_{y} \equiv \tan 2 \theta_y$ and $c_\delta \equiv \cos \delta$. Note that

$$\frac{m_1}{m_3} \approx \frac{m_2}{m_3} \approx \frac{4 s_c s_y}{s_x s_y}, \quad \sigma \approx \rho \approx \frac{2 s_c s_y}{t_{y}^2 s_x s_y},$$

(18)

in the next-to-leading order approximation, where $s_x \equiv \sin \delta$ and $s_y \equiv \sin 2 \theta_y$. Typically taking $\theta_y = 30^\circ$, $\theta_x = 40^\circ$, $\theta_e = 5^\circ$ and $\delta = 89^\circ$, we arrive at $m_1/m_3 \approx m_2/m_3 \approx 0.7$ with a difference of about 0.007, $\sigma \approx \rho \approx 179^\circ$ (or $-1^\circ$) with a difference of about $3^\circ$, $R_{e} \approx 0.02$, and $|M_{ee}|/m_3 \approx 0.7$. One can see that $|\delta| \approx 90^\circ$ is required in this texture of $M$ for plausible inputs of $\theta_y$ and $\theta_x$, such that $R_{e}$ gets suppressed sufficiently. If Pattern B1 is realistically correct, large CP-violating effects may be observable in long-baseline neutrino oscillations. It is also worth mentioning that a typical upper bound on three nearly degenerate neutrino masses can be extracted from the Heidelberg–Moscow experiment [8]: $m_1 \approx m_2 \approx 0.7 m_3 \approx |M_{ee}| < 0.34$ eV. This bound is certainly compatible with the present direct-mass-search experiments [2], in particular, for the electron neutrino.

Pattern B. $M_{e\tau} = M_{\mu\tau} = 0$ (i.e., $a = b = \tau$; $\alpha = e$ and $\beta = \mu$). We obtain

$$\frac{\lambda_1}{\lambda_3} = \frac{s_z}{c_x} \left( 2 c_x^2 s_y^2 - s_x^2 c_y^2 - s_y s_z (c_x^2 s_y^2 e^{i \delta} + c_y^2 s_x^2 e^{-i \delta}) \right),$$

$$\frac{\lambda_2}{\lambda_3} = \frac{s_z}{c_x} \left( s_x^2 c_y^2 + (s_x^2 - c_x^2) c_y^2 e^{i \delta} + s_y s_z (1 + c_x^2) e^{-i \delta} \right).$$

(19)
In the lowest-order approximation, we explicitly obtain
\[
\frac{m_1}{m_3} \approx \frac{m_2}{m_3} \approx \frac{1}{t_y^2}, \quad \rho \approx \sigma \approx \delta \pm \frac{\pi}{2}, \quad R_\nu \approx \frac{1 + t_y^2}{t_y} |t_{2y} c_3| s_z, \quad |M_{ee}| \approx \frac{m_3}{t_y^2},
\]
(20)
together with
\[
\frac{m_2}{m_3} = \frac{m_1}{m_3} \approx \frac{4 s_y c_y}{s_2 s_2 y}, \quad \sigma - \rho \approx \frac{2 t_y^2 s_z s_y}{s_2 s_2 y}.
\]
(21)
Using the same inputs as in Pattern B1, we get \( m_2/m_3 \approx m_1/m_3 \approx 1.4 \) with a difference of about 0.007, \( \sigma \approx \rho \approx 179^\circ \) (or \( -1^\circ \)) with a difference of about 1.4\(^\circ\), \( R_\nu \approx 0.02 \), and \( |M_{ee}|/m_3 \approx 1.4 \). Because of \( t_y \sim O(1) \), the phenomenological consequences of Patterns B1 and B2 are almost the same.

**Pattern B2.** \( M_{\mu\mu} = M_{e\mu} = 0 \) (i.e., \( a = b = \mu; \alpha = e \) and \( \beta = \mu \)). We obtain
\[
\frac{\lambda_1}{\lambda_3} = -\frac{s_y}{c_y}, \quad c_s s_y + c_y s_y, e^{-i\delta} e^{2i\delta}, \quad \frac{\lambda_2}{\lambda_3} = -\frac{s_y}{c_y}, \quad s_y c_y + s_y s_y e^{-i\delta} e^{2i\delta}.
\]
(22)
The approximate expressions for the neutrino mass ratios, the Majorana phases and the observables \( R_\nu \) and \( |M_{ee}| \) turn out to be
\[
\frac{m_1}{m_3} \approx \frac{m_2}{m_3} \approx \frac{t_y^2}{t_y^2}, \quad \rho \approx \sigma \approx \delta \pm \frac{\pi}{2}, \quad R_\nu \approx \frac{1 + t_y^2}{t_y} |t_{2y} c_3| s_z, \quad |M_{ee}| \approx m_3 t_y^2.
\]
(23)
In addition,
\[
\frac{m_2}{m_3} = \frac{m_1}{m_3} \approx \frac{4 t_y^2 s_z c_y}{s_2 s_2 y}, \quad \rho - \sigma \approx \frac{2 t_y^2 s_z}{s_2 s_2 y}.
\]
(24)
in the next-to-leading order approximation. Taking the same inputs as in Pattern B1, we find \( m_2/m_3 \approx m_1/m_3 \approx 0.7 \) with a difference of about 0.005, \( \rho \approx \sigma \approx 179^\circ \) (or \( -1^\circ \)) with a difference of about 2\(^\circ\), \( R_\nu \approx 0.014 \), and \( |M_{ee}|/m_3 \approx 0.7 \). One can see that the phenomenological consequences of Pattern B1 are essentially the same as those of Pattern B2. This point has been observed in Ref. [4].

**Pattern B4.** \( M_{\tau\tau} = M_{e\tau} = 0 \) (i.e., \( a = b = \tau; \alpha = e \) and \( \beta = \tau \)). We obtain
\[
\frac{\lambda_1}{\lambda_3} = -\frac{s_y}{c_y}, \quad c_s s_y + c_y s_y, e^{-i\delta} e^{2i\delta}, \quad \frac{\lambda_2}{\lambda_3} = -\frac{s_y}{c_y}, \quad s_y c_y + s_y s_y e^{-i\delta} e^{2i\delta}.
\]
(25)
To lowest order, we get the following approximate results
\[
\frac{m_1}{m_3} \approx \frac{m_2}{m_3} \approx \frac{1}{t_y^2}, \quad \rho \approx \sigma \approx \delta \pm \frac{\pi}{2}, \quad R_\nu \approx \frac{1 + t_y^2}{t_y} |t_{2y} c_3| s_z, \quad |M_{ee}| \approx \frac{m_3}{t_y^2},
\]
(26)
together with
\[
\frac{m_2}{m_3} = \frac{m_1}{m_3} \approx \frac{4 t_y^2 s_z c_y}{s_2 s_2 y t_y^2}, \quad \rho - \sigma \approx \frac{2 t_y^2 s_z}{s_2 s_2 y}.
\]
(27)
Using the same inputs as in Pattern B1, we obtain \( m_1/m_3 \approx m_2/m_3 \approx 1.4 \) with a difference of about 0.01, \( \rho \approx \sigma \approx 179^\circ \) (or \( -1^\circ \)) with a difference of about 2\(^\circ\), \( R_\nu \approx 0.03 \), and \( |M_{ee}|/m_3 \approx 1.4 \). One can see that the phenomenological consequences of Patterns B1, B2, B3 and B4 are nearly the same. Therefore, it is very difficult, even impossible, to distinguish one of them from the others in practical experiments. Nevertheless, one of the four textures might be more favored than the others in model building, when underlying flavor symmetries responsible for those texture zeros are taken into account.
To lowest order, we get

Within each category, however, category A is experimentally distinguishable from category B or C. To be specific, as well as Pattern C.

\[ M \]

\[ R_{\nu} \]

\[ \lambda_1 = \frac{-c_s c_t^2}{s_c} + 2 s_c s_t s_c e^{i \delta} \]

\[ \lambda_2 = \frac{s_s c_s c_t e^{i \delta}}{s_c} - 2 c_s s_t c_s e^{i \delta} \]

\[ \lambda_3 = \frac{s_s c_s c_t e^{i \delta}}{s_c} - 2 c_s s_t c_s e^{i \delta} \]

(28)

Assuming \( s_2^2 \ll 1 \) and \( t_x \sim t_y \sim O(1) \), we may make an analytical approximation for the exact result in Eq. (28).

To lowest order, we get

\[ \frac{m_1}{m_3} \approx \sqrt{1 - \frac{2c_8}{t_x t_2 y s_c} + \frac{1}{t_x^2 t_2 y^2 s_c^2}} \]

\[ \frac{m_2}{m_3} \approx \sqrt{1 + \frac{2 t_x c_8}{t_2 y s_c} + \frac{t_2^2}{t_x^2 t_2 y^2 s_c^2}} \]

\[ R_{\nu} = \frac{1 + t_x t_y}{t_x t_y} \left| \frac{2}{1 - t_x^2 t_2 y s_c c_8} \right| \]

\[ |M_{ee}| \approx m_3 \sqrt{1 - \frac{4c_8}{t_2 x^2 t_2 y s_c} + \frac{4}{t_2 x^2 t_2 y^2 s_c^2}} \]

(29)

as well as

\[ \rho \approx \delta + \epsilon \pm \pi \]

\[ t_x = \frac{s_8}{t_x t_2 y s_c - c_8} \]

\[ t_c = \frac{t_x s_8}{t_2 y s_c + t_x c_8} \]

(30)

One can observe that a small value of \( R_{\nu} \) is possible if and only if the condition \( t_x t_2 y s_c c_8 \approx 1 \) is satisfied [4]. Some fine tuning of the inputs seems unavoidable in this case. Taking \( \theta_x = \theta_y = 44.8^\circ \), \( \theta_z = 5^\circ \) and \( \delta = 90^\circ \), for example, we find \( R_{\nu} \approx 0.03 \), \( \rho \approx +5^\circ \) (or \( 185^\circ \)), \( \sigma \approx -5^\circ \) (or \( 175^\circ \)), and \( m_1 \approx m_2 \approx m_3 \approx |M_{ee}| \). If this texture of \( M \) is realistically correct, large CP violation may manifest itself in neutrino oscillations.

4. As shown above, the seven patterns of \( M \) can be classified into three distinct categories [4]: A (with \( A_1 \) and \( A_2 \)), B (with \( B_1, B_2, B_3 \) and \( B_4 \)), and C. It is experimentally difficult or impossible to distinguish the textures of \( M \) within each category. However, category A is experimentally distinguishable from category B or C. To be specific, let us summarize the main phenomenological consequences of each category:

1. The neutrino mass spectrum: \( m_1 \sim m_2 \ll m_3 \) in category A; \( m_1 \sim m_2 \sim m_3 \) in category B; and \( m_1 \sim m_2 \sim m_3 \) in category C.

2. The Dirac phase of CP violation: \( \delta \) is not constrained in category A; \( |\delta| \approx \pi/2 \) in category B (for plausible inputs of \( \theta_x \) and \( \theta_y \)); and \( \delta \) is sensitive to the values of three mixing angles in category C.

3. The Majorana phases of CP violation: \( |\sigma - \rho| \approx \pi/2 \) in category A; \( \sigma \approx \rho \) in category B; and \( \sigma \sim \rho \) in category C.

4. The neutrinoless double beta decay: \( |M_{ee}| \approx 0 \) in category A; \( |M_{ee}| \sim m_3 \) in category B; and \( |M_{ee}| \sim m_3 \) in category C.

We see that it is not easy to distinguish between category B and category C, unless the values of flavor mixing angles (\( \theta_x, \theta_y, \theta_z \)) and the ratio of solar and atmospheric neutrino mass-squared differences (\( R_{\nu} \)) can be accurately determined.

It is worth remarking that the “inverse” neutrino mass hierarchy \( m_1 \gg m_2 \gg m_3 \) cannot be incorporated with three categories of \( M \) discussed above. The reason is simply that such a hierarchy conflicts with our empirical hypotheses [4], i.e., \( \Delta m_{\text{sun}}^2 = |m_2^2 - m_1^2| \) and \( \Delta m_{\text{atm}}^2 = |m_3^2 - m_2^2| \). If \( m_1 \gg m_2 \gg m_3 \) were assumed, we would inevitably be led to \( R_{\nu} \equiv \frac{|m_2^2 - m_1^2|/|m_3^2 - m_2^2|}{m_2^2/m_3^2} \gg 1 \), contrary to the prerequisite \( R_{\nu} \approx \Delta m_{\text{sun}}^2/\Delta m_{\text{atm}}^2 \ll 1 \) set in Eq. (9). Therefore, we conclude that only the normal hierarchy or near degeneracy of neutrino masses is allowed for seven two-zero patterns of the neutrino mass matrix under consideration.
To give an order-of-magnitude illustration of the neutrino mass matrix, we calculate the elements of $M$ for each pattern by using the formula

$$M_{ab} = \sum_{i=1}^{3} (V_{ai}V_{bi}m_i) = \sum_{i=1}^{3} (U_{ai}U_{bi}\lambda_i)$$

(31)

and the typical inputs taken before. The rough results are listed in Table 1. We see that there is no clear hierarchy among the non-vanishing elements of $M$, unlike the familiar case of quark mass matrices [11].

Of course, the specific textures of lepton mass matrices cannot be preserved to all orders or at any energy scales in the unspecified interactions which generate lepton masses [4]. Nevertheless, those phenomenologically favored textures at low energy scales may shed light on the underlying flavor symmetries responsible for the generation of lepton masses at high energy scales. It is expected that more precise data of neutrino oscillations in the future could help select the most favorable pattern of lepton mass matrices.

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Tri-bimaximal mixing and the neutrino oscillation data

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Abstract
Following recent results from the SNO solar neutrino experiment and the K2K long-baseline neutrino experiment, the combined existing data on neutrino oscillations now point strongly to a specific form for the lepton mixing matrix, with effective bimaximal mixing of $\nu_\mu$ and $\nu_\tau$ at the atmospheric scale and effective trimaximal mixing for $\nu_e$ with $\nu_\mu$ and $\nu_\tau$ at the solar scale (hence ‘tri-bimaximal’ mixing). We give simple mass-matrices leading to tri-bimaximal mixing, and discuss its relation to the Fritzsch–Xing democratic ansatz.

1. Motivation

The first data from the Sudbury solar neutrino experiment (SNO) [1] have dramatically confirmed the long-standing HOMESTAKE [2] solar neutrino result with regard to the high-energy charged-current $\nu_e$-rate (SNO/BP2001 $= 0.347 \pm 0.029$, HOMESTAKE/BP2001 $= 0.337 \pm 0.030$). At the same time, the comparison with the existing rather precise SUPER-K (SK) result for solar-neutrino electron elastic scattering (SK/BP2001 $= 0.459 \pm 0.007$ [3]) (which includes a neutral-current contribution) has provided a significant first cross-check of the Bahcall–Pinsonneault (BP2001) [4] standard solar model calculation of the $^8$B flux in these experiments, so that it now seems reasonable to conclude that the suppression for $^8$B neutrinos is probably close to $\sim 1/3$. In detail, the SNO, HOMESTAKE and SUPER-K experiments have different thresholds and response functions (e.g., HOMESTAKE is expected to include a $\sim 15\%$ contribution from $^7$Be neutrinos) but such effects are readily taken into account [5].

By comparison, the low energy charged-current $\nu_e$ rate as sampled in the gallium-based experiments SAGE [6], GALLEX [7] and GNO [8] is known to be less suppressed (SAGE/BP2001 $= 0.59 \pm 0.07$, GALLEX GNO/BP2001 $= 0.58 \pm 0.07$). The gallium experiments are sensitive to neutrinos from the pp chain and are only marginally affected by $^8$B neutrinos (with expected signal contributions of $\sim 60\%$ pp, $\sim 30\%$ $^7$Be and $\sim 10\%$ $^8$B in the standard model [4]). We have previously emphasised, within the context of the original trimaximal mixing scenario [9], the consistency of the gallium suppression with $5/9 \simeq 0.56$ (this consistency survives today at the $1.2\sigma$ level, even...
allowing for the reduced $^8$B contribution in gallium in the LMA solution, see below).

Thus energy-dependence of the solar suppression is implicit, with the latest general fits [5] to the solar data favouring the so-called large-angle (LMA) MSW [10] solution. The long-standing small-angle (SMA) MSW solution is now essentially ruled out, while the so-called LOW and VO solutions are of marginal significance only [5]. The LMA solution is illustrated in Fig. 1 for several possible mixing angles. In the LMA solution the base of the MSW `bathtub' is arranged to account for the strong suppression at high-energy where matter effects dominate. At lower energies (for the same solar core density) the suppression reverts to its vacuum level outside the bathtub, accounting for the gallium data (the far high-energy end of the bathtub plays no role in the LMA solution). No significant day-night asymmetry is observed [3] so that the latest LMA fits [5] now prefer a mass-squared difference towards the higher end of the range $\sim 10^{-5} - 10^{-4} \text{eV}^2$ (the curves of Fig. 1 are for a representative $\Delta m^2 = 5 \times 10^{-5} \text{eV}^2$).

Interestingly, the trimaximal model [9] is known [11] to predict a ‘$5/9 - 1/3 - 5/9$’ LMA bathtub (see Fig. 3 of Ref. [11]) which could certainly be exploited to fit the current solar data in isolation. However, within the trimaximal model, the associated mass-squared difference would then necessarily be the larger mass-squared difference (compare Figs. 2 and 3 of Ref. [11]) and would thus be inconsistent with the data on atmospheric neutrinos, which seem to require a mass-squared difference some two orders of magnitude greater $\sim 10^{-3} - 10^{-2} \text{eV}^2$ [12].

Indeed, the other important new experimental input motivating the present analysis, is the currently emerging data from the K2K long-baseline experiment [13], tending to confirm [14] the mass-squared difference from the atmospheric neutrino experiments $\Delta m^2 \simeq 3 \times 10^{-3} \text{eV}^2$, clearly well above the solar mass-squared difference defined by the LMA solar fits and, in particular, subject to the CHOOZ [15] and PALO-VERDE [16] reactor limits on $\nu_\tau$-mixing, which imply $|U_{\alpha 3}|^2 \lesssim 0.03$, for $\Delta m^2 \gtrsim 10^{-5} \text{eV}^2$. ($U$ denotes the lepton mixing or MNS matrix, [17].) Note that, in this last respect, the new K2K results strongly disfavour the original trimaximal model.

An obvious solution is to sacrifice the economy of the original trimaximal model and acknowledge (in line with most other phenomenological analyses) the existence of two distinct mass-squared difference scales, $\Delta m^2 \gg \Delta m'^2$, controlling, respectively, the behaviour of atmospheric and of solar neutrinos. Within this context, the totality of the data clearly point to a particular form for the lepton mixing matrix, which turns out to be closely related to the trimaximal hypothesis, and which is given below.

2. The trend of the data

The atmospheric neutrino results are known to point strongly to twofold maximal $\nu_\mu - \nu_\tau$ mixing (or at least to effective twofold maximal $\nu_\mu - \nu_\tau$ mixing at the atmospheric scale). In particular the SUPER-K zenith angle distribution for multi-GeV ‘$\mu$-like’ events (Fig. 2(a)) clearly indicates a suppression of upward $\nu_\mu$ with respect to downward $\nu_\mu$ by a factor of $\sim 1/2$ (from Fig. 2 the up-to-down ratio for multi-GeV muons is $(U/D)_{\mu} \simeq 0.53 \pm 0.05$ for zenith angles $|\cos \theta| > 0.20$). By contrast, the corresponding

![Image](https://example.com/image.png)
distribution for ‘$e$-like’ events (Fig. 2(b)) appears to be completely unaffected, $(U/D)_{ee} \simeq 1.09 \pm 0.12$. In Fig. 2(a) the solid curve is the full oscillation curve for twofold maximal $\nu_\mu \to \nu_\tau$ mixing for $\Delta m^2 = 3 \times 10^{-3} \text{ eV}^2$ for a representative neutrino energy $E = 3$ GeV, and the dashed curve shows the effect of angular smearing with respect to the neutrino direction and averaging over neutrino energies. Tri-bimaximal mixing (or indeed any mixing hypothesis which is effectively twofold maximal $\nu_\mu - \nu_\tau$ mixing at the atmospheric scale) fits the data well.

More generally, the (locally averaged) survival probability $\langle P_\mu \rangle$ for $\nu_\mu$ at intermediate $L/E$ scales, $(\Delta m^2)^{-1} \lesssim L/E \lesssim (\Delta m^2)^{-1}$ (where $L$ is the neutrino flight path length) is given by the magnitude-squared $|U_{\mu 3}|^2$ of the MNS matrix element $U_{\mu 3}$ via

$$\langle P_\mu \rangle = (1 - |U_{\mu 3}|^2)^2 + |U_{\mu 3}|^4,$$

whereby $|U_{\mu 3}|^2 = 0.50 \pm 0.04$ whereby $0.36 \lesssim |U_{\mu 3}|^2 \lesssim 0.64$ (68% CL), certainly consistent with $|U_{\mu 3}|^2 = 1/2$ and twofold maximal mixing. Independent evidence for strong $\nu_\tau$ mixing comes from the observation of a substantive charged-current $\nu_\tau$ appearance signal, statistically separated from the neutral-current sample in SUPER-K.

As indicated in Section 1, intermediate baseline reactor experiments, such as CHOOZ [15] and PAO-VERDE [16], in fact provide the best limits on $\nu_\tau$ (actually $\bar{\nu}_\tau$) mixing at the atmospheric scale (in terms of the vacuum mixing matrix, the interpretation of the atmospheric experiments themselves can be seriously obscured by terrestrial matter effects, which tend to suppress $\nu_e$-mixing and enhance $\nu_\mu$-mixing [19] in the high-energy limit). Reactor experiments, utilising very low energy (anti) neutrinos and with existing baselines much shorter than the matter-oscillation length in the Earth, turn out to be almost completely immune to matter effects [20]. As discussed in Section 1, the best reactor limits give $|U_{\tau 3}|^2 \lesssim 0.03$ (95% CL) for $\Delta m^2 \simeq 3 \times 10^{-3} \text{ eV}^2$, consistent with $U_{\tau 3} = 0$ and thus with twofold maximal $\nu_\mu - \nu_\tau$ mixing.

We emphasise that the atmospheric and reactor data do not require $U_{\tau 3} \equiv 0$ any more than they require $|U_{\mu 3}|^2 \equiv 1/2$ (small non-zero values of $U_{\mu 3}$, and/or somewhat different values of $|U_{\mu 3}|^2$, e.g., $|U_{\mu 3}|^2 = 2/3$ [21], are more-or-less equally acceptable experimentally). It is only that $U_{\tau 3} = 0$ and $|U_{\mu 3}|^2 = 1/2$ provide a simple and adequate description of the current trend of the data, making twofold maximal $\nu_\mu - \nu_\tau$ mixing (for now) the accepted ‘default option’ [22] (at the atmospheric scale).

In a similar spirit, we turn again to the solar data displayed in Fig. 1, drawing particular attention now to the solid curve representing the ‘$5/9 - 1/3 - 5/9$’ bathtub (in the LMA solution the base of the bathtub essentially measures $|U_{e 2}|^2$ directly). In Fig. 1 the data are plotted assuming BP2001 fluxes [4]. The SNO, HOMESTAKE and SUPER-K data (after correction for the neutral-current contribution in SUPER-K) then determine the base of the bathtub, with $|U_{e 2}|^2 \sim \ldots$
1/3 clearly closely preferred. In the flux-independent approach [14] the 8B suppression is found to be \( \sim 0.33 \pm 0.10 \) (based on the measured SK–SNO difference [1]). In Fig. 1 the two broken curves correspond roughly (in the bathtub region) to the \( \pm 1 \sigma \) errors on the flux-independent suppression. Finally, note that with \( |U_{e2}|^2 = 0 \) (or small) the \( \nu_e \) survival probability outside the bathtub, \( P_e \), is (inversely) correlated with the value at its base in the LMA solution.

For vacuum mixing \( P_e = (1 - |U_{e2}|^2)^2 + |U_{e2}|^4 \), so that for \( |U_{e2}|^2 = 1/3 \) we have \( P_e = 5/9 \simeq 0.56 \). (Taking account of the 8B contribution the expected gallium suppression is actually closer to \( \simeq 0.53 \), but this is still consistent with the data at the \( \sim 1.2 \sigma \) level.) Thus the gallium data themselves provide an independent cross-check on the consistency of the LMA solution and on \( |U_{e2}|^2 \sim 1/3 \).

As above, we emphasise that the data do not require \( |U_{e2}|^2 \approx 1/3 \). If the (implicit) energy dependence of the solar suppression is real, certainly \( |U_{e2}|^2 \neq 1/2 [22] \), since there can be no MSW bathtub in that case [11]. But a somewhat more pronounced (e.g., a ’5/8 – 1/4 – 5/8’) bathtub (corresponding to \( |U_{e2}|^2 = 1/4 \)), is clearly far from excluded. As before, we regard \( |U_{e2}|^2 = 1/3 \) as a simple and adequate description of the data, which could usefully come to be seen as the default option at the solar scale.

3. ‘Tri-bimaximal’ mixing

In this section we simply take the above ’default’ values \( U_{e3} = 0, |U_{\mu 3}|^2 = 1/2 \) and \( |U_{e2}|^2 = 1/3 \) as given, and use them to evaluate the resulting lepton mixing matrix.

The lepton mixing matrix is defined with the rows labelled by the charged-lepton mass-eigenstates \((e, \mu, \tau)\) and the columns labelled by the neutrino mass-eigenstates \((\nu_1, \nu_2, \nu_3)\). Focussing on the last column (the \( \nu_3 \) column), we note that with \( U_{e3} = 0 \) and \( |U_{\mu 3}|^2 = 1/2 \), we have \( |U_{\tau 3}|^2 = 1/2 \) from unitarity, so that the last column is just as in the original bimaximal scheme [23]. Moving to the center column (the \( \nu_2 \) column), again as just in the original bimaximal scheme [23], orthogonalit requires \( |U_{e2}| = |U_{\mu 2}| \). With \( |U_{e2}|^2 = 1/3 \) (above) we then have \( |U_{e2}|^2 = |U_{\mu 2}|^2 = 1/3 \) from unitarity, so that the center column is just as in the original trimaximal scheme (see, e.g., Ref. [19]).

Finally, the first column (the \( \nu_1 \) column) follows from unitarity applied to the rows.

Indeed, it was pointed out in Ref. [19] (even before the SNO data first appeared) that a mixing scheme with the \( \nu_3 \) ’bimaximally’ mixed and the \( \nu_2 \) ’trimaximally’ mixed (hence tri-bimaximal mixing) could naturally account for the data, being also discussed in the conference literature under the name of ’optimised’ bimaximal mixing [24]:

\[
\begin{pmatrix}
\nu_1 & \nu_2 & \nu_3 \\
\frac{2}{3} & 1/3 & 0 \\
\frac{1}{6} & 1/3 & 1/2 \\
\end{pmatrix}
\]

(1)

(where the moduli-squared of the elements are given).

The name ’optimised’ bimaximal mixing reflected the scheme’s pedigree as a special case of the Altarelli–Feruglio generalised bimaximal form [25] and its close relationship to the original bimaximal form [23]. We emphasise that the mixing Eq. (1) is entirely determined by unitarity constraints once the above three ’corner’ elements are fixed to their default values.

We should also point out that the mixing Eq. (1) has much in common with the Fritzsch–Xing democratic ansatz [21] (which might in fact, see Section 4, reasonably be termed ’bi-trimaximal’ mixing, as opposed to ’tri-bimaximal’ mixing). Indeed, the Fritzsch–Xing ansatz may be viewed as a permuted form of Eq. (1) (with, somewhat remarkably, the crucial prediction \( U_{e3} = 0 \) made well before the emergence of the CHOOZ data [15]). It should be clear, however, that the phenomenologies of these two-mixing schemes are quite distinct, e.g., the Fritzsch–Xing ansatz predicted \( |U_{e2}|^2 = 1/2 \) and hence no energy dependence [11] of the solar suppression, which is now disfavoured by SNO [1]. The original bimaximal scheme [23] is likewise now disfavoured [26].

Asymptotic \((L/E \to \infty)\) predictions, specific to tri-bimaximal mixing, are the (vacuum) survival probabilities \( P_\mu = P_\tau = 7/18 \). The corresponding \( \nu_\mu \leftrightarrow \nu_\tau \) appearance probability is also 7/18. Note that \( U_{e3} = 0 \) implies no Pantaleone resonance [27] and no CP violation in neutrino oscillations, which might be considered a disappointment experimentally. Nonetheless, it is fair to say that current data point to Eq. (1), and it is therefore of interest to try to un-
understand what it might imply. In the next section we present simple mass-matrices leading to tri-bimaximal mixing.

4. Simple mass matrices

Our mass-matrices will be taken to be hermitian (i.e., we will throughout be implicitly referring to hermitian-squares of mass-matrices linking left-handed fields, $M M^\dagger = M^2$). Fermion mass-matrices are most naturally considered in a ‘weak’ basis (i.e., a basis which leaves the charged-current weak-interaction diagonal and universal).

Maximal mixing (whether trimaximal or bimaximal) undeniably suggests permutation symmetries [28]. We postulate that, in a particular weak basis, the mass-matrices take the following (permutation symmetric) forms:

$$M_l^2 = \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix}, \quad M_\nu^2 = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix},$$

where the real constants $a$, $x$, $y$, $z$ and the complex constants $b$ and $b^*$ encode the charged-lepton and neutrino masses as follows:

$$a = \frac{m_e^2}{3} + \frac{m_\mu^2}{3} + \frac{m_\tau^2}{3}, \quad x = \frac{m_e^2}{2} + \frac{m_\mu^2}{2},$$

$$b = \frac{m_e^2}{3} + \frac{m_\mu^2 \omega}{3} + \frac{m_\tau^2 \bar{\omega}}{3}, \quad y = \frac{m_e^2}{2} - \frac{m_\mu^2}{2},$$

$$b^* = \frac{m_e^2}{3} + \frac{m_\mu^2 \bar{\omega}}{3} + \frac{m_\tau^2 \omega}{3}, \quad z = m_\tau^2,$$

with $\omega = \exp(i 2 \pi / 3)$ and $\bar{\omega} = \exp(-i 2 \pi / 3)$ denoting the complex cube roots of unity.

In Eq. (2) the charged-lepton mass-matrix $M_l^2$ takes the familiar $3 \times 3$ circulant form [28], invariant under cyclic permutations of the three generation indices. The neutrino mass-matrix $M_\nu^2$ is real (i.e., symmetric, since our mass-matrices are hermitian) and is a $2 \times 2$ circulant in the 1–3 index subset, invariant under the permutation of only two out of the three generation indices (generations 1 ↔ 3). The neutrino mass-matrix has four ‘texture zeroes’ [29] enforcing the effective block-diagonal form. Note that both mass matrices (Eq. (2)) are invariant under the interchange of generation indices 1 ↔ 3 performed simultaneously with a complex conjugation. Indeed, in this basis, it is the invariance of all the leptonic terms under this combined involution, which guarantees no CP-violation (since $\text{Im} \det[M_l^2, M_\nu^2]$ changes sign).

The mass-matrices $M_l^2$ and $M_\nu^2$ are diagonalised by a threefold maximal unitary matrix $U_l$ and a twofold maximal unitary matrix $U_\nu$, respectively:

$$U_l = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{i \sqrt{2}}{\sqrt{3}} & 0 \\ \frac{i \sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$U_\nu = \begin{pmatrix} \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} \\ 0 & 1 & 0 \\ \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}} \end{pmatrix},$$

i.e., $U_l^\dagger M_l^2 U_l = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$ and $U_\nu^\dagger M_\nu^2 U_\nu = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$, so that the lepton mixing matrix (or MNS matrix) $U = U_l^\dagger U_\nu$ is given by:

$$e \left( \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \right) \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix},$$

where the RHS is the tri-bimaximal form (Eq. (1)) in a particular phase convention. For Dirac neutrinos, the factor of $i$ is readily removed by a simple rephrasing of the $v_3$ mass-eigenstate, yielding tri-bimaximal mixing expressed as an orthogonal matrix:

$$e \begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \sqrt{2} \\ -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \end{pmatrix},$$

$$U = \mu \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{\sqrt{3}}{\sqrt{2}} \end{pmatrix}.$$
While this concludes our derivation of tri-bimaximal mixing starting from Eq. (1), we take this opportunity to remark that one might easily (with perhaps equal a priori justification) have interchanged the forms of $M_2$ and $M_2^\nu$ in Eq. (1), taking the neutrino mass-matrix to be the $3 \times 3$ circulant, and the charged-lepton mass-matrix to be of the $2 \times 2$ block-diagonal circulant form. Note, however, that this leads to physically distinct mixing which, if the $2 \times 2$ circulant is chosen to be in the 1–2 index subset, is identically the Fritzsch–Xing democratic ansatz [21] (hence ‘bi-trimaximal’ mixing as a synonym for the Fritzsch–Xing ansatz, see Section 3 above). We emphasise once again that the phenomenologies of the above two mixing schemes are physically distinct, with the Fritzsch–Xing ansatz now essentially ruled out, along with many other schemes involving energy independent solar solutions [31] (including trimaximal mixing [9]) following the SNO results [1].

5. Perspective

Tri-bimaximal mixing is a specific mixing matrix (Eq. (1)/Eq. (6)) which encapsulates the trends of a broad range of experimental data (the LSND oscillation signal [32] was not considered on the grounds that it still awaits confirmation). Tri-bimaximal mixing is closely related to a number of previously suggested lepton mixing schemes, notably trimaximal mixing [9], bimaximal mixing [23], the Fritzsch–Xing democratic ansatz [21] and the Altarelli–Feruglio scheme [25] (of which tri-bimaximal mixing may be considered a special case). For the future, the couplings of the heavy neutrino, $\nu_3$, are expected to be measured more precisely in long-baseline experiments like MINOS [33] (and other projects [34]). In particular, the limits on $|U_{e3}|^2$ should continue to improve. Regarding the $\nu_2$ couplings, if tri-bimaximal mixing is right, the KAMLAND experiment [35] should confirm the LMA solution, measuring a $\bar{\nu}_e$ survival probability tending to $P_e = 5/9 \simeq 0.56$ at sufficiently low-energy. Corresponding predictions for $\nu_\mu$ disappearance and $\nu_\mu \rightarrow \nu_\tau$ appearance, see Section 3, look very hard to test (experiments with $\nu_\mu$ necessitate higher energies and hence longer baselines, with matter effects generally dominant over vacuum effects for $\nu_2$). Finally, exact tri-bimaximal mixing would imply no high-energy matter resonance and no (intrinsic) CP-violation in neutrino oscillations, which might be considered a disappointment experimentally.

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Two-loop renormalization of $\tan\beta$ and its gauge dependence

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Abstract

Renormalization of two-loop divergent corrections to the vacuum expectation values ($v_1, v_2$) of the two Higgs doublets in the minimal supersymmetric standard model, and their ratio $\tan\beta = v_2/v_1$, is discussed for general $R_\xi$ gauge fixings. When the renormalized $(v_1, v_2)$ are defined to give the minimum of the loop-corrected effective potential, it is shown that, beyond the one-loop level, the dimensionful parameters in the $R_\xi$ gauge fixing term generate gauge dependence of the renormalized $\tan\beta$. Additional shifts of the Higgs fields are necessary to realize the gauge-independent renormalization of $\tan\beta$.

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Several extensions of the standard model have more than one Higgs boson doublets. For example, the minimal supersymmetric (SUSY) standard model (MSSM) [1,2] has two Higgs doublets

$$H_1 = (H_1^0, H_1^-), \quad H_2 = (H_2^+, H_2^0).$$

Both $H_i^0$ and $H_i^0$ acquire the vacuum expectation values (VEVs) $v_i$ ($i = 1, 2$) which spontaneously break the $SU(2) \times U(1)$ gauge symmetry. $H_i^0$ are then expanded about the minimum of the Higgs potential as

$$H_i^0 = \frac{v_i}{\sqrt{2}} + \phi_i^0.$$  \hspace{1cm} (2)

$\phi_i^0$ are shifted Higgs fields with vanishing VEVs. I assume that CP violation in the Higgs sector is negligible and take $v_i$ as real and positive.

A lot of physical quantities of the theory depend on the Higgs VEVs. In calculating radiative corrections to these quantities, the VEVs have to be renormalized. In the minimal standard model with only one Higgs doublet, the renormalization of the Higgs VEV is usually substituted by that of the weak boson masses [3,4]. However, this is not enough for extended theories with two or more Higgs VEVs. For example, the renormalization of $v_i$ in the MSSM is usually performed [5–7] by specifying the weak boson masses, which are proportional to $v_1^2 + v_2^2$, and the ratio $\tan\beta = v_2/v_1$. Since $\tan\beta$ itself is not a physical observable, however, a lot of renormalization schemes for $\tan\beta$ have been proposed in the studies of the radiative corrections in the MSSM. Some of them are listed in Ref. [8]. In this Letter, I concentrate on process-independent definitions of $\tan\beta$, which are given by the ratio of the renormalized VEVs $v_i$. I discuss the renormalization of the ultraviolet (UV) divergent corrections to $v_i$ and $\tan\beta$.  

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working in the modified minimal subtraction scheme with dimensional reduction [9] (DR scheme). The results are presented as the renormalization group equations (RGEs) for \( v_i \) and \( \tan \beta \). Since they are not physical observables, they may depend on the gauge fixing in general. I therefore investigate their gauge dependence in the general \( R_\xi \) gauge fixing [10]. Although I show the results for the MSSM, the results for the gauge dependence can be generalized for other models with two or more Higgs doublets.

Even within the DR scheme, there still remains an ambiguity of the way how to cancel the radiative shifts of the Higgs VEVs, \( \Delta v_i \), by the one-point functions of \( \phi_i^0 \) by tadpole diagrams. One way is to cancel \( \Delta v_i \) entirely by the shift of \( \phi_i^0 \). As a result, the tadpole contributions have to be added to all quantities which depend on \( v_i \). The renormalized \( v_i \) give the minimum of the tree-level Higgs potential and are just tree-level functions of the gauge-symmetric quadratic and quartic couplings in the Higgs potential. These \( v_i \) are therefore independent of the gauge fixing parameters [11]. This renormalization scheme for \( v_i \) is sometimes used [12–15] to show manifest gauge independence of physical quantities. However, since the running of \( v_i \) in this scheme is very rapid [16], and the tadpole contributions appear in almost any corrections, this scheme is often inconvenient in practical calculations.

Another, more popular way [5–7,15,16] is to absorb \( \Delta v_i \) by the shift of quadratic terms in the Higgs potential. The renormalized \( v_i \) then give the minimum of the tree-level Higgs potential and are just tree-level functions of the gauge-symmetric quadratic functions of \( \phi_i^0 \). This scheme is very convenient in practical calculation, because the explicit forms of the tadpole diagrams are necessary only for two-point functions of the Higgs bosons. However, the effective potential is generally dependent on the gauge fixing parameters [17–20]. The gauge dependence of the renormalized \( v_i \) and their ratio \( \tan \beta \) then might be a serious problem in calculating radiative corrections. I will therefore discuss the gauge dependence of the running \( \tan \beta \) in this definition, in general \( R_\xi \) gauges and to the two-loop order.

The RGE for \( v_i \) can be obtained from the UV divergent corrections to \( v_i \)-dependent masses or couplings of particles. For simplicity, I use the corrections to two quark masses \( m_b \) and \( m_t \), ignoring the masses of all other quarks and leptons. These mass terms are generated from the \( b\bar{b}H_1 \) and \( t\bar{t}H_2 \) Yukawa couplings, respectively, as

\[
L_{\text{int}} = -h_b \bar{b}R b_L (v_1 / \sqrt{2} + \phi_1^0) - h_t \bar{t}R t_L (v_2 / \sqrt{2} + \phi_2^0) + \text{h.c.}
\]

The \( R_\xi \) gauge fixing term takes the form

\[
L_{\text{GF}} = -\frac{1}{2\xi_Z} (\partial^\mu Z_\mu - \rho_Z G_Z)^2
-\frac{1}{\xi_w} |\partial^\mu W_\mu^+ - i \rho_w G_w^+|^2
-\frac{1}{2\xi'_V} (\partial^\mu \gamma_\mu)^2 - \frac{1}{2\xi''_V} \sum_{a=1}^8 (\partial^\mu g_a^\mu)^2.
\]

The would-be Nambu–Goldstone bosons \( G_V \) for \( V = (Z, W) \) appear in Eq. (4). The parameters \( \rho_V \equiv \xi_V m_V \), where \( m_V^2 = g_V^2 (v_1^2 + v_2^2) / 4 \) \( (g_Z^2 = g_2^2 + g_3^2) \) are masses of \( Z \) and \( W \), are introduced in Eq. (4). This is to emphasize that the gauge symmetry breaking terms \( \xi_V m_V \) in \( L_{\text{GF}} \), and also in the accompanied Fadeev–Popov ghost term, has very different nature from \( v_i \) generated by the shifts (2), as shown later. The terms \( \rho_V G_V \) in Eq. (4) are expressed in the gauge basis (1) of the Higgs bosons as

\[
\rho_Z G_Z = \xi_Z m_Z G_Z = -\sqrt{2} \text{Im}(\rho_Z \phi_1^0 - \rho_Z \phi_2^0).
\]

\[
\rho_W G_W = \xi_w m_W G_W^\pm = - (\rho_1 W H_1^\pm - \rho_2 W H_2^\pm).
\]

with parameters \( \rho_V \). The usual form of the \( R_\xi \) gauge fixing in the MSSM is recovered by the substitution [2,6]

\[
(\rho_{1W}, \rho_{2W}) = \xi_w g_w (v_1, v_2) / 2 = \xi_w m_W (\cos \beta, \sin \beta).
\]

The UV divergent corrections to \( m_b \) contain one source for the \( SU(2) \times U(1) \) gauge symmetry breaking. It is either \( v_1 \) originated from the shift (2) of \( H_1^0 \), or \( \rho_{1W} \) in the \( R_\xi \) gauge fixing term (4) and the Fadeev–Popov ghost term. The former contribution is obtained from that to the \( b\bar{b}H b_L \phi_1^0 \) Yukawa coupling \( h_b \) by replacing external \( \phi_1^0 \) by \( v_1 / \sqrt{2} \), except for the wave function correction of \( H_1^0 \) to \( h_b \). Similar argument holds for the UV divergent corrections to \( m_t \) and to the \( tR t_L \phi_2^0 \) Yukawa coupling \( h_t \). As a result, if the \( \rho_V \) contributions are absent, the runnings of \( v_i \) are the
same as those of the wave functions of \( H_i^0 \), namely,
\[
\frac{dv_1}{dt} = \frac{1}{h_b} \left[ \sqrt{\frac{2}{\pi}} \frac{d}{dt}(m_b) - \frac{dh_b}{dt} v_1 \right] = -\gamma_1 v_1,
\]
\[
\frac{dv_2}{dt} = \frac{1}{h_t} \left[ \sqrt{\frac{2}{\pi}} \frac{d}{dt}(m_t) - \frac{dh_t}{dt} v_2 \right] = -\gamma_2 v_2,
\]
where \( t = \ln Q \) is the \( \overline{\text{DR}} \) renormalization scale. The anomalous dimensions \( \gamma_i \) are denoted as \( \gamma_i \), which generally depend on the gauge fixing parameters \( \xi \). The RGEs (8) for \( v_i \) have been widely used in the Landau gauge \( \xi = \rho_V = 0 \).

However, in general \( R_{\xi} \) gauges, \( \rho_V \) in the gauge fixing terms (4) may give additional contributions to the quark mass running, as \( \bar{b} b \rho_{1V} \) and \( t b \phi_2 \). Since they have no corresponding contributions to the \( \bar{b} b \phi_1 \) and \( t b \phi_2 \) couplings, the RGEs for \( v_i \) deviate [6,21] from Eq. (8). Their general forms are then
\[
\frac{dv_i}{dt} = -\gamma_i v_i + Y_i \rho_{1V},
\]
where \( Y_i \) are polynomials of dimensionless couplings. Therefore, the RGE for \( \tan \beta \) becomes, using Eq. (7),
\[
\frac{d}{dt} \tan \beta = \tan \beta \left( -\gamma_2 + \gamma_1 + \frac{\xi_v g_v}{2} Y_2 V - \frac{\xi_v g_v}{2} Y_1 V \right).
\]
(10)

I then give explicit form of the RGE for \( \tan \beta \) in the MSSM, to the two-loop order. First, one-loop RGEs for \( v_i \) (\( i = 1, 2 \)) are
\[
\frac{dv_i}{dt} = -\gamma_i^{(1)} v_i + \frac{1}{4 \pi^2} (g_Z \rho_i Z + 2 g_Z \rho_i W) = v_i \left[ -\gamma_i^{(1)} + \frac{1}{4 \pi^2} \left( \frac{\xi_Z g_Z^2}{2} + \xi_W g_W^2 \right) \right],
\]
with the one-loop anomalous dimensions \( \gamma_i^{(1)} \),
\[
(4 \pi)^2 \gamma_i^{(1)} = N_c h_q^2 - \frac{3}{4} g_2^2 \left( 1 - \frac{2}{3} \xi_W - \frac{1}{3} \xi_Z \right) = \frac{1}{4} g_2^2 \left( 1 - \xi_Z \right),
\]
where \( h_q^2 = (h_q^2, h_t^2) \) for \( i = 1, 2 \), respectively, and \( N_c = 3 \). The \( \rho_{1Z} \) contribution to \( m_b \) is obtained from

\[
\begin{aligned}
&b_L \rightarrow b_L \\
&H_1^0 \rightarrow \rho_{1Z} \rightarrow Z_L \\
&b_L \rightarrow b_L
\end{aligned}
\]
Fig. 1. One-loop divergent contribution of \( \rho_{1Z} \) to \( m_b \). There is another diagram obtained from this one by the interchange \((b_L \leftrightarrow b', b_L')\).
example, the $\mathcal{O}(h^2_g Z \rho_1 Z)$ contribution to $v_1$ comes from the diagram (a) in Fig. 2, while other diagrams (b), (c) cancel each other. The RGEs for $v_i$ are finally

$$\frac{dv_i}{dt}_{\text{2-loop}} = -g_{i}^{(2)} v_i - \frac{N c h_{q_i}^2}{(4\pi)^4} (g_z \rho_i Z + 2 g w \rho_i W) + P_{V}^{(i)} (g) \rho_i V,$$

where again $h_{q_i}^2 = (h_{b_i}^2, h_{t_i}^2)$ for $i = (1, 2)$, respectively. $P_{V}^{(i)} (g)$ are possibly gauge-dependent $\mathcal{O}(g^3)$ functions which are common for both $\rho_{1V}$ and $\rho_{2V}$. It is therefore seen that, due to the $\rho_{1V}$ contributions in Eq. (14), the running $\tan \beta$ has the $\mathcal{O}(h_g^2 g^2 Z, h_q^2 g^2 Y)$ gauge parameter dependence. Although existing higher-order calculations of the corrections to the MSSM Higgs sector [26–29] have not included the contributions of these orders yet, the gauge dependence of $\tan \beta$ may cause theoretical problem in future studies of the higher-order corrections in the MSSM.

One way to restore the gauge independence of renormalized running $\tan \beta$ is to introduce gauge-dependent shifts of $\phi_i^0$ such as to cancel the $\rho_{iV}$ contributions to the effective action. This modification corresponds to the addition of extra shifts of $v_i$ to all diagrams. The running $v_i$ in this new definition then obey the same RGEs as those for $H_i$, namely, Eq. (8). The modified renormalized $\tan \beta$ becomes gauge independent to the two-loop order. However, an extra two-loop shift $\delta (v_2/v_1)$ has to be added to any quantities which depend on $\tan \beta$.

Before leaving, I briefly comment on two related issues in the process-independent on-shell renormalization of $v_i$ and $\tan \beta$ which is used in Refs. [6,7]. First, they cancel the one-loop $\rho_{iV}$ contributions by extra counterterms for $v_i$, $\delta v_i$, and determine their finite parts by imposing the condition $\delta v_1/v_1 = \delta v_2/v_2$. It is clear from Eq. (14) that this condition has to be modified beyond the one-loop. Second, the gauge dependence already appears in the one-loop finite part of the on-shell counterterm $\delta (\tan \beta)$. This is similar to the gauge dependence of the on-shell renormalized mixing matrices for other particles [30].

In conclusion, I discussed the UV renormalization of the ratio $\tan \beta = v_2/v_1$ of the Higgs VEVs in
the MSSM, to the two-loop order and in general $R_\xi$ gauges. When renormalized $\tilde{v}_i$ are given by the minimum of the loop-corrected effective potential, the contributions of $\rho_i V$ in the $R_\xi$ gauge fixing term cause two-loop gauge dependence of the RGE for $\tan \beta$. To avoid this gauge dependence, the contributions of $\rho_i V$ have to be cancelled by extra shift of the Higgs boson fields $\phi^0_i$.

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References


Abstract

The squark mass-matrix from the soft supersymmetry (SUSY) breaking sector contains a rich flavor-mixing structure that allows $O(1)$ mixings among top- and charm-squarks while being consistent with all the existing theoretical and experimental bounds. We formulate a minimal flavor-changing-neutral current scheme in which the squark mixings arise from the non-diagonal scalar trilinear interactions. This feature can be realized in a class of new models with a horizontal $U(1)_H$ symmetry which generates realistic quark-mass matrices and provides a solution to the SUSY $\mu$-problem. Finally, without using the mass-insertion approximation, we analyze SUSY radiative corrections to the $H^\pm bc$ and $h^0tc$ couplings, and show that these couplings can reveal exciting new discovery channels for the Higgs boson signals at the Tevatron and the LHC.

1. Introduction

Weak-scale supersymmetry (SUSY) [1], as a leading candidate for new physics beyond the standard model (SM), sensibly explains electroweak symmetry breaking, but leaves the understanding of flavor sector as a major challenge. Being a new fundamental space–time symmetry between fermions and bosons, SUSY necessarily extends the SM flavor structure to include superpartners for all fermions and thus adds further puzzles to the flavor physics. To be consistent with the experimental data, supersymmetry has to be broken. The breaking of the SUSY lifts the mass spectrum of the superpartners and is parametrized by the soft-breaking Lagrangian of the Minimal Supersymmetric SM (MSSM) with a large number of free parameters. The soft breaking sector is often problematic with low-energy flavor changing neutral current (FCNC) data without making specific assumptions about its free parameters. One of the most popular assumptions is the proportionality of the scalar trilinear $A$-terms to the fermion Yukawa couplings. This is however not a generic feature from the string-theory constructions, and certain forms of non-diagonal $A$-terms and their interesting implications were studied recently [2,3].
In this work, we focus on the flavor-mixings of three-family squarks which originate from the scalar mass and the trilinear interaction terms. We observe that the current data mainly suppress the FCNCs associated with the first two family squarks (and in some cases, with the first and third families), but allow the flavor-mixings between the second- and third-family squarks, the scharm ($\tilde{c}$) and stop ($\tilde{t}$), to be as large as $O(1)$ [4]. Furthermore, the $O(1)$ $\tilde{t}$–$\tilde{c}$ mixings arising from the non-diagonal $A$-term are consistent with the theoretical bounds derived from avoiding the charge-color breaking (CCB) as well as maintaining the vacuum stability (VS) [5].

Taking a bottom-up approach, we first formulate a minimal SUSY FCNC scenario, called the type-A models, in which all the observable FCNC effects come from the non-diagonal trilinear $A$-term in the $\tilde{c}$–$\tilde{t}$ sector. Then, using the simplest horizontal $U(1)_H$ symmetry (à la Froggatt–Nielsen [6]), we construct a class of new models, called the type-B models which not only exhibit similar flavor-mixings in the $\tilde{t}$–$\tilde{c}$ sector but also generate realistic quark-mass/mixing pattern and provide a solution to the SUSY $\mu$-problem. Such minimal FCNC schemes can reduce the general $6 \times 6$ squark-mass matrix down to a $4 \times 4$ or $3 \times 3$ matrix involving only the $\tilde{c}$–$\tilde{t}$ sector, and therefore simplify the exact squark mass-diagonalization and rotations without invoking the crude mass-insertion approximation. This allows quantitative understanding and predictions of the relevant FCNC signatures from the squark sector, and thus provides reliable probes to the fundamental SUSY flavor structure encoded in the soft-breaking Lagrangian.

As we will show, exploring the SUSY flavor sector is also important for revealing exciting new discovery signatures from the weak-scale supersymmetry, in addition to probing the mechanism of soft SUSY breaking. Applying the minimal FCNC schemes, we analyze the SUSY radiative corrections to the $H^\pm b c$ coupling and show that this flavor-mixing coupling can be significant to provide new discovery signals of charged Higgs via charm-bottom fusion [7] at the on-going Fermilab Tevatron Collider and the CERN Large Hadron Collider (LHC). Then, we further study the flavor changing top-decays into the charm-quark and lightest neutral Higgs boson in our schemes, and their observability at the LHC.

2. Minimal supersymmetric FCNC models

2.1. Type-A minimal SUSY FCNC models with non-diagonal $A$-term

The MSSM soft-breaking squark-sector contains the following quadratic mass-terms and trilinear $A$-terms:

$$-\tilde{Q}_i^\dagger (M_Q^a)_{ij} \tilde{Q}_j - \tilde{U}_i^\dagger (M_U^a)_{ij} \tilde{U}_j - \tilde{D}_i^\dagger (M_D^a)_{ij} \tilde{D}_j + (A^u_{ij} \tilde{Q}_i H U_j - A^d_{ij} \tilde{Q}_i H d J + c.c.),$$

with $i, j = 1, 2, 3$ being the family indices. This gives a generic $6 \times 6$ mass matrix,

$$\tilde{M}_a^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^2 & M_{RR}^2 \end{pmatrix}$$

in the up-squark sector, where

$$M_{LL}^2 = M_Q^2 + M_U^2 + \frac{1}{6} \cos 2\beta (4m_w^2 - m_Z^2),$$

$$M_{RR}^2 = M_D^2 + \frac{1}{2} \cos 2\beta \sin^2 \theta_w m_Z^2,$$

$$M_{LR}^2 = A_u \sin \beta / \sqrt{2} - M_{\mu} \cot \beta,$$

with $m_{w,z}$ the masses of $(W^\pm, Z^0)$ and $M_u$ the up-quark mass matrix. For convenience, we will choose the super Cabibbo–Kobayashi–Maskawa (CKM) basis for squarks so that in Eq. (3), $A_u$ is replaced by $A_u' = K_{UL} A_u K_{LR}$ and $M_u$ by $M_u'^{\text{diag}}$, etc., with $K_{UL,R}$ the rotation matrices for diagonalizing $M_u$ to $M_u'^{\text{diag}}$. In our minimal type-A scheme, we consider all large FCNCs to solely come from non-diagonal $A_u'$ in the up-sector, and those in the down-sector to be negligible, i.e., we define, at the weak scale,

$$A_u' = \begin{pmatrix} 0 & 0 \\ 0 & x \\ y & 1 \end{pmatrix} A,$$

where, $x$ and $y$ can be of $O(1)$, representing a naturally large flavor-mixing in the $\tilde{t}$–$\tilde{c}$ sector. Such a minimal scheme of SUSY FCNC is compelling as it is fully consistent with the stringent CCB/VS theory bounds [5] as well as the existing data [4]. (In the case that the CKM matrix is generated from the down-quark sector alone [3], $A_u'$ simply reduces back to $A_u$.) Similar pattern may be also defined for $A_d$ in the down-sector, but the strong CCB/VS bounds permit $O(1)$ $\tilde{b}$–$\tilde{s}$ mixings only for very large $\tan \beta$ because $m_b \ll m_t$. Thus, to allow a full range of $\tan \beta$
value, we consider an almost diagonal $A_d$. Moreover, identifying the non-diagonal $A_u$ as the only source of the observable FCNC phenomena in the type-A models implies the squark-mass-matrices $M^{2,3}_{Q,D}$ in Eqs. (2) and (3) to be nearly diagonal. For simplicity, we define

$$M_{LL}^2 \simeq M_{RR}^2 \simeq \tilde{m}_0^2 \mathbf{1}_{3 \times 3},$$

(5)

with $\tilde{m}_0$ a common scale of scalar-masses [8].

Within this minimal type-A scheme, we observe that the first family squarks $\tilde{u}_{L,R}$ decouple from the rest in (2) so that the $6 \times 6$ mass-matrix is reduced to $4 \times 4$,

$$\tilde{M}_{ct}^2 = \begin{pmatrix}
\tilde{m}_0^2 & 0 & 0 & A_x \\
0 & \tilde{m}_0^2 & A_y & 0 \\
0 & A_y & \tilde{m}_0^2 & X_t \\
A_x & 0 & X_t & \tilde{m}_0^2
\end{pmatrix}$$

(6)

for squarks ($\tilde{c}_L, \tilde{c}_R, \tilde{t}_L, \tilde{t}_R$), where

$$A_x = x \hat{A}, \quad A_y = y \hat{A}, \quad \hat{A} = A v \sin \beta / \sqrt{2},$$

$$X_t = \hat{A} - \mu m_t / \cot \beta.$$  

(7)

In Eq. (6), we ignore tiny terms of $O(m_\tau)$ or smaller. The reduced squark mass-matrix (6) has 6 zero-entries in total and is simple enough to allow an exact diagonalization. Especially, for the cases of (i) $x \neq 0, y = 0$, (called type-A1) and (ii) $x = 0, y \neq 0$, (called type-A2), one more squark, $\tilde{c}_R$ (in type-A1) or $\tilde{c}_L$ (in type-A2), decouples, and the Eq. (6) further reduces to a $3 \times 3$ matrix. We have worked out the general diagonalization of $4 \times 4$ matrix (6) for any $(x, y)$. The mass-eigenvalues of the eigenstates ($\tilde{c}_1, \tilde{c}_2, \tilde{t}_1, \tilde{t}_2$) are,

$$M_{1,2}^2 = \tilde{m}_0^2 \pm \frac{1}{2} \left[ \sqrt{\omega_+ - \omega_-} \right].$$

$$M_{1,2}^2 = \tilde{m}_0^2 \pm \frac{1}{2} \left[ \sqrt{\omega_+ + \omega_-} \right],$$

(8)

where $\omega_{\pm} = \lambda x^2 / (A_x \pm A_y)^2$. From (8), we can deduce the mass-spectrum of the stop-scharm sector as

$$M_{1,2}^2 < M_{2,1}^2 < M_{3,2} < M_{4,2}^2.$$  

(9)

In Eq. (8), the stop $\tilde{t}_1$ can be as light as about 100–300 GeV for the typical range of $\tilde{m}_0 \lesssim 0.5$–1 TeV.

The $4 \times 4$ rotation matrix of the diagonalization is given by,

$$\begin{pmatrix}
\tilde{c}_L \\
\tilde{c}_R \\
\tilde{t}_L \\
\tilde{t}_R
\end{pmatrix} = \begin{pmatrix}
c_1 c_3 & c_1 s_3 & s_1 s_4 & s_1 c_4 \\
-c_2 s_3 & c_2 c_3 & s_2 c_4 & -s_2 s_4 \\
-s_1 s_3 & -s_1 c_3 & c_1 c_4 & c_1 s_4 \\
-s_2 s_3 & -s_2 c_3 & c_2 c_4 & -c_2 s_4
\end{pmatrix} \begin{pmatrix}
\tilde{c}_1 \\
\tilde{c}_2 \\
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix},$$

(10)

with

$$s_{1,2} = -\frac{1}{\sqrt{2}} \left[ 1 - \frac{X_t^2 + A_x^2 \pm A_y^2}{\sqrt{\omega_+ \omega_-}} \right]^{1/2}, \quad s_4 = -\frac{1}{\sqrt{2}},$$

(11)

and $s_3 = 0$ (if $x y = 0$), or, $s_3 = 1 / \sqrt{2}$ (if $x y \neq 0$), where $s_1^2 + c_1^2 = 1$.

2.2. Type-B minimal SUSY FCNC models with a horizontal $U(1)$ symmetry

The minimal type-A SUSY FCNC schemes with a non-diagonal $A_u$-term, as discussed above, are truly economical as they uniquely result from imposing all the stringent theoretical and experimental bounds. Below, we further support such type of FCNC in the $t$–$c$ sector by providing theoretically compelling constructions based upon a minimal family symmetry. An attractive approach is to make use of the simplest horizontal $U(1)_H$ symmetry to generate realistic flavor structure of both quarks and squarks, via proper powers of a single suppression factor [6,9], and to provide a solution to the SUSY $\mu$-problem. For convenience, we define the suppression factor $\epsilon = (S)/A$ to have a similar size as the Wolfenstein-parameter $\lambda$ in the CKM matrix, i.e., $\epsilon \simeq \lambda \approx 0.22$ [9]. Here, $(S)$ is the vacuum expectation value of a singlet scalar $S$, responsible for spontaneous $U(1)_H$ breaking, and $A$ is the scale at which the $U(1)_H$ breaking is mediated to light fermions. In general, the supermultiplets of three-family quarks/squarks may carry different $U(1)_H$ charges, as defined in Table 1.

Table 1 Quantum number assignments under the horizontal symmetry $U(1)_H$

| $Q_1$ | $Q_2$ | $Q_3$ | $u_1$ | $u_2$ | $u_3$ | $d_1$ | $d_2$ | $d_3$ | $H_u$ | $H_d$ | $S$ | $b_1$ | $b_2$ | $b_3$ | $a_1$ | $a_2$ | $a_3$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\xi$ | $\xi'$ |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $-$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  | $+$  |
\begin{align}
M_{ij}^u \sim \frac{v_u}{\sqrt{2}} \lambda \delta_{ij} + \xi, \\
M_{ij}^d \sim \frac{v_u}{\sqrt{2}} \tan \beta \delta_{ij} + \xi'.
\end{align}

Similarly, in the CKM matrix,
\begin{equation}
(V_{ub}, V_{cb}, V_{ub}) \sim \left( \lambda_{h_1-h_2}, \lambda_{h_2-h_3}, \lambda_{h_1-h_3} \right).
\end{equation}

Unlike Ref. [9], the key ingredient of our model-buildings is to impose a new condition,
\begin{equation}
\alpha_2 = \alpha_3,
\end{equation}

which ensures the mixings between \( \bar{c} \) and \( \bar{t} \) in the squark mass matrix to be naturally of \( O(1) \). From the condition (14) and the current data of quark-masses and CKM angles (which can all be counted in powers of \( \lambda \)), we find an almost unique solution for all quark/squark quantum numbers (cf., Table 2), which is based upon a single \( U(1)_H \) symmetry and will be called the minimal type-B scheme hereafter. In Table 2, we consider \( \tan \beta \sim O(1) \) for the down-type quark mass-matrix \( M_d \) [cf. Eq. (12)]. The extension to a larger \( \tan \beta \) only affects the quantum numbers of \( \tilde{d}_j \)'s in a trivial way as it just contributes an overall factor \( 1/\tan \beta \sim \lambda^k \) (with the integer \( k \approx 0.66 \log \tan \beta \)) to \( M_d \) in Eq. (12) and thus simply adds \( -k \) to each quantum number of \( \tilde{d}_j \) listed in Table 2.

There are some slight variations of this minimal type-B model, by allowing the quantum numbers of \( \tilde{Q}_j \)'s to have \( \xi (\xi') \)-dependence, but they all predict the same patterns for the quark masses and mixings. By this construction, we have attempted to simultaneously solve the SUSY \( \mu \)-problem from the same \( U(1)_H \). A dynamical \( \mu \)-term can originate from \( (\kappa/A^{n-1})S^n H_u H_d \), with \( n = \xi + \xi' \), such that
\begin{equation}
\mu = \kappa \lambda^{n-1}(S) \text{ is generated at a scale } (S) \ll M_{\text{Planck}}.
\end{equation}

Hence, a weak-scale value of \( \mu \) can be obtained by properly choosing \( n \) for a given \( S \). If \( U(1)_H \) is not related to the SUSY \( \mu \)-problem, the minimal type-B model becomes truly unique, corresponding to a special case of \( \xi = \xi' = 0 \) in Table 2. With Table 2, we can readily derive the structures of quark/squark mass-matrices. For instance, the up-quark mass-matrix takes the form of
\begin{equation}
M_u \sim \frac{v_u}{\sqrt{2}} \left( \begin{array}{ccc}
\lambda^7 & \lambda^4 & \lambda^4 \\
\lambda^6 & \lambda^3 & \lambda^3 \\
\lambda^3 & 1 & 1
\end{array} \right).
\end{equation}

for any \( \tan \beta \gtrsim 1 \), while the squark mass-matrices \( M^2_{LL} \) and \( M^2_{RR} \) in Eq. (2) are deduced as,
\begin{equation}
M^2_{LL} \sim \tilde{m}_0^2 \left( \begin{array}{ccc}
1 & \lambda & \lambda^4 \\
\lambda & 1 & \lambda^3 \\
\lambda^4 & \lambda^3 & \lambda^3
\end{array} \right),
\end{equation}

\begin{equation}
M^2_{RR} \sim \tilde{m}_0^2 \left( \begin{array}{ccc}
\lambda^3 & \lambda^3 & \lambda^3 \\
\lambda^3 & 1 & \lambda^3 \\
\lambda^3 & \lambda^3 & 1
\end{array} \right).
\end{equation}

Eqs. (15) and (16) show a partial quark-squark “alignment” that effectively suppresses the FCNCs between 1st and 2nd(3rd) families, while at the same time provides \( O(1) \) mixings in the \( \bar{t}-\bar{c} \) sector of \( M^2_{RR} \). Furthermore, because squarks carries the same \( U(1)_H \) charge as quarks, the \( \bar{t}-\bar{c} \) mixings originating from a non-diagonal \( A_u \) term is predicted to share the same hierarchy structure as that in \( M_u \) [cf. Eq. (15)]. This, however, does not imply an exact “proportionality” between \( A_u \) and \( M_u \) because the power-counting of \( \lambda \) allows the coefficients in the corresponding entries of \( M_u \) and \( A_u \) to differ by ratios of \( O(1) \). Ignoring the small \( O(\lambda^3) \sim 1 \% \) terms in \( M_u \), we can diagonalize \( M_u \) by a \( 2 \times 2 \) rotation of the singlet quarks \( (\bar{c}, \bar{t}) \). Under the super-CKM basis, the off-diagonal block \( M^2_{LR} \) of the Eq. (2) becomes
\begin{equation}
M^2_{LR} = A'_{u} \sin \beta/\sqrt{2} - M_u^{\text{diag}} \mu \cot \beta,
\end{equation}

where \( A'_{u} = K_{UL} A_u K_{UR} = A_u K_{UR}^{+} + O(\lambda^3) \) and the singlet-quark rotation matrix \( K_{UR} \) contains a non-trivial sub-matrix involving only the second and the third family squarks. After neglecting the tiny \( O(\lambda^3) \) terms, we can parametrize the minimal \( A'_{u} \) term of the type-B model as
\begin{equation}
A'_{u} = \left( \begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & y & 1
\end{array} \right) A,
\end{equation}

where the size of the mixing parameter \( y \) is naturally of \( O(1) \). Thus, under this construction, the squarks \((\bar{u}_L, \bar{u}_R, \bar{c}_L)\) decouple and the Eq. (2) greatly reduces to a \( 3 \times 3 \) matrix, which takes the form, under the basis...
\[ \tilde{M}_{\chi}^2[B] = \begin{pmatrix} m_0^2 & A_y & x m_0^2 \\ A_y & m_0^2 & X_i \\ x m_0^2 & X_i & m_0^2 \end{pmatrix}. \]

In the above equation, \( A_y = y A v \sin \beta / \sqrt{2} \), and the parameter \( x = O(1) \) characterizes the mixing of \( \tilde{c}_R - \tilde{t}_R \) in the mass matrix \( M_{RR}^2 \) [cf., Eq. (16)]. For convenience, we define the typical case with \( y = 0 \) as the type-B1 scheme, and that with \( x = 0 \) as the type-B2 scheme. We note that the type-B2 model is identical to the type-A2 model as they have the same form of the non-diagonal \( A_y \) and \( \tilde{c}_L \) decouples. Also, type-B2 mass matrix with \( y = 0 \) in Eq. (19) takes the same structure as that of type-A1 except that Eq. (19) involves \( \tilde{c}_R \) (not \( \tilde{c}_L \)) and its \( x \) originates from \( M_{RR}^2 \) (not \( A_y \)). Without losing generality, we will study the physical applications of the typical type-A1 and -A2 (-B2) models in the following sections.

3. SUSY radiative corrections and Higgs signatures at colliders

3.1. SUSY induced \( H^\pm bc \) vertex and \( H^\pm \) production at hadron colliders

Different from the TopColor models and the type-III of two-Higgs-doublet models discussed in Ref. [7], the MSSM Higgs sector has no FCNC at the tree level and the same is true for flavor-changing mixings in the charged sector (except the usual CKM mechanism). Thus, the new flavor-changing effects in the neutral and charged sectors of the MSSM have to be generated radiatively. From Eq. (10) and its resulted Feynman rules, we can compute the dominant SUSY-QCD radiative corrections to the \( H^\pm bc \) coupling. It contains vertex corrections [scharm(stop)-bottom-gluino loop] and self-energy corrections [scharm(stop)-gluino loop], and can be summarized as,

\[ \Gamma_{H^\pm bc} = i \bar{u}_c(k_2)(F_L P_L + F_R P_R) u_b(k_1), \]

\[ F_L + F^V_{L,R} + F^S_{R,L}. \]  \hspace{1cm} (20)

where \( P_{L,R} = (1 \mp \gamma_5)/2 \) and the tree-level results are

\[ (F_L^0, F_R^0) = \frac{g V_{cb}}{\sqrt{2} m_w} (m_t \cot \beta, m_b \tan \beta), \]  \hspace{1cm} (21)

with \( V_{cb} \) being the CKM mixing matrix element involving \( c \) and \( b \) quarks. The one-loop vertex corrections in the type-A1 model give,

\[ F_L^V = 0, \]

\[ F_R^V = \frac{\alpha_s}{3\pi} m_{\tilde{g}} \sum_{j,k} \kappa_{jk} C_0 (m_H^2, 0, 0; m_{\tilde{b}}, m_{\tilde{g}}, m_{\tilde{u}}), \]  \hspace{1cm} (22)

where \( \tilde{u}_k \in (\tilde{c}_2, \tilde{t}_1, \tilde{t}_2), \tilde{b}_j \in (\tilde{b}_1, \tilde{b}_2), \) and \( C_0 \) is the 3-point \( C \)-function of Passarino–Veltman [10], \( \kappa_{jk} \) is the product of the relevant \( H^\pm \tilde{b}_j \tilde{u}_k \tilde{b} \tilde{g} \) and \( \tilde{u}_k \tilde{g} \tilde{g} \) couplings. The \( \tilde{b} \tilde{g} \tilde{g} \) mixings are also included, which can be sizable for large \( \tan \beta \). Furthermore, the type-A1 self-energy corrections yield

\[ F_L^S = 0, \]

\[ F_R^S = \frac{\alpha_s}{3\pi} m_{\tilde{g}} \sum_{j=1,2} (-)^{j+1} B_0 (0; m_{\tilde{g}}, m_{\tilde{u}_j}). \]  \hspace{1cm} (23)

where \( B_0 \) is the 2-point Passarino–Veltman function [10] and \( s_1 \) is given in Eq. (10) for \( y = 0 \), and tree-level \( H^\pm t b \) couplings \( (F_{L,R}^0, F_{L,R}^0) = (g V_{tb}/\sqrt{2} m_w)(m_t \cot \beta, m_b \tan \beta) \). In Eqs. (22) and (23), the tiny sub-leading terms suppressed by the powers of \( m_t/m_b \) have been ignored.

The form factors \( F_{L,R}^V \) and \( F_{L,R}^S \) for the type-A2 and type-B2 models can be easily obtained from Eqs. (22) and (23) by the exchanges of \( L \leftrightarrow R \) and \( x \leftrightarrow y \). We note that \( F_L(F_R) \) vanishes in type-A1 (type-A2 and -B2) schemes because \( \tilde{c}_R (\tilde{c}_L) \) decouples.

We find that the effective \( H^\pm bc \) couplings \( F_{L,R} \) are typically around 0.03–0.1 for \( x, y \approx 0.5–0.9 \), \( (A, \tilde{m}_0) \approx 0.5–2 \) TeV, and \( \tan \beta \approx 15–50 \). Therefore, the production of a charged Higgs boson via the \( b-c \) quark fusion process can become important in a wide range of the parameter space. In Fig. 1(a), the production cross sections of \( H^\pm \) via \( p \bar{p} / pp \rightarrow H^\pm X \) at the Tevatron and LHC are shown for \( (\mu, m_\tilde{g}, \tilde{m}_0) = (300, 300, 600) \) GeV, \( (A, -A_0) = 1.5 \) TeV, \( \tan \beta = 15, 50 \), and \( x = 0.75 \) in the type-A1 model. The dotted and dash curves are the SM production rates of \( cs \rightarrow H^\pm \) and \( cb \rightarrow H^\pm \) (induced by \( V_{cb} \approx 0.04 \), respectively, after including the complete next-to-leading order (NLO) SM-QCD corrections [11]. (The NLO SM-QCD corrections include the subprocesses with one single gluon in the initial state.) The solid
Fig. 1. (a): $H^\pm$ production via $cb$ (and $cs$) fusions at hadron colliders, with sample inputs $\tan\beta = 15$ (50) shown as lower (upper) set of curves, and $x = 0.75$. (b) and (c): factor $K \equiv (F_L^2 + F_R^2)/(F_0^L + F_0^R)$ for $H^\pm bc$ vertex, as a function of parameter $x$ and for $\tan\beta = (15, 50)$.

curve is the full $cb \rightarrow H^\pm$ production rate as a function of the Higgs boson mass after including both the SM-QCD and new SUSY-QCD corrections. As indicated, the SUSY loop corrections can significantly dominate over the CKM-suppressed $F_0^{L,R}$ contributions by a factor of $\sim 2-5$. In Fig. 1(b) and (c), we show the $K$-factor, defined as the ratio of $(F_L^2 + F_R^2)$ over $(F_0^L + F_0^R)$, which characterizes the enhancement of the $H^\pm$ production rate by the SUSY loop contributions.

To study the detection of a $H^\pm$ scalar, we consider its decay modes in the following. For $m_H \lesssim 190$ GeV, the $\tau\nu$ channel dominates, and for $m_H \gtrsim 190$ GeV, $H^\pm$ mostly decays into the $tb$ channel unless $H^\pm$ mass is above the threshold of $W^\pm h^0$, in which case the $W^\pm h^0$ channel (with $W \rightarrow \ell\nu$ and $h^0 \rightarrow b\bar{b}$) can become important as well [11]. Fig. 1 suggests that the Tevatron may be sensitive to the $H^\pm$ signals for $m_H$ below $\sim 300$ GeV, with an integrated luminosity of 2–20 fb$^{-1}$ per detector, while the LHC can potentially
probe the full mass-range of $H^\pm$ with an integrated luminosity of about 100 fb$^{-1}$. A detailed Monte Carlo simulation is needed to further quantify the discovery potential of the Tevatron and the LHC, which is beyond the scope of this Letter.

We have also examined the similar SUSY-induced enhancement $K$-factor for the $H^\pm$ production rate in the type-A2 and -B2 models and found that in the low tan$\beta$ ($\lesssim 5–10$) region it can reach to about $2–5$ for $y = 0.5–0.9$. But, as tan$\beta$ increases to above $\sim 15$, the enhancement factor decreases to less than $\sim 1.3$ for Higgs mass below 1 TeV.

3.2. SUSY induced $h^0$/$c$ vertex and neutral Higgs signal from top decay

It is known that the SM branching ratio of the flavor-changing top decay $t \rightarrow ch^0$ is extremely small ($\lesssim 10^{-13}–10^{-14}$ [12]), so that this channel becomes an excellent window for probing new physics [13–15]. Our minimal SUSY FCNC models predict the branching ratio of the $t \rightarrow ch^0$ decay to be substantially above the SM value so that this decay mode becomes observable at the LHC. (Note that this decay channel is always kinematically allowed in the MSSM.)

In our minimal FCNC schemes, the one-loop SUSY QCD induced $tch^0$ coupling can be written as

$$\Gamma_{tch} = \tilde{u}_i(k_2)(F_L P_L + F_R P_R)u_i(k_1),$$

$$F_{L,R} = F^V_{L,R} + F^S_{L,R},$$

which contains the vertex corrections (from squark–stop–gluino, stop–stop–gluino and squark–scharm–gluino, triangle loops), and the self-energy corrections (from stop–gluino and scharm–gluino loops). The one-loop vertex corrections in type-A1 are,

$$F^V_{L} = \frac{\alpha_s}{3\pi} \sum_{j,k} \lambda^L_{j,k} m_t \times (C_0 + C_{11})(m_h^2, m_t^2, 0; m_{\tilde{u}_j}, m_{\tilde{z}}, m_{\tilde{u}_k}),$$

$$F^V_{R} = \frac{\alpha_s}{3\pi} \sum_{j,k} \lambda^R_{j,k} m_t C_0(m_h^2, m_t^2, 0; m_{\tilde{u}_j}, m_{\tilde{z}}, m_{\tilde{u}_k}),$$

where $\tilde{u}_{j,k} \in (\tilde{c}_2, \tilde{t}_1, \tilde{t}_2)$, and $(C_0, C_{11})$ are the 3-point $C$-function of Passarino–Veltman. $\lambda^L_{j,k}$ is the product of the relevant $h-\tilde{u}_j-\tilde{u}_k$ and $\tilde{u}_k-\tilde{t}(c)$ couplings, derived from applying the squark-rotation (10). The type-A1 self-energy corrections yield,

$$F^S_{L,R} = \tilde{F}_0 \frac{\alpha_s s_0}{3\pi m_t} \left[ B_0(0; m_{\tilde{z}}, m_{\tilde{u}_j}) - B_0(0; m_{\tilde{z}}, m_{\tilde{u}_k}) \right],$$

(26)

where $B_0$ is the 2-point Passarino–Veltman function and $s_0$ is given in Eq. (10) with $y = 0$. $\tilde{F}_0$ denotes the tree-level $h^0-t-t$ coupling, and is given by $\tilde{F}_0 = (m_t/v)(\cos \alpha/\sin \beta)$. Again, in Eqs. (25) and (26), we have ignored the tiny sub-leading terms suppressed by the powers of $m_t/m_{\tilde{u}_k}$.

The form factors $F^V_{L,R}$ in the type-A2 model can be obtained from Eqs. (25), (26) by the exchanges of $L \leftrightarrow R$ and $x \rightarrow y$ everywhere.

For the numerical study, we assume that the only dominant decay mode of the top quark is its SM decay mode, $t \rightarrow bW$. Thus, the decay branching ratio of $t \rightarrow ch^0$ is given by, $Br(t \rightarrow ch^0) \simeq \Gamma(t \rightarrow ch^0)/\Gamma(t \rightarrow bW)$, with the partial decay width

$$\Gamma(t \rightarrow ch) = \frac{m_t}{16\pi} \left[ 1 - \frac{m_h^2}{m_t^2} \right]^{1/2} (F^2_L + F^2_R),$$

(27)

where the form factors $(F_L, F_R)$ are defined in Eq. (24). As summarized by Table 3, in our minimal SUSY-FCNC schemes, the decay branching ratio $Br(t \rightarrow ch^0)$ can be as large as $10^{-3}–10^{-5}$ over a large part of the SUSY parameter space where the mass of the lightest Higgs boson $h^0$ is around 110–130 GeV. We see that these decay branching ratios are very sensitive to the mixing parameter $x$ when it varies from 0.5 to 0.9. One reason is that the branching ratio (or decay width) contains, besides other mass-diagonalization effects, a sensitive overall power factor $x^2$ which comes from the stop–scharm mixing induced flavor-changing coupling in the squark–squark–gluino triangle loop and squark–gluino self-energy loop. Another reason is that unlike

Table 3

<table>
<thead>
<tr>
<th>$m_{\tilde{g}}$ (GeV)</th>
<th>tan$\beta$</th>
<th>20</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 GeV</td>
<td>(0.011, 0.10, 0.81)</td>
<td>(0.015, 0.19, 4.6)</td>
<td>(0.016, 0.21, 7.0)</td>
</tr>
<tr>
<td>500 GeV</td>
<td>(0.011, 0.09, 0.41)</td>
<td>(0.015, 0.13, 1.0)</td>
<td>(0.016, 0.14, 1.2)</td>
</tr>
</tbody>
</table>
the usual analyses with mass-insertion approximation, we have performed exact squark mass diagonalization [cf. Eqs. (8)–(11)], so that stops and/or charm-squarks can have significant mass-splittings. For instance, Eq. (8) shows that the mass-splitting between two top-squarks is not just due to the usual left-right mixing from \(X_t\), but also arises from the non-diagonal \(A\)-term, \(x A\) and \(y A\). The latter further enhances the mass-splittings and results in a light \(t_1\) of mass \(\sim 300–100\) GeV for \(x = 0.5–0.9\), and the heavier \(t_2\) always has mass above the input \(\tilde{m}_0\) (set as 600 GeV in Table 3). The radiative vertex corrections to \(tch^0\) coupling are thus dominated by the diagram with \(\tilde{t}_1–\tilde{t}_1–\tilde{g}\) triangle loop as we have explicitly verified from the 3-point Passarino–Veltman \(C\)-functions in Eq. (25). This second reason further increases the sensitivity of our decay branching ratios to the mixing parameter \(x\). From Table 3, we also see that for moderate mixings with \(x \lesssim 0.5\), the branching ratios are generally bounded to around the order of \(10^{-5}\), consistent with other studies in the literature [13]. The similar conclusion also holds for our type-A2 and -B2 models.

Since the LHC with an integrated luminosity of 100 fb\(^{-1}\) can produce about \(10^5\) \(t\) and \(\tilde{t}\) pairs [16], it can have a great sensitivity to discover this decay channel and test the model predictions, by demanding one top decaying into the usual \(bW^\pm\) mode and another to the FCNC \(ch^0\) mode. As shown in a recent model-independent Monte Carlo analysis [14], the LHC (100 fb\(^{-1}\)) can already measure the Br[\(t \to ch^0\)] down to the level of \(4.5 \times 10^{-5}\) at the 95% C.L. The future Linear Collider, with a high luminosity, is also expected to have a good sensitivity to detect this channel.

4. Conclusions

The three-family squark mass-matrix originating from the soft SUSY breaking sector contains a rich flavor-mixing structure. In this work, we have constructed the minimal FCNC schemes for the squark mass-terms and the scalar trilinear interactions which are consistent with the existing experimental and theoretical bounds. We find that the \(O(1)\) large mixings among the top- and charm-squarks are allowed. We demonstrate that this feature can be naturally realized in a class of new models with a horizontal \(U(1)\) symmetry which also generate realistic quark-mass pattern and solve the SUSY \(\mu\)-problem. Finally, we systematically analyze the dominant supersymmetric radiative corrections to the \(bcH^\pm\) and \(tch^0\) couplings in our minimal schemes, without using the mass-insertion approximation. We show that these couplings can be significant to provide new discovery signatures of the charged and neutral Higgs bosons at the Fermilab Tevatron and the CERN LHC.

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Testing supersymmetry in the associated production of CP-odd and charged Higgs bosons

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Abstract

In the Minimal Supersymmetric Standard Model (MSSM), the masses of the charged Higgs boson ($H^\pm$) and the CP-odd scalar ($A$) are related by $M_{H^\pm}^2 = M_A^2 + m_W^2$. Furthermore, because the coupling of $W^- A - H^+$ is fixed by gauge interaction, the tree level production rate of $q\bar{q}' \rightarrow W^{\pm*} \rightarrow AH^\pm$ depends only on one supersymmetry parameter—the mass ($M_A$) of $A$. We show that to a good approximation this conclusion also holds at the one-loop level. Consequently, this process can be used to distinguish MSSM from its alternatives (such as a general two-Higgs-doublet model) by verifying the above mass relation, and to test the prediction of the MSSM on the product of the decay branching ratios of $A$ and $H^\pm$ in terms of only one single parameter—$M_A$. © 2002 Elsevier Science B.V. All rights reserved.

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One of the commonly discussed new physics models is the Minimal Supersymmetric Standard Model (MSSM). To describe an experimental data in the framework of the MSSM usually requires introducing more than one supersymmetry (SUSY) parameters. Hence, the usual practice is to compare data to a pre-selected class of MSSM in which certain well-defined relations among the SUSY parameters are assumed in order to reduce the number of independent variables needed for discussion.

An interesting question to ask is “Can one find a process to test SUSY models at colliders without making any assumptions on the choice of SUSY parameters?” To answer that, let us consider the Higgs sector of the model. In the MSSM, because of the supersymmetry, two Higgs doublets have to be introduced in its Higgs sector. Although the MSSM Higgs sector resembles the one in a type-II two-Higgs-doublet model (THDM) [1], it has a very specific feature—all the Higgs self couplings $\lambda_i$ are fixed by the electroweak gauge couplings $g$ and $g'$, as required by supersymmetry. Hence, at the tree level, only two additional free parameters appear in the Higgs sector of the MSSM. We may take $M_A$ (the mass of the CP-odd Higgs boson $A$) and $\tan \beta$ (the ratio of the two vacuum expectation values) as these two free parameters.

One of the striking features resulted from the requirement of supersymmetry is that the mass ($m_h$) of the lightest CP-even Higgs boson ($h$) has to be less than the mass ($m_Z$) of the weak gauge boson $Z$ at
the Born level, although a large radiative correction due to a heavy top quark can push this bound up to about 130 GeV in the MSSM [2]. This result is interesting when compared to the theoretical bounds on the mass of the SM Higgs boson. Requiring the SM be a well defined theory up to the Planck scale (about $10^{19}$ GeV), the Higgs boson mass has to be approximately between 130 and 180 GeV [3,4]. Therefore, a light Higgs boson with its mass less than about 130 GeV can be a signal of the supersymmetric models, especially the MSSM. However, it is also known that such a light Higgs boson can exist in various non-SUSY models, such as a general THDM or the Zee model, even when the cutoff scale of the model is close to the Planck scale [5]. Hence, the existence of a light Higgs boson by itself cannot rule out models other than the MSSM (or its extensions).

Another striking feature resulted from the requirement of supersymmetry is that the masses of the charged Higgs boson $H^\pm$ and the CP-odd scalar $A$ are strongly correlated. At the Born level, they are related by the mass of the $W^\pm$ boson ($m_W$) as

$$M^2_{H^\pm} = M^2_A + m^2_W. \quad (1)$$

For comparison, the corresponding mass relation in a general THDM is $M^2_{H^\pm} = M^2_A + \frac{1}{2}(\lambda_5 - \lambda_4)\nu^2$, where $\nu$ is the weak scale (246 GeV) and $\lambda_4,5$ are two free parameters of the model [6]. Therefore, the mass relation (1) can be a strong criterion to discriminate the MSSM from its alternatives, e.g., a general THDM.

To test the mass relation (1), we propose to study the associated production of $A$ and $H^\pm$ at high energy hadron colliders, e.g., $p\bar{p} \to AH^\pm$ at the Fermilab Tevatron (a 2 TeV proton–antiproton collider) and $pp \to AH^\pm$ at the CERN LHC (a 14 TeV proton–proton collider). This process has the following unique features: (i) its Born level rate generally depends on the masses of $A$ and $H^\pm$. Because of the mass relation (1), the MSSM prediction of the Born level rate only depends on one (in contrast to two or more) SUSY parameter—$M_A$; (ii) the kinematic acceptance (therefore, the detection efficiency) of the signal events do not depend on the choice of other SUSY parameters because both $A$ and $H^\pm$ are spin-0 (pseudo-)scalar particles so that the kinematic distributions of their decay particles can be accurately modeled; (iii) it can constrain MSSM parameters by examining the product of the Higgs boson decay branching ratios (in contrast to the product of decay branching ratios and production rate); (iv) both $M_A$ and $M_{H^\pm}$ can be reconstructed from its final state to test the mass relation (1); (v) finally, the electroweak radiative corrections to its production rate and to the mass relation (1) are generally smaller than the expected experimental errors, such as the di-jet invariant mass resolution.

Either in the MSSM or the THDM, the coupling of $W^\pm A H^\pm$ is induced from the gauge invariant kinetic term of the Higgs sector [1]:

$$\mathcal{L}_{\text{int}} = \frac{g}{2} W^\pm_\mu \left( A \partial^\mu H^- - H^- \partial^\mu A \right) + \text{h.c.}, \quad (2)$$

so that the coupling strength of $W^+ H^- A$ is completely determined by the weak gauge coupling $g$. (In contrast, the coupling constants relevant to the interactions of $W$-boson (or $Z$-boson) and neutral Higgs bosons depend on $\beta$ and $\alpha$, where $\alpha$ characterizes the mixing between the two CP-even Higgs bosons $h$ and $H_0$.) Thus, the Born level production rate of $p\bar{p}, pp \to AH^\pm$ only depends on $M_A$ and $M_{H^\pm}$. Moreover, in the MSSM these masses are strongly correlated, cf. Eq. (1). Consequently, the production rate depends on only one SUSY parameter, which can be taken as $M_A$.

At the Tevatron and the LHC, the dominant constituent process for the production of a $AH^\pm$ pair is $q\bar{q}' \to W^\pm s \to AH^\pm$. For a given $M_A$, the cross section $\sigma(p\bar{p}, pp \to AH^\pm)$ is completely determined. Its squared amplitude, after averaging over the spins and colors, is

$$|\mathcal{M}|^2 = \frac{4}{3} m_W^2 G_F^2 \frac{s}{(s - m^2_W)^2 + m^2_W P^2} p^2 \sin^2 \theta, \quad (3)$$

where $P = \sqrt{E^2_A - m^2_A}$ with

$$E_A = (s + M^2_A - M^2_{H^\pm})/(2\sqrt{s})$$

and $\theta$ is the polar angle of $A$ in the center-of-mass (c.m.) frame of $A$ and $H^\pm$. We note that for the $c\bar{b} \to AH^+$ subprocess, in addition to the CKM (Cabibbo–Kobayashi–Maskawa) suppressed $s$-channel $W$-boson diagram, there is a $t$-channel diagram that depends on $\tan \beta$. However, the $c\bar{b} \to AH^+$ contribution to the inclusive $AH^+$ rate is small. For example, its contribution to the total rate is less than 0.01% and 0.1% at the Tevatron and the LHC, respectively, for
tan $\beta = 40$ and $M_A = 90$ GeV. For a smaller $\tan \beta$, its contribution becomes negligible. Hence, we shall ignore its contribution in the following discussion. In Fig. 1, we show the inclusive production rate of $\tilde{A}H$ and $\tilde{A}H^-$ as a function of $M_A$. The cross sections for $\tilde{A}H^+$ and $\tilde{A}H^-$ coincide at the Tevatron for being a $p\bar{p}$ collider.

It is trivial to model the kinematic acceptance (therefore, the detection efficiency) of the signal event. This is because both $A$ and $H^\pm$ are spin-0 bosons. Therefore, if the signal is not found, knowing the luminosity of the collider, the detection efficiency, and the theoretical production rate, one can induce from the data a constraint on the product of the decay branching ratio of $A$ and $H^\pm$ as a function of $M_A$.

For example, if the decay mode of $A \rightarrow b\bar{b}$ and $H^\pm \rightarrow \tau^+\tau^-$ is studied and no excess is found for a given mass bin of $M_A$ (hence, $M_{H^\pm}$) when comparing with the experimental data, then one can constrain the MSSM by demanding the product of the branching ratios, $\text{Br}(A \rightarrow b\bar{b}) \times \text{Br}(H^\pm \rightarrow \tau^+\tau^-)$, to be bounded from above as a function of $M_A$. Needless to say that applying the same strategy, one can constrain the product $\text{Br}(A \rightarrow X) \times \text{Br}(H^\pm \rightarrow Y)$ for any decay mode $X$ and $Y$ predicted by the MSSM as a function of only one SUSY parameter—$M_A$.

In case that a signal is found, the analysis is slightly more complicated. In the MSSM, the mass of the heavier CP-even Higgs boson ($H$) is not very different (less than about 10 GeV) from $M_A$ for $M_A \gtrsim 120$ GeV and $\tan \beta \gtrsim 10$. In this case, $q\bar{q}^\prime \rightarrow HH^\pm$ can produce the similar final states as $q\bar{q}^\prime \rightarrow AH^\pm$. Generally, the coupling of $Wq\bar{q}H^\pm$ depends on $g$ and $\sin(\alpha - \beta)$. However, for $M_A \gtrsim 190$ GeV and $\tan \beta \gtrsim 10$, $\sin^2(\alpha - \beta) \simeq 1$ and the production rate of $HH^\pm$ is almost the same as $AH^\pm$ in the MSSM. When both of them decay into the same decay channels, it will be difficult to separate the production of $AH^\pm$ from $HH^\pm$ unless a fine mass resolution can be achieved experimentally. Nevertheless, studying different decay channels can help to separate these two production modes. For instance, a heavy $H$ can decay into a $ZZ$ pair at the Born level, but $A$ cannot.

In conclusion, if no signal is found experimentally, a conservative bound on the product of the decay branching ratios of $A$ and $H^\pm$ can be derived for a CP-conserving model. This is because in a CP-conserving model, the $AH^\pm$ and $HH^\pm$ production modes do not interfere even if the masses of $A$ and $H$ are about the same. (We note that $A$ is a CP-odd scalar, while $H$ is CP-even.)

To test the MSSM relation (1) via the $AH^\pm$ production process is straightforward. For example, let us again consider the $b\tau\nu$ mode. If the signal is sufficiently large as compared to the backgrounds, a resonance bump should be observed (at a value $\langle M_{b\bar{b}} \rangle$) in the distribution of the $b\bar{b}$ invariant mass. Then, by searching for the corresponding Jacobian peak (at a value $\langle M_T(\tau\nu) \rangle$) in the distribution of the transverse mass of the $\tau\nu$ pair, one can test the MSSM by examining whether $\langle M_T(\tau\nu) \rangle$ is consistent with $\sqrt{(M_{b\bar{b}})^2 + m_W^2}$ within the accuracy of the mass resolution of the detector. By testing this mass relation via the process $p\bar{p}, pp \rightarrow AH^\pm$, one can discriminate the MSSM from its alternative (e.g., a general THDM).

In order to prove that the proposed process can be used to test the MSSM and is sensitive to only one SUSY parameter—$M_A$, we have to show that the SUSY electroweak correction, which occurs at the one-loop level, to the production rate is small. (The final state SUSY QCD correction does not contribute

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1 This is similar to the NLO QCD correction to the $W$-boson production at hadron colliders, except at a different invariant mass.
until the two-loop order.) Specifically, we need to consider two essential points: (a) what is the typical size of the radiative correction to the coupling of \( W^\mp A H^\mp \)? (b) does the mass relation (1) hold beyond the Born level? In the following, we shall have a more detailed discussion on these important questions. A brief summary is that as long as the typical SUSY mass scales are at the order of a couple of TeV or below, the radiative correction to the effective coupling \( \bar{g} \) can be written as

\[
M_{WHA}^\mu(q^2) = -\frac{\tilde{g}}{2}(p_A - p_H)^\mu [1 + F^{(1)}(q^2)],
\]

where \( q^\mu, p_A^\mu \) and \( p_H^\mu \) are the incoming momenta of \( W^+, A \) and \( H^- \), respectively, and \( \tilde{g} \) is the effective weak gauge coupling evaluated at \( q^2 \). Hence, the radiative correction to the cross section of the subprocess \( q\bar{q} \to A H^+ \) at the one-loop order is

\[
K^{(1)}(q^2) = 2 \text{Re} F^{(1)}(q^2).
\]

The one-loop correction to the renormalized form factor \( F^{(1)}(q^2) \), apart from the effective weak gauge coupling \( \tilde{g} \), can be written as

\[
F^{(1)}(q^2) = Z_{AA}^{1/2} Z_{H+H^-}^{1/2} \times \left\{ 1 + \delta F_{WHA} + F_{WHA}^{(1\text{PI})} \left( M_A^2, M_H^2, q^2 \right) \right\} - 1,
\]

where \( \tilde{Z}_{AA} \) and \( \tilde{Z}_{H+H^-} \) are the finite wavefunction factors for the renormalization of the external Higgs bosons \( A \) and \( H^\pm \). In our scheme,

\[
\tilde{Z}_{AA} = 1, \quad \tilde{Z}_{H+H^-} = 1 - \Pi_{H+H^-} \left( M_A^2 + m^2_W \right) + \Pi_{AA} \left( M_A^2 \right),
\]

where \( \Pi_{AA} \left( M_A^2 \right) \) denotes taking the derivative of the two point function \( \Pi_{AA}(k^2) \) of the CP-odd scalar \( A \) with respect to \( k^2 \) at \( k^2 = M_A^2 \), etc. The terms inside the curly bracket of Eq. (6) arise from the renormalized vertex function of \( WHA \), \( F_{WHA}^{(1\text{PI})} \left( p_A^2, p_H^2, q^2 \right) \) represents the one-loop contribution of the one-particle-irreducible (1PI) diagrams with \( p_A^2, p_H^2, q^2 \) as the four-momentum square of the incoming \( A \), \( H^\pm \) and \( W^\pm \) particles, respectively. \( \delta F_{WHA} \) is the counterterm contribution resulting from the field renormalization of \( H^+ \) and \( A \). The transformation (from the bare to the renormalized fields)

\[
H^+ A \to H^+ A \left( 1 + \frac{1}{2} \delta Z_{H^+} + \frac{1}{2} \delta Z_A \right)
\]

implies that \( \delta F_{WHA} = \frac{1}{2} \delta Z_{H^+} + \frac{1}{2} \delta Z_A \) which is found to be equal to \( -\Pi_{AA} \left( M_A^2 \right) \) in our scheme. We note that in \( \delta F_{WHA} \) the contributions from the counterterms of the weak gauge coupling and the wavefunction renormalization of the \( W \)-boson are not included, because they should be combined with the \( W \)-boson self energy contribution to derive the running weak gauge coupling \( \tilde{g}(q^2) \). In our numerical calculation, we use

\[
\tilde{g}^2 = 4 \sqrt{2} m^2_W G_F.
\]

\footnote{The other form factor, \( (p_A + p_H)\mu \), does not contribute to this process for massless quarks.}
In summary, the one-loop electroweak correction to the form factor $F^{(1)}(q^2)$ is found to be
\[
F^{(1)}(q^2) = F^{\text{WHA}}_{W^+ H^-}(M_A^2, M_B^2, q^2) - \frac{1}{2} \Pi_{H^+ H^-}(M_A^2 + m_c^2) - \frac{1}{2} \Pi_{A A}(M_A^2). \tag{10}
\]
In Eq. (10), the top- and bottom-quark loop contribution to $F^{\text{WHA}}_{W^+ H^-}$ is given by
\[
F^{\text{WHA}}_{W^+ H^-}(p_A^2, p_B^2, q^2) = \sum_{f = \text{ttb, bbt}} F^{\text{WHA}}_{W^+ H^-}(p_A^2, p_B^2, q^2), \tag{11}
\]
with
\[
F^{\text{WHA}}_{W^+ H^-}(p_A^2, p_B^2, q^2) = + \frac{N_c}{16\pi^2} \left[ p_A^2 C_{31}^{\text{WHA}} - p_B^2 C_{32}^{\text{WHA}} \right] + (2p_A \cdot p_B - p_A^2) C_{11}^{\text{WHA}} - (2p_A \cdot p_B + p_B^2) C_{22}^{\text{WHA}} - (D + 2)(C_{23}^{\text{WHA}} - C_{24}^{\text{WHA}} - m_f^2 C_{11}^{\text{WHA}} - (q^2 + m_f^2)) C_{12}^{\text{WHA}} - \frac{c_f}{16\pi^2} \gamma_f y_f m_f \left[ C_{0}^{\text{WHA}} \right]. \tag{12}
\]
where $N_c (= 3)$ is the number of colors, $c_f = +1$ and $-1$ for $f = \text{ttb}$ and $\text{bbt}$, respectively, and $C_{ijkl}^{\text{WHA}}$ are defined in terms of the Passarino–Veltman functions [10].

\[
C_{ij}^{\text{WHA}} = C_{ij}(p_A^2, p_B^2, (p_A + p_B)^2; m_f, m_f, f). \tag{13}
\]

The top- and bottom-loop contribution to $\Pi_{A A}(q^2)$ and $\Pi_{H^+ H^-}(q^2)$ is given by [10]
\[
\Pi_{A A}^{\text{quark}}(q^2) = -\frac{N_c}{16\pi^2} \sum_{f = \text{ttb, bbt}} 2 \sum_{i,j=1}^{2} \left[ q^2 (B_i(q^2, m_f, m_f) + B_{21}(q^2, m_f, m_f) + m_f^2 B_0(q^2, m_f, m_f)) \right], \tag{14}
\]
\[
\Pi_{H^+ H^-}^{\text{quark}}(q^2) = -\frac{N_c}{16\pi^2} \sum_{f = \text{ttb, bbt}} 2 \sum_{i,j=1}^{2} \left[ \lambda f_i f_j A f_i A \right] A(m_f^2). \tag{15}
\]

As shown in Eqs. (12), (14) and (15), the quark-loop contribution is proportional to the squared Yukawa coupling $y_i$, and $y_i y_b$ with $y_i = \sqrt{2} m_i \cot \beta / v$ and $y_b = \sqrt{2} m_b \tan \beta / v$. In the large $m_t^2$ or large $(m_t^2 \tan^2 \beta)$ limit, it is written as
\[
F^{(1)}_{\text{quark}} \sim \frac{N_c}{16\pi^2} \left[ -\frac{1}{4} y_t^2 + \frac{2}{3} \left( \frac{\ln m_t^2}{m_t^2} - \frac{3}{2} \right) y_b^2 \right]. \tag{16}
\]
Since $y_t^2$ and $y_b^2$ are at most $O(1)$ for $\tan \beta \approx 1$ and $m_t/m_b$, respectively, $F^{(1)}_{\text{quark}}$ is less than a few percent for $1 \lesssim \tan \beta \lesssim m_t/m_b$.

We also calculate the top- and bottom-loop contribution to $\Pi_{W^+ H^-}$, $\Pi_{A A}$ and $\Pi_{H^+ H^-}$ is given by
\[
\Pi_{W^+ H^-}^{\text{quark}}(q^2) = -\frac{N_c}{16\pi^2} \sum_{f = \text{ttb, bbt}} 2 \sum_{i,j=1}^{2} \left[ \lambda f_i f_j A f_i A \right] A(m_f^2). \tag{18}
\]
The squarks are obtained from the weak eigenstates \( \tilde{f} \). The mass eigenstates \( f \) are compared to the quark effects, the squark effects and the electric charge of the quark \( f_L \). In this expression, \( \lambda \) represents the coefficient of the \( \tilde{f}_i \tilde{f}_j \phi_k \) interaction in the MSSM Lagrangian [11], and

\[
\Pi_{H^+H^-}(q^2) = -\frac{N_c}{16\pi^2} \sum_{i,j=1}^{2} \lambda [\tilde{t}_i^*, \tilde{b}_j, H^+] \lambda [\tilde{b}_i^*, \tilde{t}_j, H^-] \times \frac{2}{16}\bar{m}_{\tilde{f}_i} \bar{m}_{\tilde{f}_j} A(m_{\tilde{f}_i}),
\]

(19)

where \( U_{1i}, D_{1j} \) are the rotation matrices for stops and sbottoms between the weak eigenstate basis \( (I = L, R) \) and the mass eigenstates basis \( (i = 1, 2) \), respectively. \( \lambda [\tilde{f}_i^*, \tilde{f}_j^*, \phi_k] \) represents the coefficient of the \( \tilde{f}_i^* \tilde{f}_j^* \phi_k \) interaction in the MSSM Lagrangian [11], and

\[
\bar{C}_{\tilde{f}_i^* \tilde{f}_j^* \phi_k} = (C_{11} - C_{12}) (p_{f_i}^2, p_{f_j}^2, q^2; m_{\tilde{f}_i}, m_{\tilde{f}_j}, m_{\phi_k}).
\]

(20)

As compared to the quark effects, the squark effects are rather complex due to the additional free (SUSY) parameters. The mass eigenstates \( \tilde{f}_{1,2} \) (\( \tilde{f} = \tilde{t} \) or \( \tilde{b} \)) of the squarks are obtained from the weak eigenstates \( \tilde{f}_{L,R} \) by diagonalizing the mass matrices defined through [11]

\[
\mathcal{L}_{\text{mass}} = -\bar{f}_L^* \tilde{f}_R \left( M_f^2 + m_f X_f M_R^2 \right) \left( \tilde{f}_L \tilde{f}_R \right),
\]

(21)

where,

\[
M_f^2 = M_{ij}^2 + m_f^2 + (m_Z^2 \cos 2\beta) (T_f L - Q_f s_W^2)
\]

and

\[
M_R^2 = M_{ij}^2 - m_f^2 + (m_Z^2 \cos 2\beta) Q_f s_W^2.
\]

In this expression, \( M_{ij}^2 \) is the soft-breaking masses for \( f_{L,R} \) and \( b_{L,R} \), respectively; \( s_W = \sin \theta_W \) with \( \theta_W \) being the weak mixing angle; \( T_f L \) and \( Q_f \) are the isospin and the electric charge of the quark \( f_L \). Moreover, \( X_f = A_f - \mu \cot \beta \) and \( X_b = A_b - \mu \tan \beta \), where \( A_f(A_b) \) is the trilinear \( A \)-term for \( t(b) \), and \( \mu \) is the SUSY invariant higgsino mass [11].

To examine the numerical effect of the one-loop electroweak corrections, we shall discuss two limiting cases below. Firstly, we consider the cases with \( \mu = A_t = A_b = 0 \), i.e., the cases without stop mixing (\( |X_t| = 0 \)) and sbottom mixing (\( |X_b| = 0 \)). Under this scenario, the squark masses are proportional to the soft-breaking masses \( M_{\tilde{Q}}, M_{\tilde{U}} \) or \( M_{\tilde{D}} \), and the couplings of squarks to Higgs bosons are independent of these soft-breaking masses. Thus, the squark-loop effect is decoupled and its contribution is very small for a large value of \( M \), where \( M \equiv M_{\tilde{Q}} \approx M_{\tilde{U}} \approx M_{\tilde{D}} \).

(Throughout this Letter we denote \( M \) as the typical scale of the soft-breaking masses.) For a smaller \( M \), \( F_{\text{sqark}}^{(1)} \) becomes larger. However, \( M \) cannot be too small because a small \( M \) implies light squarks whose masses are already bounded from below by the direct search results [12]. Furthermore, as to be shown later, the case with a small \( M \) is also strongly constrained by the \( \rho \) parameter measurement. Secondly, we examine the case with a large stop mixing, assuming \( |m_{\tilde{t}_1}| \sim M^2 \gg m_{\tilde{t}_2}^2 \). Such a large stop mixing leads to a large mass splitting between \( \tilde{t}_1 \) and \( \tilde{t}_2 \) so that \( m_{\tilde{t}_1} \approx 0(\tilde{m}_Z) \) and \( m_{\tilde{t}_2} \approx \sqrt{2} M \), while \( m_{\tilde{b}_{1,2}} \approx 0(\tilde{M}) \). The leading squark contribution to \( F_{\text{sqark}}^{(1)}(q^2) \) is

\[
F_{\text{sqark}}^{(1)} \sim -\frac{N_e}{16\pi^2} \left[ \frac{3}{4} \ln \frac{2}{|Y_t|^2} \left( \frac{Y_t}{M} \right)^2 + \frac{13}{6} - 3 \ln 2 \left( \frac{Y_b}{M} \right)^2 \right],
\]

(22)

with

\[
Y_t = \frac{m_t}{v} (A_t \cot \beta + \mu)
\]

and \( Y_b = \frac{m_b}{v} (A_b \tan \beta + \mu) \). Since in this case \( |A_t| \approx |M^2/m_t| \pm |\mu| \cot \beta \), we have \( |Y_t| \lesssim O(M^2/v) \) for \( |\mu| \lesssim M \) and \( 1 \lesssim \tan \beta \). When \( |A_b| \approx |A_t| \approx |\mu| \) and \( \tan \beta \lesssim m_t/m_b \), we find \( |Y_b| \lesssim O(M^2/v) \). Thus, with a large stop mixing \( (m_{\tilde{t}_1}|X_t| \approx M^2) \), \( F_{\text{sqark}}^{(1)} \) is proportional to the soft-breaking mass scale \( M \), and does not decouple in the large \( M \) limit. However, the experimental bound on the \( \rho \)-parameter (or the \( T \)-parameter) can also strongly constrain such kind of model. With a large stop mixing \( (M^2 \approx m_{\tilde{t}_1}|X_t|) \), the squark contribution to the \( \rho \)-parameter is

\[
\Delta \rho_{\text{sqark}} \approx (2.2 \times 10^{-3}) \frac{M^2}{v^2}.
\]

(23)

\(^3\) Eq. (23) is deduced from the expression given in Ref. [15] under the assumption that \( M^2 = M_{\tilde{Q}}^2 = M_{\tilde{U}}^2 = M_{\tilde{D}}^2 \gg m_{\tilde{t}}^2 \) and the stop mixing is large \( (m_{\tilde{t}_1}|X_t| \approx M^2 \) and \( m_{\tilde{b}_1}|X_b| \approx 0 \).
Since any new physics contribution to the $\rho$-parameter has to be bounded by data as [13]

$$-1.7 < \Delta \rho_{\text{new}} \times 10^3 < 2.7, \quad \text{at 2}\sigma \text{ level}, \quad (24)$$

the scale $M$ cannot be too large in this case. Consequently, the $F_{\text{squark}}^{(1)}$ is constrained to be smaller than a few percent as long as $\mu^2$ is not much larger than $M^2$.

To examine the effect from the stop and sbottom loops to the production rate of $A\tilde{H}^\pm$, we consider 4 sets of SUSY parameters, as listed in Table 1, which give the largest allowed deviation in the $\rho$-parameter. Set 1 and Set 2 represent the cases without either a stop mixing ($X_t = 0$) or a sbottom mixing ($X_b = 0$), and Set 3 and Set 4 are the cases with a large stop mixing ($m_t |X_t| \simeq M^2$) and $m_b \simeq 100$ GeV. The $K^{(1)}(s)$ factor, as defined in Eq. (5), is shown in Fig. 2 as a function of the invariant mass ($\sqrt{s}$) of the constituent process for $M_A = 90$ GeV. It is clear that the quark-loop contribution to $K^{(1)}(s)$ dominates the squark-loop contribution. Generally, the squark contributions are at most a few percent, unless $|\mu|$ is taken to be very large as compared to the scale $M$. This conclusion does not change when our assumption of $M_Q^2 \simeq M_U^2 \simeq M_D^2$ is relaxed to some extent. Including both the quark- and squark-loop contributions to $K^{(1)}(s)$, we found that the correction to the hadronic cross section of $H^+A$ production in the invariant mass region just above the $H^+A$ threshold, where the constituent cross section is the largest, is at a percent level. In summary, we illustrated that to be consistent with the low-energy data and the direct search results for stops and sbottoms, the one-loop electroweak correction to the production rate of $pp, p\bar{p} \rightarrow A\tilde{H}^\pm$ is small (at most a few percent).

Next, we discuss the one-loop corrections to the mass relation (1). Let us parameterize the deviation from the tree-level relation by $\delta$, so that at the one-loop order

$$M_{H^\pm} = \sqrt{M_A^2 + m_W^2 (1 + \delta)}. \quad (25)$$

We note that in our renormalization scheme, $M_A$ and $m_W$ are the input parameters, but $M_{H^\pm}$ is not. The one-loop corrected mass of the charged Higgs boson $M_{H^\pm}$ can be obtained by solving

$$\begin{vmatrix}
\Gamma^{(2)}_{G^+G^-} (p^2) & \Gamma^{(2)}_{G^+H^-} (p^2) \\
\Gamma^{(2)}_{H^+G^-} (p^2) & \Gamma^{(2)}_{H^+H^-} (p^2)
\end{vmatrix} = 0, \quad (26)$$

where $\Gamma^{(2)}_i (p^2)$ represent the renormalized two-point functions in the basis of the renormalized Goldstone boson ($G^\pm$) and charged Higgs boson ($H^\pm$) fields. The notation “Det” denotes taking the determinant of the $2 \times 2$ matrix. One of the solution of the above equation is $p^2 = 0$, which corresponds to the charged Nambu–Goldstone mode, and the other gives the pole mass of the charged-Higgs boson, $M_{H^\pm}$. In our scheme, $M_{H^\pm}^2$ can be expressed at the one-loop level by a compact formula as a function of the input
parameters as
\[
M_{H^\pm}^2 = M_A^2 + m_W^2 + \Pi_{AA}(M_A^2) - \Pi_{H^+H^-}(M_A^2 + m_W^2) + \Pi_{WW}(m_W^2),
\]
(27)
where \(\Pi_{\phi\phi}(q^2)\) (\(\phi = A, H^\pm\), and \(W\)) are the self-energies. The quark- and squark-loop contribution to \(\Pi_{AA}\) and \(\Pi_{H^+H^-}\) are given in Eqs. (14), (15), (18) and (19). The quark- and squark-loop contributions to \(\Pi_{WW}\) were presented, for example, in Ref. [14] and Ref. [15], respectively. The above result is to be compared with a more complicated expression of \(M_{H^\pm}\) derived from the effective Lagrangian approach [16].

When \(A_{t,b}\) and \(\mu\) are zero (i.e., no-mixing case), the leading contribution (which is proportional to the forth power of heavy quark mass) to \(\delta\) is found to be
\[
\delta \sim \frac{N_c}{8\pi^2 v^2} \frac{1}{M_A^2 + m_W^2} \left(\frac{m_t^2 m_b^2}{m_W^4} \right) \left(1 + \ln \frac{M^2}{m_t^2}\right) \sin^2 \beta \cos^2 \beta.
\]
(28)
This correction is substantial for \(\tan \beta \simeq m_t/m_b\) and \(M^2 \gg m_t^2\). Applying Eq. (27) with the complete expression of \(\Pi_{\phi\phi}(q^2)\), we found \(\delta\) to be less than 4.9% for \(2 < \tan \beta < 40\), \(M < 2\) TeV and \(M_A > 90\) GeV. Our result agrees well with Ref. [17], in which the approximate formula were presented for \(M^2 \gg m_t^2\).

For non-zero \(A_{t,b}\) and \(\mu\), \(\delta\) receives extra contributions which are proportional to \(A_{t,b}^4/M^4\), \(A_{t,b}^2\mu^2/M^4\) and \(\mu^4/M^4\), and are originated from the squark couplings and squark masses [6,17]. For the parameter sets of Set 3 and Set 4, \(\delta\) is less than 5.3% and 3.6% for \(M_A > 90\) GeV, respectively. In summary, as long as \(|A_{t,b}|\) and \(|\mu|\) are not too large as compared to \(M\), in a wide range of the parameter space that is allowed by the available experimental and theoretical constraints, \(\delta\) does not exceed 7–10%.

Supported by the finding that the one-loop electroweak corrections to the \(W^\pm A H^\mp\) coupling and to the mass relation (1) are generally smaller than the other theoretical errors (such as the parton distribution function uncertainties) and the expected experimental errors (such as the mass resolution of a Higgs boson decaying into jets), we anticipate that our conclusions based upon a Born level analysis should also hold well at the loop level. Namely, studying the process \(p\bar{p}, pp \rightarrow W^{\pm*} \rightarrow AH^{\mp}\) allows us to distinguish the MSSM from its alternatives by verifying the mass relation (1) and checking its production rate. If a signal is not found, studying this process provides an upper bound on the product of the decay branching ratios of \(A\) and \(H^\mp\) as a function of the only one SUSY parameter—\(M_A\).

To detect the signal event, it is necessary to suppress its potentially large backgrounds. For example, the \(\tau vbb\) backgrounds can be largely reduced by having a good \(b\)-tagging and tau selection (by using the nature of \(\tau\) polarization, which differs between a parent \(H^\pm\) and \(W^\pm\) [18]). We expect that its observability is relatively easy at the LHC and is a challenging task at the Tevatron (because of its small signal event rate). A detailed Monte Carlo analysis is needed to calculate the significance of the signal event. This will be deferred to a future study.

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**References**

Twisted parafermions

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Abstract

A new type of nonlocal currents (quasi-particles), which we call twisted parafermions, and its corresponding twisted $Z$-algebra are found. The system consists of one spin-1 bosonic field and six nonlocal fields of fractional spins. Jacobi-type identities for the twisted parafermions are derived, and a new conformal field theory is constructed from these currents. As an application, a parafermionic representation of the twisted affine current algebra $A_2^{(2)}$ is given.

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used in the study of D-branes. The operator product expansions (OPEs) and the corresponding $Z$-algebra of the untwisted parafermions were studied in [20,21].

In this Letter, we find a new type of nonlocal currents (quasi-particles), which will be referred to as twisted parafermions. The system contains a bosonic spin-1 field and six nonlocal fields with fractional spins. Some of the fields are in the Ramond sector and some are in the Neveu–Schwarz (NS) sector. They correspond to a new type of quasi-particles or generalized Majorana fermions. We derive the corresponding twisted $Z$-algebra and the Jacobi-type identities for the twisted parafermion currents. From the twisted parafermions, we construct a new conformal field theory which is different from the known ones. As an application, we obtain a parafermionic representation of the twisted affine current algebra $A_{(2)}^{(2)}$, which we expect to have application in the description of the entropy of the $AdS_3$ black hole [22].

It is well known that Euclidean correlation functions are defined only if operators in the correlators are time-ordered [23]. In the radial picture, $|z| > |w|$ means that $z$ is later than $w$. In the Euclidean functional integral definition of correlation functions, the time ordering is automatic. So, in Euclidean field theory the operator products $A(z)B(w)$ are only defined for $|z| > |w|$. Therefore the radial ordering is implied throughout this Letter.

Now we propose the twisted parafermion current algebra:

$$\psi_l(z)\psi_{l'}(w)(z-w)^{\ell/2k} = \frac{\delta_{l,l+1}}{(z-w)^2} + \frac{\delta_{l,0}}{z-w}\psi_{l+1}(w) + \cdots,$$

$$\psi_l(z)\psi_{l'}(w)(z-w)^{\ell/2k} = \frac{\delta_{l,l+1}}{(z-w)^2} + \frac{\delta_{l,0}}{z-w}\psi_{l+1}(w) + \cdots,$$

$$\psi_l(z)\psi_{l'}(w)(z-w)^{\ell/2k} = \frac{\delta_{l,l+1}}{z-w}\psi_{l+1}(w) + \cdots,$$

where $l, l' = \pm 1$ and $\bar{l}, \bar{l}' = 0, \pm 1, \pm 2$; $\delta_{l,0}$ and $\delta_{l,l'}$ are structure constants. If we denote $\psi_l$ or $\psi_{l'}$ by $\Phi_l$, then we can rewrite the above relations as:

$$\Phi_a(z)\Phi_b(w)(z-w)^{ab/2k} = \sum_{n=-2}^{\infty} (z-w)^n[\Phi_a\Phi_b]_n,$$  \hspace{1cm} (2)

where $a, b = 0, \pm 1, \pm 1, \pm 2$. So we have $[\Phi_0\Phi_0]_l = 0 (l \geq 3)$, $[\Phi_0\Phi_1]_2 = \delta_{a+b,0}$ and $[\Phi_0\Phi_2]_1 = \delta_{a+b,0}$. For consistency, $\delta_{a,b}$ must have the properties: $\delta_{a,b} = -\delta_{b,a} = -\delta_{-a,-b} = \delta_{-a,a+b}$ and $\delta_{a,-a} = 0$. Due to the mutually semilocal property between two parafermions, the radial ordering products are multivalued functions. So we define the radial ordering product of (generating) twisted parafermions (TPFs) as

$$\Phi_a(z)\Phi_b(w)(z-w)^{ab/2k} = \Phi_b(w)\Phi_a(z)(z-w)^{ab/2k},$$  \hspace{1cm} (3)

which, like the untwisted case, is an extension of the fermion (i.e., $ab = 1, k = 1$) and boson (i.e., $k \to \infty$) theories.

For every field in the parafermion theory there is a pair of charges $(\lambda, \bar{\lambda})$, which take values in the weight lattice. We denote such a field by $\phi_{\lambda,\bar{\lambda}}(z, \bar{z})$ [1,8,20]. The OPE of $\Phi_a$ with $\phi_{\lambda,\bar{\lambda}}(z, \bar{z})$ is given by

$$\Phi_a(z)\phi_{\lambda,\bar{\lambda}}(w, \bar{w}) = \sum_{m=-\infty}^{\infty} (z-w)^{-m-1-\lambda/2k} A_m^{a,\lambda}\phi_{\lambda,\bar{\lambda}}(w, \bar{w}),$$  \hspace{1cm} (4)

where $m \in Z$ (Ramond sector) for $a = l$ and $m \in Z + 1/2$ (Neveu–Schwarz sector) for $a = \bar{l}$. The action of $A_m^{a,\lambda}$ on $\phi_{\lambda,\bar{\lambda}}(z)$ is defined by the integration

$$A_m^{a,\lambda}\phi_{\lambda,\bar{\lambda}}(w, \bar{w}) = \oint_{c_w} \int d\bar{z} (z-w)^{m+a\lambda/2k} \Phi_a(z)\phi_{\lambda,\bar{\lambda}}(w, \bar{w}),$$  \hspace{1cm} (5)

where $c_w$ is a contour around $w$. Consider the difference of integrals

$$\oint_{c_u} \int d\bar{z} \Phi_a(u)\phi_{\lambda,\bar{\lambda}}(w, \bar{w})(u-z)^{-p-1+ab/2k} \times \Phi_a(u-w)^{m+q+ab/2k} (z-w)^{p-q+ab/2k},$$  \hspace{1cm} (6)

along two contours satisfying $|u-w| > |z-w|$ and $|u-w| < |z-w|$, respectively. The difference of the two contour integrals can be expressed as a two-fold integration of a $u$-contour enclosing $z$ once followed by a $z$-contour enclosing $w$ once. Properly carrying out the Taylor expansion of $(u-z)^s$, we then obtain
the so-called twisted Z-algebra relations,
\[
\sum_{l=0}^{\infty} C^{(l)}_{p-1+ab/2k} \left[A^0_{m-l-p+q} A^0_{n+l+p-q} + (-1)^p A^0_{m+l-q-1} A^0_{n+l+q+1}\right] \\
= C^{(p+2)}_{m+q+1+a/2k} \delta_{m,-b} \delta_{m,-n} \\
+ C^{(p+1)}_{m+q+1+ab/2k} e_a h b \delta_{m+n} \\
+ \sum_{r=0}^{\infty} C^{p-r}_{m+q+1+ab/2k} \left[\Psi_a \Psi_b\right]_{r-m+n},
\]
(7)
where \(p = 2q \) or \(2q + 1\) and \([\Psi_a \Psi_b]_{-m,n}\) is defined by
\[
[\Psi_a \Psi_b]_{-m,n}(w, \bar{w}) = \oint \frac{dz}{c_w} (z - w)^{m+n+1+(a+b)\lambda/2k} \\
\times [\Psi_a \Psi_b]_{-m}(z) \phi_{\lambda, \lambda}(w, \bar{w}).
\]
Let \(A_\lambda\) and \(B_\lambda\) be two arbitrary functions of the twisted parafermions with charges \(a\) and \(b\), respectively. These fields are local (\(a \neq 0\) or semilocal \((a \neq 0)\) and \(b = 0\)). The OPEs can be written as
\[
A_\lambda(z) B_\lambda(w)(z - w)^{ab/2k} = \sum_{n = -[h_A + h_B]} C_{\lambda,T} \lambda A_\lambda(z) B_\lambda(w)(z - w)^n \\
= \delta_{\lambda, \lambda} \frac{h_A + h_B}{2},
\]
(8)
where \([h_A]\) stands for the integral part of the dimension of \(A\). Hence we have \([A_\lambda B_\lambda](w) = \oint dz \times A_\lambda(z) B_\lambda(w)(z - w)^{n+1+ab/2k}\). It is easy to find the following relation between the three-fold radial ordering products
\[
\left\{ \oint dw \oint dw d\bar{w} R(A(u) R(B(z) C(w))) \right\} \\
- \oint dw \oint dw \oint dw (-a/2k) R (B(z) R(A(u) C(w))) \\
- \oint dw \oint dw \oint dw (R(A(u) B(z)) C(w)) \\
\times (z - w)^{P-1+ab/2k} (a - w)^{q-1+ac/2k} \\
\times (a - z)^{r-1+ab/2k} = 0,
\]
(9)
where integers \(p, q, r\) are in the regions: \(-\infty < p \leq [h_B + h_C], -\infty < q \leq [h_C + h_A], -\infty < r \leq [h_A + h_B]\); and \(a, b, c\) are parafermionic charges of the fields \(A, B, C\), respectively. The above equation is an extension of \(A(BC) - B(AC) - [A, B]C = 0\). Performing the binomial expansion, we obtain the following twisted Jacobi-like identities:
\[
\left\{ \sum_{i=0}^{p} \frac{\left((-1)^p C_{p-i}^{(i-p)} A[B(BC)]_i\right)}{r-i} \right\} Q_{-l}(w) + (-)^q \\
\times \left\{ \sum_{j=q}^{p} \frac{\left((-1)^{j+1} C_{j-q}^{(j-q)} B[A(BC)]_j\right)}{r-j} \right\} Q_{-j}(w) \\
= \sum_{l=r}^{\infty} \frac{(-)^{l-r} C_{l-r}^{(l-r)} (AB)_l C_{r-l}(w)}{(l-r)!},
\]
(10)
where \(Q = p + q + r - 1\), \(C_x = (-)^{x(x-1)}(-1)^{x-l+1}\) \((x = l)\) and \(C_0 = C_0 = C_{-1} = 1\), \(C_{l+1} = 0\), for \(l > p > 0\). This identity will be used extensively for our purpose. Performing analytic continuation one obtains
\[
[BA]_r(w) = \sum_{j=r}^{\infty} \frac{(-)^j}{(l-r)!} r! d^{-r} [AB]_l(w).
\]
(11)
We remark that \(A, B, C\) can be any composite operators and we can calculate any coefficient in the OPEs from the fundamental Eq. (2).
For the twisted parafermion theory to be a conformal field theory, we require that the spin-2 terms in the OPEs contain an energy–momentum tensor. It is obvious that the spin-2 terms in the OPEs are \([\Psi_a \Psi_b]_0\).
Since the parafermionic charge for the energy–momentum tensor should be zero, so the relevant terms are \([\Psi_a \Psi_a]_0\). Note that \([\Psi_a \Psi_a]_0(z) = [\Psi_a]_0(z)\), we calculate the OPE of \([\Psi_a \Psi_a]_0\) with \(\Psi_a\) and \([\Psi_b \Psi_b]_0\). Setting \(Q = p = 2, q = 1\) and \(r = 0\) in (10), we have
\[
\left\{ [\Psi_a \Psi_a]_0 \Psi_b \right\}_2 \\
= \delta_{a, b} \Psi_a + \delta_{a, b} \epsilon_{a, b} E_{a, b} \Psi_b \\
+ (1 + a^2/2k) \delta_{a, b} \Psi_a + \frac{ab}{4k} \left(1 - \frac{ab}{2k}\right) \Psi_b.
\]
(12)
From the general theory of conformal field theory [2], the conformal dimension of \(\Psi_a\) should be
Carrying out a similar program for parafermion is 1, or equivalently
\[
T\psi(z)\Psi_b(w) = \frac{6 - b^2}{k}.
\]
\[
\sum_a ab = 0, \quad \sum_a (ab)^2 = 12b^2. \quad (13)
\]
One solution to these constraints is given by
\[
\varepsilon_{1,1} = 1 = \varepsilon_{-1,1}, \quad \varepsilon_{1,-1} = -\frac{1}{2\pi}.
\]
Then we have
\[
\sum_a [T\psi_a\psi_b]_2 = \frac{2k + 6}{k} \left(1 - \frac{b^2}{4k}\right) \psi_b.
\]
Choose a proper normalization and write
\[
T\psi(z)\psi_b(w) = \frac{k}{2\pi r_0} \times \sum_a [T\psi_a\psi_a]_0. \quad \text{Then} \quad [T\psi_\psi]_2 = (1 - \frac{b^2}{2k})\psi_b.
\]
Repeating the above process, we obtain
\[
[T\psi_a\psi_a]_1 = T\psi_a\psi_a, \quad \text{or equivalently} \quad [T\psi_\psi]_1 = \psi_a\psi_a.
\]
These results can be written in the form of OPEs
\[
T\psi(z)\psi_b(w) = \frac{1 - b^2/4k}{(z-w)^2} \psi_b(w) + \frac{1}{z-w} - \psi_b(w) + \cdots. \quad (15)
\]
It follows that the conformal dimension of the twisted parafermion is 1, 1 - \frac{1}{4k} or 1 - \frac{1}{k}, for a given level k. Carrying out a similar program for T, we obtain the OPE:
\[
T\psi(z)T\psi(w) = \frac{c_{\text{pf}}/2}{(z-w)^2} + \frac{2T\psi(w)}{(z-w)^2} + \frac{\partial T\psi(w)}{z-w} + \cdots, \quad (16)
\]
where \( c_{\text{pf}} = 7 - \frac{24}{k(k+3)} = \frac{8k}{k+3} - 1 \) is the central charge of the twisted parafermion theory.

One of the applications of the twisted parafermionic currents is that they give a representation of the twisted affine current algebra \( A_2(2) \). Introduce eight currents:
\[
j^+(z) = 2\sqrt{k} \psi_1(z)e^{\sqrt{2k}\phi_0(z)}, \quad j^0(z) = 2\sqrt{2k} \psi_0(z),
\]
where \( \phi_0 \) is an U(1) current obeying \( \phi_0(z)\phi_0(w) = -\ln(z-w) \). Then it can be checked that the above currents satisfy the OPEs of the twisted affine currents algebra \( A_2(2) \) [24].

In summary, we have found a new type of nonlocal currents (quasi-particles) and corresponding twisted \( Z \)-algebra. We derive the Jacobi-type identities for the twisted parafermion currents. These Jacobi-type identities are expected to be useful in proving some of the Ramanujan identities, which play an important role in statistical physics. From the twisted parafermions, we construct a new conformal field theory, and give a parafermionic representation of twisted current algebra \( A_2(2) \). This representation is expected to be useful in the description of entropy of the \( AdS_3 \) black hole.

Note added

P. Mathieu pointed out to us Ref. [25], where graded parafermions associated with the \( osp(1|2)^{(1)} \) algebra were studied and fields with conformal dimensions of \( 1 - \frac{1}{k} \) and \( 1 - 1/k \) also appeared. However, our twisted parafermion algebra is quite different from the graded parafermion algebra in [25]. Our theory contains more fields, is unitary and has a different central charge.

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References

Standard-like models from intersecting D4-branes

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Abstract

We construct a one-parameter set of intersecting D4-brane models, with six stacks, that yield the (non-supersymmetric) standard model plus extra vector-like matter. Twisted tadpoles and gauge anomalies are cancelled, and the model contains all of the Yukawa couplings to the tachyonic Higgs doublets that are needed to generate mass terms for the fermions. A string scale in the range 1–10 TeV and a Higgs mass not much greater than the current bound is obtained for certain values of the parameters, consistently with the observed values of the gauge coupling constants. © 2002 Elsevier Science B.V. All rights reserved.

The use of D-branes in Type II string theories has led to exciting new possibilities for constructing models with realistic gauge groups and three generations of chiral matter [1]. Since D-branes carry gauge fields localised on their world volumes, an attractive approach is to start from a configuration of D-branes designed to give the standard model gauge group, possibly augmented by additional $U(1)$ factors, and to introduce further D-branes to produce at least the required matter content. The branes are located at an $R^6/Z_N$ orbifold fixed point, to ensure $N = 1$ supersymmetry, and the whole system is then embedded in a global compact space, an orbifold or orientifold, for example. Further D-branes and/or Wilson lines as well as anti-D-branes are added to ensure the cancellation of twisted tadpoles and the overall consistency of the theory. This is the “bottom-up” approach pioneered in [2], who also showed that only the $Z_3$ point group can yield three chiral generations of matter. Extra vector-like matter is generic to such models, and it was shown in [3] that this leads to gauge unification at an intermediate string scale between about $10^{10}$ and $10^{12}$ GeV, consistently with the measured value of $\sin^2 \theta_W (m_Z)$. Despite their attractions these models are not without defects. Specifically, the unavoidable extra D-branes needed to construct the model generate additional gauge groups under which some of the standard model matter is charged. So these gauge groups are not hidden from the observable sector, and the absence of data for such extra gauge groups must be taken as evidence against such models. Further, some of the Yukawa terms needed to generate masses for the quarks and leptons are forbidden by surviving global $U(1)$ symmetries. Also, these global symmetries are spontaneously broken when electroweak symmetry breaking occurs, thereby generating a keV-
scale axion\textsuperscript{1} which is unambiguously excluded by axion searches.

A different route to the standard model was motivated by Berkooz, Douglas and Leigh’s observation\textsuperscript{4} that intersecting D-branes can give rise to chiral fermions propagating in the intersection of their world volumes. D-branes at angles afford a T-dual interpretation as D-branes with magnetic flux\textsuperscript{5}, and based on that paper the stringy methods for what are now called “intersecting brane models” were developed in\textsuperscript{6}. This last paper led Aldazabal et al.\textsuperscript{7,8} to develop (non-supersymmetric) four-dimensional chiral models from intersecting D4-branes wrapped on 1-cycles of a 2-dimensional torus \( T^2 \) sitting at a singular point in the transverse 4-dimensional space \( B \).\textsuperscript{2}

The local geometry of \( B \) near the singularity is modelled as \( C^2/Z_N \). This gives an \( \mathcal{N} = 2 \) supersymmetric gauge sector. Since the models are generically non-supersymmetric, by virtue of their matter content, solution of the hierarchy problem requires a string scale of not more than 10 TeV or so, although the usual four-dimensional Planck mass can be obtained by making the volume of \( B \) large enough. The generic appearance at tree level of scalar tachyons at some intersections allows the interpretation of doublets as electroweak Higgs scalars, although colour-triplet and/or charged singlet tachyons are potentially lethal for the weak scale. We find that these constraints cannot be simultaneously satisfied using at most five stacks of D-branes, but that certain six-stack models can do so. these models predict interesting values of the Higgs scalar mass, not far from the current lower bound.

As in\textsuperscript{8}, we take the space \( B \) transverse to the D4-branes to be \( C^2/Z_3 \), where the point group generator \( \theta \) of \( Z_3 \) acts on the two complex coordinates of \( B \) with twist vector \( v = \frac{1}{3}(1, -1) \). We too choose the first stack to have \( N_1 = 3 \) D4-branes and to be sitting at a singular point of \( B \) with wrapping numbers \( (n_1, m_1) = (1, 0) \). In general, the integers \( n_a \) and \( m_a \) count the number of times that the 1-cycle wrapped by the stack \( a \) wraps the two basis 1-cycles defining the torus \( T^2 \); there is no loss of generality in this choice of wrapping numbers for the first stack. \( \theta \) is embedded in the stack as \( \gamma_0 = \alpha^p I_{3} \) which generates a \( U(3) \supset SU(3)_c \) gauge group. Here \( \alpha = e^{2\pi i/3} \) and \( p = 0, 1 \) or 2. As in\textsuperscript{8}, we also choose the second stack to generate a \( U(2) \supset SU(2)_L \) gauge group by taking \( N_2 = 2 \) D4-branes and embedding \( \theta \) as \( \gamma_0 = \alpha^q I_{2} \), where \( q \not\equiv p \mod 3 \). The wrapping numbers are \( (n_2, 3) \), so that the \( (12) \) intersection produces \( I_{12} \equiv n_1 m_2 - n_2 m_1 = 3 \) quark doublets \( Q_L \). Note that, unlike in the bottom-up approach\textsuperscript{2}, the number

\textsuperscript{1} We are grateful to Mark Hindmarsh for pointing this out.

\textsuperscript{2} For other recent work on intersecting brane models, both supersymmetric and non-supersymmetric, and their phenomenological implications, see Ref.\textsuperscript{9}. 
of generations is not determined by the choice of the point group \( \mathbb{Z}_3 \). Since further non-abelian gauge groups are not required, all remaining stacks have just one D4-brane. Without loss of generality these remaining stacks are split into three sets \( I, J \) and \( K \) having \( \gamma_p = \alpha_q, \alpha_r^a \) and \( \alpha_r^b \), respectively, where \( p, q, r \) are a permutation of 0, 1, 2. The wrapping numbers for the stack \( i \in I \) are \((n_i, m_i)\), and similarly for \( j \in J \) and \( k \in K \). Table 1 summarizes the assignments.

The gauge group resulting from these stacks of branes is at least

\[
\prod_a U(N_a) = U(3) \times U(2) \times \prod_i U(1)_i \times \prod_j U(1)_j \times \prod_k U(1)_k.
\]

When \( n_a \) and \( m_a \) have a greatest common divisor greater than 1 or one of the \( n_a \) or \( m_a \) is zero, there may be additional gauge group factors. The wrapping numbers are constrained by the requirements of twisted tadpole cancellation, which for the system described above gives

\[
3 + \sum_k n_k = 2n_2 + \sum_i n_i = \sum_j n_j,
\]

\[
\sum_k m_k = 6 + \sum_i m_i = \sum_j m_j.
\]

The spectrum of these models is generically chiral, so potentially there are gauge anomalies. In general, tadpole cancellation is sufficient to ensure cancellation of the cubic non-abelian anomalies, which in our case arise only for the \( SU(3) \) group. Each of the stacks provides a \( U(1) \) gauge group, and the weak hypercharge \( Y \) is in general a linear combination of the \( U(1) \) charges \( Q_a \) associated with the \( a \)th stack:

\[
Y = c_1 Q_1 + c_2 Q_2 + \sum_i c_i Q_i + \sum_j c_j Q_j + \sum_k c_k Q_k.
\]

(4)

For convenience we have suppressed any reference to any extra \( U(1) \) factors which have been discussed after Eq. (1). These are trivial to incorporate.

So we must also ensure that mixed \( U(1)_Y - SU(3)_C \) and \( U(1)_Y - SU(2)_L \) anomalies are absent, and their cancellation imposes further constraints on the wrapping numbers. To find them we must first fix the coefficients \( c_a \). The quark doublets \( Q_i \) that arise from the (12) intersection have \( Q_1 = 1, Q_2 = -1 \) and \( Q_3 = 0 \), \( a = 1, 2 \). Thus we require

\[
c_1 - c_2 = \frac{1}{6}.
\]

(5)

The (1i) intersections, with \( i \in I \), yield colour triplet, \( SU(2) \) singlet fermions, which we require to be right-chiral \( u \) or \( d \) quarks, so

\[
c_1 - c_{i_1, j_1} = \frac{2}{3} \text{ or } -\frac{1}{3},
\]

(6)

similarly for the sets \( J, K \). Thus we split the sets \( I, J \) and \( K \) into \( I_{1,2}, J_{1,2} \) and \( K_{1,2} \) with

\[
c_1 - c_{i_1, j_1, k_1} = \frac{2}{3},
\]

\[
c_1 - c_{i_2, j_2, k_2} = -\frac{1}{3},
\]

(7)

(8)

where \( i_1 \in I_1, \) etc. Then

\[
Y = \left( c_1 + \frac{1}{3} \right)
\]

\[
\times \left( Q_1 + Q_2 + \sum_i Q_i + \sum_j Q_j + \sum_k Q_k \right)
\]

\[
- \left( \frac{1}{3} Q_1 + \frac{1}{2} Q_2 \right)
\]

\[
+ \sum_{i_1} Q_{i_1} + \sum_{j_1} Q_{j_1} + \sum_{k_1} Q_{k_1}. \]

(9)

It is easy to see that the combination \( Q_1 + Q_2 + \sum_i Q_i + \sum_j Q_j + \sum_k Q_k \) is zero for all of the chiral
fermions, so henceforth we take

\[ Y = -\left(\frac{1}{3} Q_1 + \frac{1}{2} Q_2 \right) + \sum_{i_1} Q_{i_1} + \sum_{j_1} Q_{j_1} + \sum_{k_1} Q_{k_1} \]  

(10)

Using (3), the cancellation of the mixed \( U(1)_Y - SU(3)_c \) anomaly then requires that the wrapping numbers satisfy

\[ \sum_{i_1} m_{i_1} + \sum_{j_1} m_{j_1} = 0 \]  

(11)

Chiral \( SU(2) \) doublet fermions arise from the \( I_{2a} = \sum_{i_1} m_{i_1} = 0 \) intersections between the second stack and the \( a \)th stack, and cancellation of the mixed \( U(1)_Y - SU(2)_L \) anomaly requires that the wrapping numbers also satisfy

\[ \sum_{i_1} n_{i_1} + \sum_{j_1} n_{j_1} = 0 \]  

(12)

where we have used Eqs. (2), (3) and (11).

All intersecting branes give chiral fermions, but in addition tachyonic scalars arise whenever the Chan–Paton phases of the intersecting branes coincide. Thus, we get \( SU(2)_L \) doublet tachyons at \( (2i_1) \) and \( (2j_2) \) intersections, charged singlet tachyons from \( (i_1 j_2) \) and \( (k_1 k_2) \) intersections, and \( SU(3)_c \) colour triplet tachyons from \( (1k_1) \) and \( (1k_2) \) intersections. Doublet tachyons may be interpreted as electroweak Higgs scalars, but the charged singlet and colour triplet tachyons are less welcome. To avoid the latter, the stacks in the sets \( K_{1,2} \) must not intersect with those in the first stack, and this requires

\[ m_{k_1} = 0 = m_{k_2} \quad \forall k_1 \in K_1, \quad \forall k_2 \in K_2. \]  

(13)

Similarly, to avoid charged singlet tachyons we require

\[ I_{i_1 i_2} = 0 = I_{j_1 j_2} \quad \forall i_1 \in I_1, \quad \forall j_1 \in J_1, \quad \forall j_2 \in J_2. \]  

(14)

As usual, the Yukawa couplings of the Higgs doublets generate fermion mass terms when the electroweak symmetry is spontaneously broken. The existence of

<table>
<thead>
<tr>
<th>Stack ( a )</th>
<th>( N_a )</th>
<th>( (n_{i_1}, m_{a}) )</th>
<th>( \gamma_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>(1, 0)</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>(1, 0)</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>3</td>
<td>( i_1 )</td>
<td>(1, 0)</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>4</td>
<td>( i_2 )</td>
<td>(1, 0)</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>5</td>
<td>( J_1 )</td>
<td>(1, 0)</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>6</td>
<td>( k_1 )</td>
<td>(1, 0)</td>
<td>( \alpha )</td>
</tr>
</tbody>
</table>

Table 2

Multiplicities, wrapping numbers and Chan–Paton phases for the six-stack models \( (p \neq q \neq r \neq p) \)

Higgs doublets requires that

\[ \sum_{i_1} |I_{2i_1}| + \sum_{i_2} |I_{2i_2}| \neq 0, \]  

(15)

and we also demand that the model allows the (renormalizable) Yukawa couplings needed to give all quarks masses. This requires that

\[ I_1 \neq 0 \neq I_2. \]  

(16)

Yukawa couplings arise from a disk-shaped world sheet with three open-string vertex operators attached to its boundary. The boundaries of the disk are the relevant D4-branes. For example, the insertion of a \( Q_1 \) vertex operator turns an \( a = 1 \) stack D4-brane boundary into a \( b = 2 \) stack D4-brane boundary [10]. The allowed Yukawas satisfy selection rules that derive from a \( Z_2 \) symmetry associated with each stack of D4-branes. A state associated with a string between the \( a \)th and \( b \)th stack of D4 branes is odd under the \( ab \)th and \( bc \)th \( Z_2 \) and even under any other \( Z_2 \). It is straightforward to show that models with at most five stacks cannot satisfy all of the above constraints. The only six-stack models that do are parametrized by a single integer \( n_2 \) and are given in Table 2.

The matter content of these models is easily determined using the results of [7]. In general, there are \( n_G = 3 \) generations of (massless) chiral fermions plus vector-like fermionic matter and (tachyonic) Higgs doublets of the form

\[ \alpha(e^c + \bar{e}^c) + \beta(L + \bar{L}) + \gamma_u(u^c + \bar{u}^c) + \gamma_d(d^c + \bar{d}^c) + hH. \]  

(17)

In our six-stack models this matter content turns out to be independent of \( n_2 \). We find

\[ \alpha = 3, \quad \beta = 9, \quad \gamma_u = 0 = \gamma_d, \quad h = 6. \]  

(18)
The gauge group is that of the standard model apart from some extra $U(1)$ factors provided $n_2 \neq 0 \text{ mod } 3$. The weak hypercharge $Y$ is given by the superposition of $U(1)$ factors associated with the 6-stacks:
\[ Y = \frac{1}{3} Q_1 + \frac{1}{2} Q_2 + Q_3 + (Q_5^{(1)} + Q_5^{(2)}) + Q_6. \]
(19)

The mass of the tachyonic Higgs doublets \cite{8} does depend on $n_2$ and is given by
\[ m_H^2 = \frac{m_s^2 \epsilon (n_2 - \delta)}{2\pi |\delta| |1 - \delta|}. \]
(20)

where $m_s$ is the string scale; $\epsilon$ and $\delta$ are related to the parameters defining the torus wrapped by the D4-branes. If $R_1$ and $R_2$ are the radii of the two fundamental 1-cycles, and $\theta$ is the angle between the two vectors defining the lattice, then
\[ \epsilon \equiv 2|\cos \theta/2|, \]
(21)
\[ \delta \equiv n_2 - \frac{3R_2}{R_1}. \]
(22)

The above formula for $m_H^2$ is valid so long as $\epsilon \ll 1$, but in any case $m_H^2 \ll m_s^2$ is required for consistency of the standard model without major contamination by string effects. $m_H^2$ also sets the scale for the various vector-like gonions that arise below the string scale. Specifically, we find the states whose masses and multiplicities are given in Table 3.

The matter content detailed above contributes to the running of the coupling strengths $\alpha_i(\mu)$ ($i = 3, 2, Y$) when the renormalization scale $\mu$ is greater than the mass of the relevant matter, and we can hence evaluate the coupling strengths at the (unknown) string scale $\mu = m_s$. There are also Kaluza–Klein and winding modes partners of gauge bosons. The former will have masses above the string scale $m_s$ provided
\[ \frac{\alpha_i(m_s)}{\lambda_H} \geq 1 \quad \forall i. \]
(23)

The latter will have masses above $m_s$ if in addition
\[ \frac{4\pi}{3} |\epsilon|(n_2 - \delta)(m_s R_1)^2 \geq 1. \]
(24)

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Multiplicity</th>
<th>Gonion</th>
<th>Mass$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(12)</td>
<td>3</td>
<td>Fermions $Q_L + \bar{Q}_L$</td>
<td>$2n</td>
</tr>
<tr>
<td>(13)</td>
<td>3</td>
<td>Fermions $\alpha^c + \bar{\alpha}^c$</td>
<td>$2n</td>
</tr>
<tr>
<td>(14)</td>
<td>3</td>
<td>Fermions $\alpha^c + \bar{\alpha}^c$</td>
<td>$2n</td>
</tr>
<tr>
<td>(23)</td>
<td>6</td>
<td>Scalars $H$</td>
<td>$(2n + 1)m_H^2$</td>
</tr>
<tr>
<td>(24)</td>
<td>6</td>
<td>Scalars $H$</td>
<td>$(2n + 1)m_H^2$</td>
</tr>
<tr>
<td>(25)</td>
<td>6</td>
<td>Scalars $H$</td>
<td>$(2n + 1)m_H^2$</td>
</tr>
<tr>
<td>(26)</td>
<td>3</td>
<td>Scalars $H$</td>
<td>$(2n + 1)m_H^2$</td>
</tr>
<tr>
<td>(27)</td>
<td>6</td>
<td>Scalars $H$</td>
<td>$(2n + 1)m_H^2$</td>
</tr>
<tr>
<td>(46)</td>
<td>3</td>
<td>Fermions $\epsilon^c + \bar{\epsilon}^c$</td>
<td>$2n</td>
</tr>
</tbody>
</table>

We shall assume Eqs. (23) and (24) in what follows. In practice, we find values of $|\epsilon| \sim 10^{-1}$ so that Eq. (24) is satisfied when $R_1 \geq m_s^{-1}$. Then, at the string scale the gauge couplings are given by:
\[ \alpha_i^{-1}(m_s) = \alpha_3^{-1}(m_z) + \frac{1}{\pi} \ln \frac{m_s}{m_z} \]
\[ \left[- \frac{1}{\pi}(N_3 \ln X_3 - \ln N_3!) - \frac{1}{\pi}(N_2 \ln X_2 - \ln N_2!) \right], \]
(25)

where
\[ X_3 = \left| \frac{\delta \pi}{\epsilon(n_2 - \delta)} \right| \text{ and } N_3 = [X_3]. \]
(26)
\[ X_2 = \left| \frac{(1 - \delta) \pi}{\epsilon(n_2 - \delta)} \right| \text{ and } N_2 = [X_2]. \]
(27)

The integers $N_2, N_3$ count the numbers of colour triplet gonions with masses below $m_s$. In Eq. (25), and Eqs. (28), (30) below, we have not included the effects of running the gonion and Higgs masses. We find that the gonion contributions to our solutions are small, so there is post hoc justification for our approximation.
\[ \alpha_2^{-1}(m_s) = \alpha_2^{-1}(m_z) - \frac{6}{2\pi} \ln \frac{m_h}{m_z} - \frac{7}{2\pi} \ln \frac{m_s}{m_h} \]
\[ -\frac{3}{\pi}(N_3 \ln X_3 - \ln N_3!) - \frac{2}{\pi}(N_1 \ln X_1 - \ln N_1!) \]
\[ -\frac{3}{4\pi} \left[ (N_0 + 1) \ln 2X_1 - \ln \left( \frac{2N_0 + 1}{2N_0 N_0!} \right) \right], \]
(28)
where
\[ X_1 \equiv \left| \frac{\delta(1-\delta)\pi}{\epsilon(n_2-\delta)} \right|. \]
\[ N_1 \equiv [X_1] \quad \text{and} \quad N_0 \equiv [X_1 - 1/2]. \tag{29} \]

\( N_1 \) counts the number of lepton doublet quarkons below the string scale, and \( 2N_0 + 1 \) is the number of scalar and vector quarkons with masses below this scale.

\[
\alpha_Y^{-1}(m_s) = \frac{50}{6 \pi} \ln \frac{m_h}{m_Z} - \frac{53}{6 \pi} \ln \frac{m_s}{m_h} - \frac{5}{3 \pi} (N_3 \ln X_3 - \ln N_3!) - \frac{14}{3 \pi} (N_2 \ln X_2 - \ln N_2!) \]
\[ - \frac{2}{\pi} (N_1 \ln X_1 - \ln N_1!) \]
\[ - \frac{3}{4 \pi^2} \left( N_0 + 1 \right) \ln 2X_1 - \ln \left( \frac{2N_0 + 1!}{2^{N_0} N_0!} \right) \tag{30} \]

The ratios of the gauge coupling strengths, which depend on the length of the cycles \((n_s, m_s)\) and on the superposition of the U(1) factors in the weak hypercharge \([7]\), are independent of the unknown \( \lambda_H \):

\[
\frac{\alpha_Y^{-1}(m_s)}{\alpha_Z^{-1}(m_s)} = \left[ \delta^2 + n_2(n_2 - \delta)\epsilon^2 \right]^{1/2}, \tag{31} \]
\[
\frac{\alpha_Y^{-1}(m_s)}{\alpha_\lambda^{-1}(m_s)} = \frac{10}{3} + \frac{\alpha_Y^{-1}(m_s)}{2\alpha_Z^{-1}(m_s)}
+ \left[ (1-\delta)^2 + (n_2 - 1)(n_2 - \delta)\epsilon^2 \right]^{1/2}, \tag{32} \]

where \( \epsilon \) and \( \delta \) are defined in Eqs. (21) and (22). Thus, by substituting the solutions (25), (28) and (30) of the renormalization group equations into (31) and (30), we obtain two constraint equations on \( \epsilon \) and \( \delta \).

We also considered the case when the \( \mathcal{N} = 2 \) supersymmetric partners of the gauge fields acquire masses on the scale of \( m_s \) or greater so they do not contribute to the renormalization group equations. In that case, the coefficient of \( \ln \frac{m_h}{m_Z} \) in Eq. (25) is replaced by \( \frac{50}{6 \pi} \), the coefficients of \( \ln \frac{m_h}{m_Z} \) and \( \ln \frac{m_s}{m_h} \) in Eq. (28) become \( -\frac{4}{25} \) and \( -\frac{12}{25} \), respectively, and in Eq. (30) become \( -\frac{2}{\pi} \) and \( -\frac{3}{4 \pi^2} \), respectively.

We solve the constraints (31) and (32) numerically for a range of values of the parameters \( n_2 \) and \( m_s \), and using the latest values [11] of the standard model parameters which yield

\[ \alpha_Y^{-1}(m_Z) = 8.403, \tag{33} \]

<table>
<thead>
<tr>
<th>( n_2 )</th>
<th>( m_s ) (TeV)</th>
<th>( \epsilon )</th>
<th>( m_h ) (GeV)</th>
<th>( a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.0</td>
<td>0.185</td>
<td>137</td>
<td>0.294</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>0.123</td>
<td>137</td>
<td>0.198</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
<td>0.074</td>
<td>137</td>
<td>0.119</td>
</tr>
<tr>
<td>30</td>
<td>1.0</td>
<td>0.066</td>
<td>425</td>
<td>0.122</td>
</tr>
</tbody>
</table>

The ratio \( a \) of the compactification radii is then determined from Eq. (22). Our results are summarised in Table 4 for the case of an \( \mathcal{N} = 2 \) supersymmetric gauge sector, and in Table 5 when the \( \mathcal{N} = 2 \) gauge superpartners are assumed to have masses of order \( m_s \).

We find that it is easy to obtain values of \( \epsilon \ll 1 \) and \( m_H^2 \ll m_s^2 \) consistently with a string scale \( m_s \) not more than a few TeV. Physical Higgs masses not much greater than the current LEP bound \( (m_h > 114 \text{ GeV}) \) [12] are found for some values of the parameters, but these are probably excluded because they are necessarily accompanied by charged, vector-like quarkons with similar masses. The general behaviour is that with \( m_s \) fixed, \( m_h \) falls slowly as \( n_2 \) increases, whereas with \( n_2 \) fixed, \( m_h \) increases as \( m_s \) increases. As in [2,3], with D-branes located at \( R/Z_3 \) orbifold fixed points, the mixed U(1) anomalies in the present intersecting brane model are cancelled by a generalized Green–Schwarz mechanism mediated by
Table 5

Predicted values of the physical Higgs mass $m_h \equiv |m_H^2|^{1/2}$ and the ratio $a = R_1/R_2$ of the compactification radii in the case when the $N = 2$ gauge superpartners have masses of order $m_s$.

<table>
<thead>
<tr>
<th>$n_2$</th>
<th>$m_s$ (TeV)</th>
<th>$\epsilon$</th>
<th>$m_h$ (GeV)</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
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<td>145</td>
<td>0.345</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>0.125</td>
<td>175</td>
<td>0.353</td>
</tr>
<tr>
<td>5</td>
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<td>0.112</td>
<td>286</td>
<td>0.370</td>
</tr>
<tr>
<td>5</td>
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<td>0.099</td>
<td>466</td>
<td>0.389</td>
</tr>
<tr>
<td>10</td>
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<td>0.082</td>
<td>145</td>
<td>0.219</td>
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<tr>
<td>10</td>
<td>1.2</td>
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<td>10</td>
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<td>286</td>
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<td>10</td>
<td>3.0</td>
<td>0.060</td>
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</tr>
<tr>
<td>20</td>
<td>1.2</td>
<td>0.045</td>
<td>175</td>
<td>0.128</td>
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<td>20</td>
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<td>0.044</td>
<td>191</td>
<td>0.128</td>
</tr>
<tr>
<td>20</td>
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<td>20</td>
<td>3.0</td>
<td>0.034</td>
<td>466</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Twisted Ramond–Ramond fields. However, it is unclear whether the anomalous $U(1)$s survive as global symmetries because of the lack of supersymmetry in the present set-up. Certainly, the Yukawa terms needed to generate the required masses conserve all of the $U(1)$ symmetries, so it seems likely that they are global symmetries. If so, keV-scale axions will remain a problem in these models too. Unlike the models of [2,3], the present model possesses lepton mass terms as well as quark mass terms, at renormalizable level provided we assign the lepton doublets to (25) intersections rather than the (24) intersection. Also unlike these models our model is free from lepton number violating terms at renormalizable level.

In conclusion, we have found a unique class of six-stack standard-like models that give masses to all of the matter, and are free of charged singlet (and colour triplet) tachyons. We find it relatively easy to ensure that the Higgs mass is small compared with the 1–3 TeV string scale, while consistently reproducing the observed values of the gauge coupling constants at the electroweak scale, and achieving the values required by the string theory at the string scale. However, our models do have extra light, vector-like lepton states (but not quark states), and Higgs doublets, as well as the towers of gones characteristic of all theories of this type. There are also three extra non-anomalous $U(1)$ gauge groups under which the matter is charged.

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Bouncing branes

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Abstract


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1. Introduction

Theories with extra dimensions where our four-dimensional world is a hypersurface (three-brane) embedded in a higher-dimensional spacetime and at which gravity is localised have been intensely studied since the work of Randall and Sundrum [1]. This Letter investigates a model of a single brane, embedded in a five-dimensional spacetime. In particular, we study cosmological solutions of (4 + 1)- and (5 + 0)-dimensional gravity coupled to a scalar field sigma-model. In much of the current literature it is assumed that such scalars depend only on the fifth dimension and that the target space metric is of Euclidean signature. By contrast, we consider a non-compact sigma-model and allow the scalars to depend on time, as well as the fifth dimension, which we take to be infinite.
in extent. We also include a bulk fluid with energy-
momentum tensor $\tilde{T}_R^A(\rho) = \text{diag}(-\rho, p, p, p, P)$ and
equation of state $p = \omega \rho$, $P = \omega \rho$ and show that the
fluid exists, provided $\omega = \tilde{\omega} = 1$ (i.e., the fluid is
isotropic and stiff). A family of warp factors that in-
cludes both the original RS solution [1] and the self-
tuning solution of Kachru, Schulz and Silverstein [3]
is found. Conventional cosmology is also obtained.

Further, we simplify our model by taking a projec-
tion in the target space (onto a dilatonic degree of free-
dom) and by making the fifth radius static. This results
in a Kachru–Schulz–Silverstein warp factor [2,3]. We
show that the cosmology on this self-tuning brane is
isotropic and stiff). A family of warp factors that in-
cludes both the original RS solution [1] and the self-
tuning solution of Kachru, Schulz and Silverstein [3]
is found. Conventional cosmology is also obtained.

In this section, following [4], we shall present our
calculations in $(4 + 1)$-dimensional spacetime with
flat spatial three-sections on the brane and only quote
analogous results for the $5 + 0$ case. The action for
gravity coupled to a scalar field sigma-model is:

$$S = \int d^4x\ dr\ (L_{\text{MATTER}}^{(5)} + L_{\text{GRAVITY}}^{(5)}),$$

where:

$$L_{\text{MATTER}}^{(5)} = -\frac{1}{2}\sqrt{-g^{(5)}}\nabla^\mu \phi_i \nabla^\nu \phi_i G^{ij}(\phi) g^{(5)}_{\mu\nu}$$

$$-\sqrt{-g^{(5)}} V(\phi) - \sqrt{-g^{(4)}} V(\phi) \delta(r),$$

$$L_{\text{GRAVITY}}^{(5)} = \frac{1}{k^2} \sqrt{-g^{(5)}} R.$$  

Here, $g^{(4)}_{\mu\nu}$ is the pull-back of the five-dimensional
metric $g^{(5)}_{\mu\nu}$ to the (thin) domain wall taken to be at
$r = 0$. The wall is represented by a delta function
source with coefficient $V(\phi)$ parametrising its tension.
We take $G_{ij} = \text{diag}(1, -1)$. The “correctly-signed”
scalar, $\phi^1$, may be interpreted as the dilaton and
the “wrongly-signed” scalar, $\phi^2$, as an axion. (It is
possible to consider a non-trivial coupling between the
two—for example, $G_{ij} = \text{diag}(1, -e^{r\phi^1}$ is discussed in
[10].)

We assume a separable metric of RS type with flat
spatial three-sections on the wall and a ‘rolling’ fifth
radius:

$$ds^2 = -e^{-A(r)} dt^2 + e^{-A(r)} g(t)(dx^2 + dy^2 + dz^2)$$

$$+ f(t) dr^2.$$  

Given the above ansatz, it is not unreasonable to
assume scalars of the form

$$\phi(t, r) = a^i \psi(t) + b^i \chi(r).$$
The linear independence of the scalar fields $\phi^i$ (which are coordinates on the target spacetime) leads to:

$$\det\begin{pmatrix} a^1 & b^1 \\ a^2 & b^2 \end{pmatrix} \neq 0$$

and, consequently, to the Schwarz inequality

$$\frac{(a \cdot a)(b \cdot b)}{(a \cdot b)} < 1.$$ 

We also make the ansatz that both the potentials $U$ and $V$ are of Liouville type (see, for instance, [2]):

$$U(\phi) = U_0 e^{\alpha \phi},$$
$$V(\phi) = V_0 e^{\beta \phi}.$$ 

The energy–momentum tensor for the scalar fields is:

$$T_{\mu\nu} = \frac{1}{2} \nabla_{\mu} \phi^i \nabla_{\nu} \phi^j G_{ij}$$
$$= \frac{1}{2} g_{\mu\nu} \left( \frac{1}{2} \nabla_{\mu} \phi^i \nabla_{\nu} \phi^j G_{ij} \delta^{ab} + U(\phi) \right)$$
$$= \frac{1}{2} \sqrt{-g(4)} \nabla(\phi) \delta(r) g(4) \delta^{ab} \mu_{\mu}.$$

We introduce a bulk fluid via its energy–momentum tensor [11]:

$$\tilde{T}_{\mu\nu} = \text{diag}(-\rho, p, p, p, P)$$

with $\rho$ the density and $p$ and $P$ the pressures in the three spatial directions on the brane and in the fifth dimension, respectively. We assume that the equations of state are $P = \tilde{\omega} \rho$ and $p = \omega \rho$. The preferred coordinate system (3) is taken as the rest frame of the fluid. The anisotropy can be considered as a result of the mixing of two interacting perfect fluids [12].

Einstein’s equations $G_{\mu\nu} = k^2 (T_{\mu\nu} + \tilde{T}_{\mu\nu})$ reduce to:

$$\frac{1}{4} \frac{\dot{f}}{f} g + \frac{g^2}{4 f^2} + \frac{1}{2} \frac{\dot{f}^2}{f^2} - \frac{\dot{g}}{2 f} - \frac{k^2}{2} a \cdot a \psi^2$$
$$- k^2 e^{-A} (\rho + p) = 0,$$
$$\frac{3}{4} \frac{\dot{f}}{f} g + \frac{3 g^2}{4 f^2} - \frac{k^2}{4} a \cdot a \psi^2 - k^2 e^{-A} \rho = 0,$$
$$\frac{3}{2} (A^2 - A'') + \frac{k^2}{4} b \cdot b \chi^2$$
$$+ \frac{k^2}{2} f U + \frac{k^2}{2} f^{1/2} V \delta(r) = 0,$$
$$\frac{3 \dot{g}}{2 g} + \frac{k^2}{4} a \cdot a \psi^2 + k^2 e^{-A} P = 0.$$ 

In the above equations we have assumed separability and we have set all separation constants equal to zero (thus losing classes of solutions). In the next sections we will analyse cases with non-zero separation constants.

The density and pressures are each of the form $e^{A(r)}$ times a function of $t$.

The equations of motion for the scalar fields

$$\nabla^2 \phi^i G_{jk} = - \frac{\partial U(\phi)}{\partial \phi^k} - \sqrt{-g(4)} \frac{\partial V(\phi)}{\partial \phi^k} \delta(r) = 0$$

result in the following bulk equations [4]:

$$\dot{a}_i \left( f^{1/2} g^{3/2} \dot{\psi} \right) = 0,$$
$$b_i (2 A' \chi' - \chi'') + f a_i U_0 = 0,$$

and the jump condition [4]:

$$\lim_{\epsilon \to 0^+} \left[ b_i (\chi'(-\epsilon) - \chi'(\epsilon)) \right] = \beta_i f^{1/2} V (\psi(t, 0)).$$

Eq. (15) implies that we can make the following choice for the scalar fields [4]:

$$k \chi'(r) = \sqrt{6} A'(r),$$
$$k \dot{\psi}(t) = - \frac{\sqrt{6}}{4} \frac{1}{a \cdot b} \frac{\dot{f}(t)}{f(t)}.$$

Inserting (22) into (14) gives $U(\phi)$ as:

$$U = \frac{3}{k^2} f A''(1 - b \cdot b).$$

\[1\] In principle, Einstein’s equations can handle $\rho$ and $p$ in the form

$$\rho(t, r) = e^{A(r)} (\dot{\rho}(t) + F(t, r)),$$
$$p(t, r) = e^{A(r)} (\dot{p}(t) - F(t, r)),$$

for arbitrary $F(t, r)$. However, the constant $\omega$ in the equations of state is in the range $-1 \leq \omega \leq 1$. The generic case $\omega \neq -1$ implies that $F$ should be zero. We shall assume this also to be so in the special case $\omega = -1$. 

$$\frac{3}{2} A''^2 - \frac{k^2}{4} b \cdot b \chi'^2 + \frac{k^2}{2} f U = 0,$$
$$\frac{3}{2} A' \frac{\dot{f}}{f} + k^2 a \cdot b \dot{\psi} \chi' = 0.$$
Expressing the domain wall potential as \( V_0 f(t)^{-1/2} \times \delta(r) \), we get the following equation

\[
A'' - 2b \cdot b A' - \frac{k^2}{3} V_0 \delta(r) = 0 \tag{25}
\]

and options for \( A(r) \) and \( V_0 [4] \):

1. If \( b \cdot b = 0 \), we find \( A(r) = 2\alpha k |r| \), where \( \alpha = \pm 1 \). Then \( V_0 = 12\alpha k \), \( \alpha = -1 \) is the RS1 solution and \( \alpha = +1 \) is the RS2 solution, as described in [13].

2. If \( b \cdot b \neq 0 \), we find \( A(r) = c \ln(k |r| + 1) \) where \( c = -1/2b \cdot b \) and \( V_0 = -3k^2/b \cdot b \). If \( b \cdot b \) and \( k \) are both positive, then this represents the self-tuning solution of Kachru, Schulz and Silverstein [3].

The above forms for \( U \) and \( V \) are consistent with (6) if \( b = a \), \( b = b^2 \), and \( a = a^2 \). It can now be verified that (20) is equivalent to (25) in the bulk, whilst (21) yields no further information.

The equation of motion (19) implies that [4]:

\[
\frac{f'(t)}{f(t)^{3/2}} = \mu g(t)^{-3/2},
\]

where \( \mu = -4a \cdot b/\sqrt{6} \).

This equation, together with the time-dependent Einstein’s equations and the above equations of state, leads to the following three relations [4]:

\[
\omega \rho = p = \frac{1}{3} \rho + \frac{2}{3} P = \frac{1}{3} (1 + 2\tilde{\omega}) \rho, \tag{27}
\]

the equation for the density:

\[
\rho(t, r) = \frac{3e^A}{4\kappa^2} \left( \frac{f'}{f^2} + \frac{g'}{g^2} - \frac{a \cdot a}{8(a \cdot b)^2} \frac{f^2}{f' \cdot f} \right), \tag{28}
\]

and the cosmology defining equation:

\[
\tilde{g} \frac{g'}{g^n} + 2 \frac{g'}{g} + \frac{f'}{f} + (1 - \tilde{\omega}) \frac{a \cdot a}{8(a \cdot b)^2} \frac{f^2}{f'} = 0. \tag{29}
\]

We seek either power law, \( f \sim t^q \), or exponential (inflationary), \( f \sim e^{\gamma t} \), solutions of (29). The corresponding solutions for \( g(t) \) are \( g \sim t^{(2q-3)/3} \) and \( g \sim e^{-\gamma t/3} \), respectively. The exponents \( q \) and \( \gamma \) are non-zero but otherwise arbitrary. The density is positive if \( a \cdot a/(a \cdot b)^2 < -2 \).

It can be shown [4] that the fluid exists if, and only if, \( \tilde{\omega} = 1 \). This implies that \( \omega = 1 \), that is \( P = p \). Thus the fluid, if exists, is isotropic (perfect) \( (P = p) \) and stiff \( (\omega = \tilde{\omega} = 1) \). The attribute “stiff” refers to the fact that the velocity of sound in the fluid is equal to the velocity of light.

The only essential difference between the 5+0 case and the 4+1 case considered above is that \( \tilde{T}^{\mu \nu} \) flips sign. This changes the sign of \( \rho \) in (28) so that the density is positive if \( a \cdot a/(a \cdot b)^2 > -2 \).

We note in passing that the scalar field equations of motion, (18), imply that \( \nabla^\mu T_{\mu \nu} = 0 \) (and conversely off the brane only). This, in turn, implies that the fluid equation of motion \( \nabla_\mu \tilde{T}^{\mu \nu} = 0 \) is automatically satisfied. In this sense, the same results in the bulk can be obtained from Einstein’s equations and \( \nabla_\mu \tilde{T}^{\mu \nu} = 0 \).

### 3. Standard cosmology on a self-tuning domain wall

In this section we will make a projection in the target space onto a dilatonic degree of freedom (i.e., set \( a^1 = a^2 = b^2 = 0 \), \( b^1 = 1 \)), consider the case of a static fifth radius (i.e., set \( f(t) = \text{const} \)), and, again, take Liouville type potentials:

\[
U(\phi) = U_0 e^{a\chi}, \tag{30}
\]

\[
V(\phi) = V_0 e^{b\phi}. \tag{31}
\]

(Note that in this section the scalar field \( \psi(t) \) will be excluded from the analysis.)

Introducing non-zero separation constants \( N \) and \( M \) on the right-hand-sides of Einstein’s equations (11), (12) and (13), (14), respectively, we get the solutions [5]:

\[
g \sim \begin{cases} 
\sin^2 q \left( \sqrt{\frac{M}{2\omega}} t \right), & M > 0, \\
\frac{t^{2q}}{\sin^2 \left( \sqrt{\frac{M}{2\omega}} t \right)}, & M = 0, \\
\frac{t^{2q}}{\sin^2 \left( \sqrt{\frac{M}{2\omega}} t \right)}, & M < 0,
\end{cases} \tag{32}
\]

where \( q = 1/(2 + \tilde{\omega}) = 2/(3(1 + \omega)) = q_{\text{standard}} \).

---

\(2\) Einstein’s equations lead to (see [5]) \( N = 2M \) and \( \omega = \frac{1}{2}(1 + 2\tilde{\omega}) \).
Defining $A(0) = 0$, we see that the brane density is [5]:

$$\tilde{\rho}(t) = \begin{cases} 
\kappa^{-2}M \sinh^{-2}\left(\frac{M}{3q^2t}\right), & M > 0, \\
\frac{3\kappa^{-2}q^2}{t^2}, & M = 0, \\
\kappa^{-2}|M| \sin^{-2}\left(\frac{|M|}{3q^2t}\right), & M < 0.
\end{cases}$$

When $M \geq 0$, we also obtain the de-Sitter solutions $g = e^{\pm 2N/\sqrt{M}}$. These solutions have vanishing density $\tilde{\rho}$ and were discussed in [3,14–16].

For the case $M = 0$, we obtain conventional cosmology $H = \dot{a}/a \propto \sqrt{\tilde{\rho}}$ on the brane with evolution at the standard rate.

Of particular note is the case of radiation-dominated fluid on the brane ($\omega = 1/3$), for which the pressure in the fifth direction vanishes and the stress tensor is given by:

$$\tilde{T}_{\mu\nu}(\rho) = e^{A(r)} \tilde{\rho}(t) \text{diag}(-1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0),$$

with $q_{\text{standard}} = 1/2$.

It should be noted that the case of a bulk cosmological constant ($\omega = \bar{\omega} = -1$) is not covered here; however, it corresponds to the choice $U(\chi) = \text{const}$ instead.

The self-tuning domain wall (solution (I) of [3]) is given by

$$U = M = 0, \quad \beta \neq \pm \frac{1}{a},$$

$$\chi(r) = a \tau \ln(d - cr),$$

$$A(r) = -\frac{1}{2} \ln(d - cr) - e,$$

where $a = \frac{\tau}{\chi}$ and $\tau$ is a sign that takes opposite values either side of the brane at $r = 0$. The parameters $c, d$, and $e$ are constants of integration such that:

$$c_+ = -\frac{2}{3} \kappa^2 d_+(\alpha \beta \tau_+ - 1)V_0e^{\alpha \beta \tau_+ \log d_+},$$

$$c_- = -\frac{2}{3} \kappa^2 d_-(\alpha \beta \tau_+ + 1)V_0e^{\alpha \beta \tau_+ \log d_-},$$

$$d_+ > 0,$$

$$e_+ = -\frac{1}{2} \ln d_+,$$

$$e_- = -\frac{1}{2} \ln d_-,$$

with the convention $A(0) = 0$ and the ± subscript denoting the right (left) side of the brane. The solution is self-tuning because given $d_+, \tau_+ = \pm 1$ and $\beta \neq \pm 1/a$, there is a Poincaré-invariant four-dimensional domain wall for any value of the brane tension $V_0$; $V_0$ does not need to be fine-tuned to find a solution.

Other warp factors are possible both when $M = 0$ and when $M \neq 0$. Solution (II) of [3] with $U = 0$ and solution (III) of the same reference with $U \neq 0$ are examples of the former case. The solution presented in [14] with $U = 0$ provides an example of the latter.

To summarize this section, we state that the self-tuning domain wall, with warp factor given by (37), has vanishing separation constant $M$ and therefore expands according to the power law (32) at the standard rate and exhibits conventional cosmology when coupled to a bulk anisotropic fluid. The pressure of the fluid in the fifth direction, $P$, vanishes for a radiation-dominated brane.

### 4. Bouncing branes

In view of the Giddings–Strominger theorem [6] (stated in the introduction), which allows negative kinetic energy associated with a free time-dependent only scalar field, minimally coupled to gravity, and in view of the possibility of negative kinetic energy, associated with the sigma-model, we will try to find a wormhole solution for our set-up. For this purpose, we will take another projection in our target space, namely, $a^1 = b^1 = 1, a^2 = b^2 = 0$. In other words, we will consider only one of the scalar fields:

$$\phi(t, r) = \psi(t) + \chi(r).$$

Again, both the potentials $U$ and $V$ will be of Liouville type:

$$U(\phi) = U_0 e^{\alpha \phi},$$

$$V(\phi) = V_0 e^{\beta \phi}.$$

We will exclude (for simplicity) the fluid from the analysis, consider again a ‘rolling’ fifth radius case, and introduce non-flat spatial three-sections on the brane. That is, the metric will be:

$$ds^2 = se^{-A(r)} dt^2$$

$$+ \frac{e^{-A(r)} g(t)}{1 + \frac{4}{3}(x^2 + y^2 + z^2))} (dx^2 + dy^2 + dz^2)$$
+ q f(t) \, dt^2. \quad (46)

This is a natural generalisation of the most general four-dimensional homogeneous and isotropic Robertson–Walker metric [7] to a five-dimensional Randall–Sundrum context [1]. The scale factor \( g(t) \) is a strictly positive function (we are working with a mostly-plus metric) and the function \( f(t) \) is strictly positive as well (the metric is never degenerate). The factors \( s \) and \( q \) are signs \((s^2 = q^2 = 1)\). The curvature parameter is \( \epsilon = +1 \) (for spherical spatial three-sections) or \( \epsilon = -1 \) (for hyperbolic spatial three-sections).

Einstein’s equations for this case, \( G_{\mu \nu} = \kappa^2 T_{\mu \nu} \), equivalently written in terms of the Ricci tensor as \( R_{\mu \nu} = \kappa^2(T_{\mu \nu} - \frac{1}{2}g(\mu \nu)T^\alpha_\alpha) \), are:

\begin{align*}
- s \frac{1}{q} \frac{1}{f} e^{-A} A'^2 + s \frac{1}{2q} e^{-A} A'' & \quad + \frac{1}{4q^2} \left( \frac{f'}{f} \right)^2 - \frac{1}{2q} \left( \frac{f'}{f} \right) \left( \frac{g'}{g} \right) + \frac{3}{4} \frac{g'}{g} - \frac{3}{2} \frac{\hat{g}}{\hat{g}} \\
= \frac{\kappa^2}{2} \hat{\psi}^2 + \frac{\kappa^2}{4} s e^{-A} U + \frac{\kappa^2}{6} \frac{s}{f^{1/2}} e^{-A} V \delta(r), & \quad (47)
\end{align*}

\begin{align*}
2 \epsilon - \frac{1}{q} \frac{\hat{g}}{\hat{g}} e^{-A} A'^2 + \frac{1}{2q} \frac{\hat{g}}{\hat{g}} e^{-A} A'' & \quad - \frac{1}{4s} \frac{\hat{g}}{\hat{g}} - \frac{1}{2s} \frac{\hat{g}}{\hat{g}} \\
= \frac{\kappa^2}{2} e^{-A} g U + \frac{\kappa^2}{6} \frac{g}{f^{1/2}} V \delta(r), & \quad (48)
\end{align*}

\begin{align*}
-A'^2 + 2A'' + \frac{q}{4s} \frac{f'}{f} e^A - \frac{q}{2s} \frac{f'}{f} e^A - \frac{3q}{4s} \frac{\hat{g}}{\hat{g}} e^A & \quad = \frac{\kappa^2}{2} \chi' \chi + \frac{\kappa^2}{3} q f U + \frac{2\kappa^2}{3} q f^{1/2} V \delta(r), & \quad (49)
\end{align*}

\begin{align*}
- \frac{3}{4} \frac{A'}{f} & \quad = \frac{\kappa^2}{2} \hat{\psi}^\prime. & \quad (50)
\end{align*}

Similarly to the sigma-model case, the \( t^r \)-equation, (50), implies:

\begin{align*}
\kappa \hat{\psi} (t) & \quad = - \sqrt{3} \frac{\hat{f}(t)}{f(t)}, \quad (51)
\kappa \chi' (r) & \quad = \sqrt{3} \frac{1}{2} A'(r). \quad (52)
\end{align*}

When \( \beta = \frac{\kappa}{\sqrt{3}} \), the potential \( V(\phi) \) can be written in the form:

\begin{equation}
V(\phi(t, r)) = \frac{1}{\kappa^2} \left[ \frac{1}{2} \frac{1}{q f^{1/2}} W \right], \quad (53)
\end{equation}

where \( W \) is a constant.

Let us assume that the potential \( U(\phi) \) can be written as a function of \( t \) and \( r \) in the separable form:

\begin{equation}
U(\phi(t, r)) = \frac{1}{\kappa^2} \left[ \frac{1}{q f^{1/2}} U_1(r) + e^A U_2(t) \right]. \quad (54)
\end{equation}

At the end we will recast the potential \( U \) back into the original exponential form (44).

Einstein’s equations then reduce to:

\begin{align*}
\frac{1}{4} f^2 - \frac{1}{2} \frac{\hat{f}}{f} + \frac{1}{4} \frac{\hat{g}}{g} + \frac{g^2}{g} - \frac{2\epsilon s}{g} - \frac{3}{2} \frac{f^2}{f} & \quad = 0, \quad (55)
\end{align*}

\begin{align*}
\frac{1}{4s} \frac{\hat{g}}{\hat{g}} - \frac{1}{4s} \frac{\hat{g}}{\hat{g}} - \frac{1}{2s} \frac{\hat{g}}{\hat{g}} + \frac{2\epsilon s}{g} - \frac{1}{3} U_2 & \quad = \frac{C}{q f}, \quad (56)
\end{align*}

\begin{align*}
\frac{1}{2} f^2 - \frac{3}{2} \frac{\hat{f}}{f} - \frac{3}{f^2} - \frac{2\epsilon s}{3} U_2 & \quad = \frac{2s D}{q f}, \quad (57)
\end{align*}

\begin{align*}
\frac{1}{2} A'' + \frac{1}{3} U_1 + \frac{1}{6} \frac{W \delta(r)}{2} & \quad = Ce^A, \quad (58)
\end{align*}

\begin{align*}
\frac{11}{8} A'^2 - 2A'' + \frac{1}{3} U_1 + \frac{2}{3} \frac{W \delta(r)}{2} & \quad = De^A, \quad (59)
\end{align*}

where \( C \) and \( D \) are separation constants.

We will be looking for a bounce solution in the form:

\begin{equation}
g(t) = f(t) = B t^2 + h > 0, \quad (60)
\end{equation}

where \( B \) is a positive constant, not equal to 1 (so that the pull-back of the metric to 4 dimensions is flat only asymptotically\(^3\)), and \( h \) is another strictly positive constant.

Upon substitution of the solutions (60) into the Einstein’s equations, (55) gives:

\begin{equation}
B = - \frac{2\epsilon s}{3}. \quad (61)
\end{equation}

On the other hand, \( B \) must be positive. Therefore, \( B = 2/3 \) and \( \epsilon \) and \( s \) must have opposite signs. Thus the solution is either a brane with spherical three-sections and Lorentz metric or a brane with hyperbolic three-sections and positive definite metric. Clearly, these

\(^3\) The curvature of the brane is \( R = \frac{6 \kappa^2 \delta B}{B^3 + \frac{3}{h}} \).
two solutions can also be related by a Wick rotation (time \( t \) is changed to \( it \) and the positive-curvature spacetime becomes a negative-curvature spacetime).

The last term in (55) is the kinetic energy of the scalar field \( \psi \). One can easily see from (55) that it is strictly positive, unlike the Giddings–Strominger case [6].

Einstein’s equations (56) and (57) are consistent if we choose

\[
U_2(t) = \sigma \frac{g^2(t)}{g^2(t)}
\]

(62)

where \( \sigma \) is a constant.

The next Einstein’s equation, (56), yields that the separation constant \( C \) is \( \frac{3\epsilon q}{\kappa^2} \) and that \( \sigma = -\frac{\epsilon q}{\kappa^2} \).

The remaining time-dependent Einstein’s equations (58) and (59) yield:

\[
U(t) = 10\epsilon q e^{A(r)} - \frac{21}{8} A'(r)^2
\]

(63)

and these two equations reduce to a single equation:

\[
\frac{1}{8} A'^2 - \frac{1}{2} A'' + \frac{1}{6} W\delta(r) = -\frac{2\epsilon q}{3} e^A.
\]

(64)

A solution of this equation is of KSS [3] type:

\[
A(r) = \ln \left( \frac{1}{k|r| + 1} \right)^2,
\]

(65)

where \( k \) is a constant, such that \( k^2 = 4\epsilon q \). Therefore, \( \epsilon \) and \( q \) must have the same signs (\( k^2 = 4/3 \)). The constant \( W \) in the brane tension \( V \) is \(-12k\).

The equation of motion for the scalar field:

\[
\nabla^2 \phi - \frac{\delta U(\phi)}{\delta \phi} - \frac{\sqrt{|g^{(4)}|}}{\sqrt{|g^{(5)}|}} \frac{\delta V(\phi)}{\delta \phi} \delta(r) = 0,
\]

(66)

after integration over the fifth dimension in an infinitesimal interval, gives a jump condition across the brane:

\[
A'(+0) - A'(-0) = -4k.
\]

(67)

Let us now write the potential

\[
U(\phi(t,r)) = \frac{1}{\kappa^2} \left[ \frac{1}{qf} U_1(r) + e^A U_2(t) \right]
\]

back in the exponential form \( U = U_0 e^{\alpha \phi} \). Substituting the solution (65) for \( A(r) \) into (63) gives:

\[
U_1(r) = -4\epsilon q e^{A(r)}.
\]

(68)

Using this form of \( U_1(r) \), together with (62) for \( U_2(t) \) and the value of \( B \), we easily find that:

\[
U(\phi(t,r)) = \frac{4\epsilon h e^A}{\kappa^2 g^2} = U_0 e^{\frac{2\phi}{\kappa^2}}.
\]

(69)

For a realistic model, one could choose \( h \) sufficiently small.

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References

Abstract

We study the $N=2$ supersymmetric $E_6$ models on the 6-dimensional space–time where the supersymmetry and gauge symmetry can be broken by the discrete symmetry. On the space–time $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2')$, for the zero modes, we obtain the 4-dimensional $N=1$ supersymmetric models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, $SU(4) \times SU(2) \times SU(2) \times U(1)$, and $SU(3) \times SU(2) \times U(1)^3$ with one extra pair of Higgs doublets from the vector multiplet. In addition, considering that the extra space manifold is the annulus $A^2$ and disc $D^2$, we list all the constraints on constructing the 4-dimensional $N=1$ supersymmetric $SU(3) \times SU(2) \times U(1)^3$ models for the zero modes, and give the simplest model with $Z_9$ symmetry. We also comment on the extra gauge symmetry breaking and its generalization. © 2002 Elsevier Science B.V. All rights reserved.

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Keywords: $E_6$ model; Symmetry breaking; Extra dimensions

1. Introduction

Grand Unified Theory (GUT) gives us an simple and elegant understanding of the quantum numbers of quarks and leptons, and the success of gauge coupling unification in the Minimal Supersymmetric Standard Model strongly supports this idea. Although the Grand Unified Theory at high energy scale has been widely accepted now, there are some problems in GUT: the grand unified gauge symmetry breaking mechanism, the doublet–triplet splitting problem, and the proton decay, etc.

Recently, a new scenario proposed to address above questions in GUT has been discussed extensively [1–3]. The key point is that the GUT gauge symmetry exists in 5 or higher dimensions and is broken down to the 4-dimensional $N=1$ supersymmetric Standard Model like gauge symmetry for the zero modes due to the discrete symmetries in the neighborhoods of the branes or on the extra space manifolds, which become non-trivial constraints on the multiplets and gauge generators in GUT [3]. The attractive models have been constructed explicitly, where the supersym-
metric 5-dimensional and 6-dimensional GUT models are broken down to the 4-dimensional $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)^{n-3}$ model, where $n$ is the rank of GUT group, through the compactification on various orbifolds and manifolds. The GUT gauge symmetry breaking and doublet–triplet splitting problems have been solved neatly by the discrete symmetry projections. Other interesting phenomenology, like $\mu$ problems, gauge coupling unifications, non-supersymmetric GUT, gauge-Higgs unification, proton decay, etc., have also been discussed [1–3].

All of the models [1–3] discussed previously have gauge group $SU(N)$ or $SO(N)$. So, we study the $E_6$ model in the present Letter, which is as interesting as $SU(5)$ and $SO(10)$ GUT models. Because $E_6$ is a rank 6 exceptional group, in order to break the gauge symmetry and supersymmetry, we need to consider at least two extra dimensions. In addition, the 6-dimensional $N = 1$ supersymmetric theory is chiral, where the gaugino (and gravitino) has positive chirality and the matters (hypermultiplets) have negative chirality, so, it often has anomaly unless we put the Standard Model fermions on the brane, and add one multiplet in the adjoint representation of the gauge group or some suitable matter contents in the bulk to cancel the gauge anomaly. And the 6-dimensional non-supersymmetric $E_6$ models and $N = 1$ supersymmetric $E_6$ models can be considered as special cases of $N = 2$ supersymmetric $E_6$ models, therefore, we only discuss the 6-dimensional $N = 2$ supersymmetric $E_6$ models. Moreover, because $N = 2$ 6-dimensional supersymmetric theory has 16 real supercharges, which corresponds to $N = 4$ 4-dimensional supersymmetric theory, we cannot have hypermultiplets in the bulk. Therefore, we have to put the Standard Model fermions on the brane or brane intersection.

In this Letter, we first review the discussions of $E_6$ breaking by Wilson line in our context [4]. Then, we study $E_6$ breaking on the space–time $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2'')$, where $M^4$ is the 4-dimensional Minkowski space–time. For the zero modes, we obtain the 4-dimensional $N = 1$ supersymmetric models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, $SU(4) \times SU(2) \times SU(2) \times U(1)$, and $SU(3) \times SU(2) \times U(1)^3$ with one extra pair of Higgs doublets from the vector multiplet. In addition, considering that the extra space manifold is an annulus $A^2$ and a disc $D^2$, we can define $Z_n$ symmetry on the extra space manifold. We list all the constraints on constructing the 4-dimensional $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)^3$ model for the zero modes, and give the simplest model with $Z_9$ symmetry. Furthermore, we comment on the extra gauge symmetry breaking and its generalization.

We would like to explain our convention. For simplicity, we define

\[
(\alpha, \beta, \gamma) \equiv \begin{pmatrix}
\alpha & 0 & 0 \\
0 & \beta & 0 \\
0 & 0 & \gamma
\end{pmatrix}.
\]

In addition, suppose $G$ is a Lie group and $H$ is a subgroup of $G$. In general, for $G = SU(N)$ and $G = SO(N)$, $H$ can be the subgroup of $U(N)$ and $O(N)$, respectively. We denote the commutant of $H$ in $G$ as $G/H$,

\[
G/H \equiv \{ g \in G | gh = hg, \text{ for any } h \in H \}.
\]

And if $H_1$ and $H_2$ are the subgroups of $G$, we define

\[
G/[H_1 \cup H_2] \equiv \{ G/H_1 \} \cap \{ G/H_2 \}.
\]

2. Background of $E_6$ breaking

In 1985, a lot of work has been done on $E_6$ breaking because the $E_6$ model can be obtained from the compactification of the weakly coupled heterotic $E_8 \times E_8$ string theory on the Calabi–Yau manifold by spin connection embedding [4,5]. We would like to review the discussions of $E_6$ breaking by Wilson line in our context [4], which will be used to discuss $E_6$ breaking in this Letter.

Suppose the space–time manifold is $M^4 \times K$ where $K$ is the $k$-dimensional extra space manifold, and we can define a discrete symmetry $\Gamma$ on $K$. In general, $\Gamma$ can be the product of discrete groups, and $\Gamma$ may not act freely on $K$. And when it does not act freely on $K$, there exists a brane at each fixed point, line or hypersurface, where the Standard Model fermions can be located.

The gauge fields of $E_6$ are in the adjoint representation of $E_6$ with dimension 78, and $E_6$ contains a maximal subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$ where $SU(3)_C$ is color $SU(3)$, $SU(3)_L$ and $SU(3)_R$ describe the weak interactions of left-handed and right-handed quarks, respectively [6]. Under the gauge groups
SU(3)_C \times SU(3)_L \times SU(3)_R$, the $E_6$ gauge fields decompose to $(\mathbf{8}, \mathbf{1}, 1), (\mathbf{1}, \mathbf{8}, 1), (\mathbf{1}, \mathbf{1}, \mathbf{8}), (\mathbf{3}, \mathbf{3}, 3), (\mathbf{3}, \mathbf{\bar{3}}, \mathbf{3})$ [6].

For simplicity, we assume that $\Gamma$ is $Z_n$ in the following discussions, where $Z_n$ is generated by the $n$th roots of unity. Since the discussions for the product of cyclic groups are similar, we do not repeat them here. Let $\gamma$ be a generator of $\Gamma$, we choose the following matrix representation for $\gamma$, which will give us the representations of all the elements in $\Gamma$,

$$R_\gamma = (+1, +1, +1) \otimes (\alpha, \alpha, \beta) \otimes (\delta, \rho, \sigma),$$

where $\alpha^a = \beta^a = \delta^a = \rho^a = \sigma^a = 1$. And the map $R : \Gamma \rightarrow R_\Gamma \subset SU(3) \times (SU(3) \times Z_n) \times (SU(3) \times Z_n) \subset SU(3) \times U(3) \times U(3)$ must be a homomorphism.

We would like to make a remark here. If we discussed the $SU(N)$ breaking on the extra space manifold with $Z_n$ symmetry, we could choose the representation of $Z_n$ in $SU(N) \times Z_n \subset U(N)$. And if we discussed the $SO(N)$ breaking on the extra space manifold with $Z_n$ symmetry, we could choose the representation of $Z_n$ in $SO(N)$ if $n$ is odd and in $SO(N) \times Z_2 \approx O(N)$ if $n$ is even. The same rule applies for the product of cyclic groups. However, we are interested in $E_6$ breaking, not $SU(3) \times SU(3)_L \times SU(3)_R$ breaking in this Letter. For an arbitrarily choice of $R_\gamma$, the commutant of $R_\Gamma$ in $E_6$ ($E_6/R_\Gamma$) might not form a group if $R_\gamma$ were not a subgroup of $E_6$. Because we require $E_6/R_\Gamma$ to be a group, we choose that $R_\Gamma \subset SU(3)_C \times SU(3)_L \times SU(3)_R$ in the following discussions, i.e., $\alpha^2 = \beta^2 = \delta^2 = \sigma = \rho = 1$. With the choice of $R_\Gamma \subset SU(3)_C \times SU(3)_L \times SU(3)_R$, we can embed all the generators back to those of $E_6$. Because $R_\Gamma$ is an abelian subgroup of $E_6$, it is easy to prove that $E_6/R_\Gamma$ also forms a subgroup of $E_6$ with rank 6. However, there are two exceptions in our discussions. For $Z_2$ case, in order to break $E_6$ down to $SU(3)_C \times SU(3)_L \times SU(3)_R$, we choose $R = (+1, +1, +1) \otimes (+1, +1, +1) \otimes (+1, +1, +1)$ or $R = (+1, +1, +1) \otimes (-1, -1, -1) \otimes (+1, +1, +1)$.

Now, we discuss the $E_6$ breaking. For simplicity, we just consider $E_6$ gauge field $A_\mu = A_\mu^B T^B$, where $\mu = 0, 1, 2, 3$ and $B = 1, 2, \ldots, 78$. Let us denote the $E_6$ gauge fields in $SU(3)_C \times SU(3)_L \times SU(3)_R$ as $A_\mu = A_\mu^B T^B$, where $T^B$ is the product of three $3 \times 3$ matrices, for example, if $T^B$ was a Lie algebra for $SU(3)_C$, the corresponding $T^B = T^B \otimes (1, +1, +1)$, and the other field is $(+1, +1, +1)$, we also denote the $E_6$ gauge fields in $(\mathbf{3}, \mathbf{3}, 3)$ and $(\mathbf{3}, \mathbf{\bar{3}}, 3)$ as $A_\mu = A_\mu^B T^B$ and $A_\mu = A_\mu^B T^B$, respectively, where $T^B$ and $T^B$ are the products of three $3 \times 1$ columns.

Because the extra space manifold has $\Gamma = Z_n$ symmetry, for any $\gamma \in \Gamma$, we have

$$A_\mu^B(\gamma) = (R_\gamma)^{\lambda A} A_\mu^B(\gamma, y_1, y_2, \ldots, y_k) T^B (R_\gamma^{-1})^{\mu A},$$

where $y_i$ for $i = 1, 2, \ldots, k$ are the coordinates for the extra space manifold, and $(l_A, m_A)$ are equal to $(1, 1), (1, 0)$, and $(0, 1)$ for $B = a, \bar{a}, \bar{a}$, respectively. The $E_6$ gauge fields $A_\mu^B$ will have zero modes only if

$$A_\mu^B(\gamma, y_1, y_2, \ldots, y_k) = A_\mu^B(\gamma, y_1, y_2, \ldots, y_k).$$

The zero modes of gauge fields form the group $E_6/R_\Gamma$ with rank 6. From the phenomenological point of view [4], for the zero modes, we require that: (1) $SU(3)_L$ and $SU(3)_R$ cannot be completely unbroken for otherwise their couplings would evolve at low energy to be as strong as that of $SU(3)_C$; (2) to avoid the proton decay, the unbroken subgroups do not contain $SU(5), SU(6)$ and $SO(10)$. The first requirement implies that $\alpha \neq \beta$, and that $\delta, \rho, \sigma$ are not all equal. The second requirement implies that we can have at most one pair of color triplets.

As an example, if $\delta \neq \rho \neq \sigma$, and $\alpha \delta, \alpha \rho, \alpha \sigma, \beta \delta, \beta \rho, \beta \sigma$ are all not equal to one, for the zero modes, the gauge group is $SU(3)_C \times SU(2)_{L_1} \times U(1)_3$. And for $Z_2$, if we chose $R = (+1, +1, +1) \otimes (+1, +1, +1) \otimes (-1, -1, -1)$ or $R = (+1, +1, +1) \otimes (-1, -1, -1) \otimes (+1, +1, +1)$, we break $E_6$ down to $SU(3)_C \times SU(3)_L \times SU(3)_R$.

3. $E_6$ breaking on $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2')$

In this section, we will discuss $E_6$ breaking on the space–time $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2')$. We consider the 6-dimensional space–time which can be factorized into the product of the ordinary 4-dimensional Minkowski space–time $M^4$, and the torus $T^2$ which is homeomorphic to $S^1 \times S^1$. The corresponding coordinates for the space–time are $x^\mu$,
The corresponding operators for the $Z_2$ symmetries $y \sim -y$, $z \sim -z$, $y' \sim -y'$ and $z' \sim -z'$ are $P^y$, $P^z$, $P^{y'}$ and $P^{z'}$, respectively. Allowing a little abuse of notation, we also denote the matrix representations of the $\Sigma$ chiral multiplets as $P^y$, $P^z$, $P^{y'}$ and $P^{z'}$, as used in the literature [1–3].

Let us explain the 6-dimensional gauge theory with $N = 2$ supersymmetry. $N = 2$ supersymmetric theory in 6 dimension has 16 real supercharges, corresponding to $N = 4$ supersymmetry in 4 dimension. Therefore, only the vector multiplet can be introduced in the bulk, and the Standard Model fermions are confined on the 4-branes, 3-branes or 4-brane intersections. In terms of the 4-dimensional $N = 1$ supersymmetry language, the theory contains a vector multiplet $V(A_\mu, \lambda_1)$ in which $\lambda_1$ is the gaugino, and three chiral multiplets $\Sigma_5, \Sigma_6, \Phi$. All of them are in the adjoint representation of the gauge group. In addition, the $\Sigma_5$ and $\Sigma_6$ chiral multiplets contain the gauge fields $A_5$ and $A_6$ in their lowest components, respectively.

In the Wess–Zumino gauge and 4-dimensional $N = 1$ supersymmetry language, the bulk action is [7]

\[ S = \int d^8x \left\{ \frac{1}{4g^2} \text{Tr} \left[ \int d^2\theta \left( \frac{1}{2} W^\ell \{ \Sigma_5, \Sigma_6 \} \right) \right] + \frac{1}{k_g^2} \left( \Phi \partial_5 \Sigma_6 - \Phi \partial_6 \Sigma_5 - \frac{1}{\sqrt{2}} \Phi [\Sigma_5, \Sigma_6] \right) \right\} + \text{h.c.} \\
+ \int d^2\theta \frac{1}{k_g^2} \text{Tr} \left[ \sum_{i=5}^{6} \left( \frac{1}{2} \partial_i + \Sigma_i \right) e^{-V} \times \left( -\sqrt{2} \partial_i + \Sigma_i \right) e^{V} + \partial_i e^{-V} \partial_i e^{V} \right] + \Phi^i e^{-V} \Phi e^{V} \right\} \right\}. \] (8)

And the gauge transformation is given by

\[ e^V \rightarrow e^{A_1} e^V e^{A_1^\dagger}, \]
\[ \Sigma_i \rightarrow e^{A_1} (\Sigma_i - \sqrt{2} \partial_i) e^{-A_1}, \]
\[ \Phi \rightarrow e^{A_1} \Phi e^{-A_1}, \]

where $i = 5, 6$.

From the action, we obtain the transformations of vector multiplet under the $Z_2$ operators $P^y, P^z$

\[ V(x^\mu, -y, z) = (P^y)^{i\mu} V(x^\mu, y, z)((P^y)^{-1})^{m\nu}, \]
\[ \Sigma_5(x^\mu, -y, z) = -(P^y)^{i\xi} \Sigma_5(x^\mu, y, z)((P^y)^{-1})^{m\xi}, \]
\[ \Sigma_6(x^\mu, -y, z) = (P^y)^{i\xi} \Sigma_6(x^\mu, y, z)((P^y)^{-1})^{m\xi}, \]
\[ \Phi(x^\mu, -y, z) = -(P^y)^{i\phi} \Phi(x^\mu, y, z)((P^y)^{-1})^{m\phi}, \]
\[ V(x^\mu, y, -z) = (P^z)^{i\mu} V(x^\mu, y, z)((P^z)^{-1})^{m\nu}, \]
\[ \Sigma_5(x^\mu, y, -z) = -(P^z)^{i\xi} \Sigma_5(x^\mu, y, z)((P^z)^{-1})^{m\xi}, \]
\[ \Sigma_6(x^\mu, y, -z) = (P^z)^{i\xi} \Sigma_6(x^\mu, y, z)((P^z)^{-1})^{m\xi}, \]
\[ \Phi(x^\mu, y, -z) = -(P^z)^{i\phi} \Phi(x^\mu, y, z)((P^z)^{-1})^{m\phi}, \]

where $(l_V, m_V), (l_{\Sigma_5}, m_{\Sigma_5}), (l_{\Sigma_6}, m_{\Sigma_6})$, and $(l_{\Phi}, m_{\Phi})$ are equal to $(1, 1)$ if the gauge fields were in the representations $(8, 1, 1), (1, 8, 1), (1, 1, 8)$, and $(l_V, m_V), (l_{\Sigma_5}, m_{\Sigma_5}), (l_{\Sigma_6}, m_{\Sigma_6})$, and $(l_{\Phi}, m_{\Phi})$ are equal to $(0, 0)$ if the gauge fields were in the representation $(1, 1, 8)$. Moreover, the transformations of vector multiplet under the $Z_2$ operators $P^y$ and $P^z$ are similar.

In the following models, for the zero modes, we will break the 4-dimensional $N = 4$ supersymmetry down to $N = 1$ supersymmetry, and break the $E_6$ gauge group down to $E_6/(P^y \cup P^z \cup P^{y'} \cup P^{z'})$. Including the KK modes, the intersection 3-branes and...
boundary 4-branes preserve the 4-dimensional $N = 1$ and $N = 2$ supersymmetry, respectively. The general 4-dimensional supersymmetry and gauge groups on the intersection 3-branes and boundary 4-branes are given in Table 1. The KK mode expansions and the detail of this set-up can be found in Ref. [2].

### 3.1. Models without the zero modes of $\Sigma_5$, $\Sigma_6$, and $\Phi$

We will first discuss the models without the zero modes of $\Sigma_5$, $\Sigma_6$, and $\Phi$. In order to project out all the zero modes of $\Sigma_5$, $\Sigma_6$, and $\Phi$, we choose the matrix representations of $P^\gamma$ and $P^x$ as product of three $3 \times 3$ unit matrices

$$P^\gamma = P^x = (+1, +1, +1) \otimes (1, +1, +1)$$

$$\otimes (+1, +1, +1). \quad (20)$$

So, considering the zero modes, under $P^\gamma$ projection, we can break the 4-dimensional $N = 4$ supersymmetry down to the $N = 2$ supersymmetry with $(V, \Sigma_6)$ forming a vector multiplet and $(\Sigma_5, \Phi)$ forming a hypermultiplet, and we can break the 4-dimensional $N = 2$ supersymmetry down to the $N = 1$ supersymmetry further by the $P^x$ projection.

We define 5 matrices which will be used in the following discussions

$$A = (+1, +1, +1) \otimes (-1, -1, +1)$$

$$\otimes (-1, -1, +1), \quad (21)$$

$$B = (+1, +1, +1) \otimes (+1, +1, +1)$$

$$\otimes (-1, -1, +1), \quad (22)$$

$$C = (+1, +1, +1) \otimes (-1, -1, +1)$$

$$\otimes (+1, +1, +1), \quad (23)$$

$$D = (+1, +1, +1) \otimes (+1, +1, +1)$$

$$\otimes (-1, -1, -1), \quad (24)$$

$$E = (+1, +1, +1) \otimes (-1, -1, -1)$$

$$\otimes (+1, +1, +1). \quad (25)$$

Because $A, B, C, D, E$ are order 2 elements and the unit element (or identity) $e$ commutes with all the elements in the group, we define $E_6/A \equiv E_6/\{e, A\}$ for simplicity and similarly for the others. As an example, we will explain how to obtain $E_6/A$. As we know, $E_6$ has three maximal subgroups with rank 6: $SO(10) \times U(1)$, $SU(6) \times SU(2)$ and $SU(3) \times SU(3) \times SU(3)$ $\approx SU(3)$ $\times SU(3)$ $\times SU(3)$ $\times SU(3)$ $\times SU(3)$ $\times SU(3)$. Because $A$ is an order 2 subgroup of $SU(3) \times SU(3) \times SU(3)$, $E_6/A$ forms a maximal subgroup and must be one of the three $E_6$ maximal subgroups with rank 6. $E_6/A$ has 46 gauge generators by simple counting, therefore, we obtain that $E_6/A$ is $SO(10) \times U(1)$. Similarly, we can calculate the other commutants. In short, we have

$$E_6/A \approx SO(10) \times U(1), \quad (26)$$

$$E_6/B \approx E_6/C \approx SU(6) \times SU(2), \quad (27)$$

$$E_6/D \approx E_6/E \approx SU(3) \times SU(3) \times SU(3), \quad (28)$$

$$E_6/[A \cup B] \approx E_6/[A \cup C] \approx E_6/[B \cup C] \approx SU(4) \times SU(2) \times SU(2) \times SU(2) \times U(1), \quad (29)$$

$$E_6/[A \cup D] \approx E_6/[A \cup E] \approx SU(3) \times SU(2) \times SU(2) \times U(1)^2. \quad (30)$$

### Model I

We choose the matrix representations for $P^{\gamma'}$ and $P^{x'}$ as

$$P^{\gamma'} = A, \quad P^{x'} = B. \quad (31)$$

For the zero modes, the bulk 4-dimensional $N = 4$ supersymmetric $E_6$ model is broken down to the $N = 1$ supersymmetric $SU(4) \times SU(2) \times SU(2) \times U(1)$ model. Including the KK modes, the gauge groups on the intersection 3-branes at $(y = 0, z = 0)$, $(y = 0, z = \pi R_2/2)$, $(y = \pi R_1/2, z = 0)$ and $(y = \pi R_1/2, z = \pi R_2/2)$ are $E_6$, $SU(6) \times SU(2)$, $SO(10) \times U(1)$ and $SU(4) \times SU(2) \times SU(2) \times SU(2)$, respectively. And the gauge groups on the 4-branes at $y = 0, z = 0, y = \pi R_1/2$ and $z = \pi R_2/2$ are $E_6, E_6, E_6, E_6$. 

---

Table 1

<table>
<thead>
<tr>
<th>Brane position</th>
<th>SUSY</th>
<th>Gauge symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 0)$</td>
<td>$N = 1$</td>
<td>$G/(P^x \cup P^z) / D$</td>
</tr>
<tr>
<td>$(0, \pi R_2/2)$</td>
<td>$N = 1$</td>
<td>$G/(P^x \cup P^z)$</td>
</tr>
<tr>
<td>$(\pi R_1/2, 0)$</td>
<td>$N = 1$</td>
<td>$G/(P^y \cup P^z) / A$</td>
</tr>
<tr>
<td>$(\pi R_1/2, \pi R_2/2)$</td>
<td>$N = 1$</td>
<td>$G/(P^y \cup P^z)$</td>
</tr>
<tr>
<td>$y = 0$</td>
<td>$N = 2$</td>
<td>$G/P^z$</td>
</tr>
<tr>
<td>$z = 0$</td>
<td>$N = 2$</td>
<td>$G/P^z$</td>
</tr>
<tr>
<td>$y = \pi R_1/2$</td>
<td>$N = 2$</td>
<td>$G/P^x$</td>
</tr>
<tr>
<td>$z = \pi R_2/2$</td>
<td>$N = 2$</td>
<td>$G/P^x$</td>
</tr>
</tbody>
</table>
\[ SO(10) \times U(1) \text{ and } SU(6) \times SU(2) \text{, respectively. Similarly, one can discuss the model by choosing } P^\gamma = A \text{ and } P^{\bar{c}} = C. \]

**Model II.** We choose the matrix representations for \( P^\gamma \) and \( P^{\bar{c}} \) as
\[ P^\gamma = B, \quad P^{\bar{c}} = C. \quad (32) \]

For the zero modes, the bulk 4-dimensional \( N = 4 \) supersymmetric \( E_6 \) model is broken down to the \( N = 1 \) supersymmetric \( SU(4) \times SU(2) \times SU(2) \times U(1) \) model. Including the KK modes, the gauge groups on the intersection 3-branes at \( (y = 0, z = 0) \), \( (y = 0, z = \pi R_2/2) \), \( (y = \pi R_1/2, z = 0) \) and \( (y = \pi R_1/2, z = \pi R_2/2) \) are \( E_6, SU(6) \times SU(2), SU(6) \times SU(2) \) and \( SU(4) \times SU(2) \times SU(2) \times U(1) \), respectively. And the gauge groups on the 4-branes at \( y = 0, z = 0 \), \( y = \pi R_1/2 \) and \( z = \pi R_2/2 \) are \( E_6, SU(6) \times SU(2) \) and \( SU(6) \times SU(2) \), respectively.

**Model III.** We choose the matrix representations for \( P^\gamma \) and \( P^{\bar{c}} \) as
\[ P^\gamma = A, \quad P^{\bar{c}} = D. \quad (33) \]

For the zero modes, the bulk 4-dimensional \( N = 4 \) supersymmetric \( E_6 \) model is broken down to the \( N = 1 \) supersymmetric \( SU(3) \times SU(2) \times SU(2) \times U(1)^2 \) model. Including the KK modes, the gauge groups on the intersection 3-branes at \( (y = 0, z = 0) \), \( (y = 0, z = \pi R_2/2) \), \( (y = \pi R_1/2, z = 0) \) and \( (y = \pi R_1/2, z = \pi R_2/2) \) are \( E_6, SU(3) \times SU(3) \times SU(3), SO(10) \times U(1) \) and \( SU(3) \times SU(2) \times SU(2) \times U(1)^2 \), respectively. And the gauge groups on the 4-branes at \( y = 0, z = 0 \), \( y = \pi R_1/2 \) and \( z = \pi R_2/2 \) are \( E_6, SO(10) \times U(1) \) and \( SU(3) \times SU(3) \times SU(3) \), respectively. Similarly, one can discuss the model by choosing \( P^\gamma = A \) and \( P^{\bar{c}} = E \).

3.2. Model with gauge-Higgs unification

In this subsection, we will present the model with \( SU(3) \times SU(2) \times U(1)^3 \) gauge symmetry and one pair of \( SU(2)_L \) Higgs doublets from \( \Phi \). We choose the matrix representations for \( P^\gamma, P^\bar{c}, P^\gamma \) and \( P^{\bar{c}} \) as
\[ P^\gamma = P^{\bar{c}} = A = (+1, +1, +1) \otimes (-1, -1, +1) \]
\[ \otimes (-1, -1, +1), \quad (34) \]
\[ P^\gamma = (+1, +1, +1) \otimes (-1, -1, +1) \]
\[ \otimes (-1, -1, +1), \quad (35) \]
\[ P^{\bar{c}} = (+1, +1, +1) \otimes (-1, -1, +1) \]
\[ \otimes (+1, -1, -1). \quad (36) \]

And we would like to point out the commutant groups
\[ E_6 / A \approx E_6 / P^\gamma \approx E_6 / P^{\bar{c}} \approx SO(10) \times U(1). \quad (37) \]
\[ E_6 / [ A \cup P^\gamma ] \approx E_6 / [ A \cup P^{\bar{c}} ] \]
\[ \approx SU(5) \times U(1)^2, \quad (38) \]
\[ E_6 / [ A \cup P^\gamma ] \cup P^{\bar{c}} ] \approx SU(3) \times SU(2) \times U(1)^3. \quad (39) \]

We project out all the zero modes of \( \Sigma_3 \) and \( \Sigma_6 \) by choosing \( P^\gamma = P^\bar{c} \). And we project out all the zero modes of \( \Phi \) except one pair of \( SU(2)_L \) doublets, which can be viewed as one pair of Higgs doublets. Considering the zero modes, the bulk 4-dimensional \( N = 4 \) supersymmetric \( E_6 \) model is broken down to the \( N = 1 \) supersymmetric \( SU(3) \times SU(2) \times U(1)^3 \) model. Including the KK modes, the gauge groups on the intersection 3-branes at \( (y = 0, z = 0) \), \( (y = 0, z = \pi R_2/2) \), \( (y = \pi R_1/2, z = 0) \) and \( (y = \pi R_1/2, z = \pi R_2/2) \) are \( SO(10) \times U(1), SU(5) \times U(1)^2, SU(5) \times U(1)^2 \), and \( SU(5) \times U(1)^2 \), respectively. In addition, the gauge groups on the 4-branes at \( y = 0, z = 0 \), \( y = \pi R_1/2 \) and \( z = \pi R_2/2 \) are all \( SO(10) \times U(1) \).

4. \( E_6 \) breaking on \( M^4 \times A^2 \) and \( M^4 \times D^2 \)

In this section, we would like to discuss \( E_6 \) breaking on the space–time \( M^4 \times A^2 \) and \( M^4 \times D^2 \), where \( A^2 \) and \( D^2 \) are the two-dimensional annulus and disc, respectively. And we only show the models with \( SU(3) \times SU(2) \times U(1)^3 \) gauge symmetry and 4-dimensional \( N = 1 \) supersymmetry for the zero modes. Similarly, one can discuss the models with gauge groups \( SU(3) \times SU(2) \times SU(2) \times U(1)^2 \), or \( SU(4) \times SU(2) \times U(1)^2 \), or \( SU(4) \times SU(2) \times SU(2) \times U(1) \) and 4-dimensional \( N = 1 \) supersymmetry for the zero modes.

The convenient coordinates for the annulus \( A^2 \) is polar coordinates \( (r, \theta) \), and it is easy to change them to the complex coordinates by \( z = re^{i \theta} \). We call the inner radius of the annulus as \( R_1 \), and the outer radius of the annulus as \( R_2 \). When \( R_1 = 0 \), the annulus
becomes the disc $D^2$, which is a special case of $A^2$. We can define the $Z_n$ symmetry on the annulus $A^2$ by the equivalent class

$$z \sim \omega z,$$

(40)

where $\omega = e^{2\pi i/n}$. And we denote the corresponding generator for $Z_n$ as $\Omega$ which satisfies $\Omega^n = 1$. The KK mode expansions and the detail of this set-up can be found in Ref. [3].

The $N = 2$ supersymmetry in 6 dimension corresponds to the $N = 4$ supersymmetry in 4 dimension, thus, only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under the 4-dimensional $N = 1$ supersymmetry into a vector multiplet $V$ and three chiral multiplets $\Sigma, \Phi$, and $\Phi^c$ in the adjoint representation, with the fifth and sixth components of the gauge field, $A_5$ and $A_6$, contained in the lowest component of $\Sigma$. The Standard Model fermions are on the boundary 4-brane at $r = R_1$ or $r = R_2$ for the annulus $A^2$ scenario, and on the 3-brane at origin or on the boundary 4-brane at $r = R_2$ for the disc $D^2$ scenario.

In the Wess–Zumino gauge and 4-dimensional $N = 1$ supersymmetry language, the bulk action is [7]

$$S = \int d^6x \left[ \int d^2\theta \left( \frac{1}{4k^2} M^\alpha N_{\alpha} \right) + \frac{1}{k^2} \left( \Phi^c \partial \Phi - \frac{1}{2} \Sigma \left[ \Phi, \Phi^c \right] \right) + \text{h.c.} \right]$$

$$+ \int d^2\theta \frac{1}{k^2} \text{Tr} \left[ (\sqrt{2} \partial + \Sigma^I) e^{-V} \right.$$  

$$\times \left( - (\sqrt{2} \partial + \Sigma^I) e^{V} \right)$$

$$+ \int d^2\theta \frac{1}{k^2} \text{Tr} \left[ + \Phi^I e^{-V} \Phi^c e^{V} \right.$$  

$$\left. + \Phi^c e^{-V} \Phi^I e^{V} \right].$$  

(41)

From above action, we obtain the transformations of gauge multiplet under $\Omega$ as

$$V (\omega z, \omega^n z) = (R_\Omega)^{\nu \nu} V(z, \bar{z})(R_\Omega^{-1})^{\nu \nu},$$

(42)

$$\Sigma(\omega z, \omega^n z) = \omega^{-1} (R_\Omega)^{\nu \nu} \Sigma(z, \bar{z})(R_\Omega^{-1})^{\nu \nu},$$

(43)

$$\Phi(\omega z, \omega^n z) = \omega^{-1} (R_\Omega)^{\nu \nu} \Phi(z, \bar{z})(R_\Omega^{-1})^{\nu \nu},$$

(44)

$$\Phi^c(\omega z, \omega^n z) = \omega^{2} (R_\Omega)^{\nu \nu} \Phi^c(z, \bar{z})(R_\Omega^{-1})^{\nu \nu},$$

(45)

where $(l_V, m_V)$, $(l_\Sigma, m_\Sigma)$, $(l_\Phi, m_\Phi)$, and $(l_{\Phi^c}, m_{\Phi^c})$ are equal to $(1, 1)$ if the gauge fields were in the representations $(8, 1, 1)$, $(1, 8, 1)$, $(1, 1, 8)$, and $(l_V, m_V)$, $(l_\Sigma, m_\Sigma)$, $(l_\Phi, m_\Phi)$, and $(l_{\Phi^c}, m_{\Phi^c})$ are equal to $(1, 0)$ if the gauge fields were in the representation $(3, 3, 3)$, and $(l_V, m_V)$, $(l_\Sigma, m_\Sigma)$, $(l_\Phi, m_\Phi)$, and $(l_{\Phi^c}, m_{\Phi^c})$ are equal to $(0, 1)$ if the gauge fields were in the representation $(3, 3, 3)$.

Moreover, we choose the following matrix representation for $R_\Omega$

$$R_\Omega = (+1, +1, +1) \otimes (\omega^{n_1}, \omega^{n_2}, \omega^{n_3})$$

$$\otimes (\omega^{n_4}, \omega^{n_5}).$$

(46)

In order to have the models with $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry and 4-dimensional $N = 1$ supersymmetry for the zero modes, we obtain the following constraints on $n_i$

$$(a) \quad 2n_1 + n_2 = 0 \text{ mod } n,$$

(47)

$$(b) \quad n_3 + n_4 + n_5 = 0 \text{ mod } n,$$

(48)

$$(c) \quad n_1 \neq n_2 \text{ mod } n,$$

(49)

$$(d) \quad n_3 \neq n_4 \neq n_5 \text{ mod } n,$$

(50)

$$(e) \quad |n_1 - n_2| \neq 1 \quad \text{and} \quad n - 2 \text{ mod } n,$$

(51)

$$(f) \quad |n_i - n_j| \neq 1 \quad \text{and} \quad n - 2 \text{ mod } n,$$

for $i, j = 3, 4, 5$ and $i \neq j$.

(52)

$$(g) \quad |n_i + n_j| \neq 0, 1 \quad \text{and} \quad n - 2 \text{ mod } n,$$

for $i, j = 1, 2$ and $j = 3, 4, 5$.

(53)

Because $R_\Omega \subset SU(3)_L \times SU(2)_L \times SU(3)_R$, we obtain the constraints (a) and (b). And the constraints (c) and (d) will break the $SU(3)_L$ down to $SU(2)_L \times U(1)$ and $SU(3)_R$ down to $U(1)^2$, respectively. In addition, the constraints (e) and (f) will project out all the zero modes of $\Sigma, \Phi$ and $\Phi^c$ in the representations $(8, 1, 1)$, $(1, 8, 1)$, $(1, 1, 8)$, and the constraint (g) will project out all the zero modes of $V, \Sigma, \Phi$ and $\Phi^c$ in the representations $(3, 3, 3)$ and $(3, 3, 3)$.

Let us give the simplest model with $Z_9$ symmetry, the matrix representation for $R_\Omega$ is

$$R_\Omega = (+1, +1, +1) \otimes (\omega^2, \omega^2, \omega^5)$$

$$\otimes (+1, \omega^2, \omega^5).$$

(54)

It is easy to check that all the constraints are satisfied.

First, we consider that the extra space manifold is the annulus $A^2$. For the zero modes, we have
4-dimensional $N=1$ supersymmetry and $SU(3) \times SU(3) \times SU(2) \times U(1)^3$ gauge symmetry in the bulk and on the 4-branes at $r = R_1$ and $r = R_2$. Including the KK states, we will have the 4-dimensional $N=4$ supersymmetry and $E_6$ gauge symmetry in the bulk, and on the 4-branes at $r = R_1$ and $r = R_2$.

Second, we consider that the extra space manifold is the disc $D^2$. For the zero modes, we have 4-dimensional $N=1$ supersymmetry and $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry in the bulk and on the 4-brane at $r = R_2$. Including all the KK states, we will have the 4-dimensional $N=4$ supersymmetry and $E_6$ gauge symmetry in the bulk, and on the 4-brane at $r = R_2$. In addition, because the origin $(r = 0)$ is the fixed point under the $Z_9$ symmetry, we always have the 4-dimensional $N=1$ supersymmetry and $SU(3) \times SU(2) \times U(1)^3$ gauge symmetry on the 3-brane at origin in which only the zero modes exist. And if we put the Standard Model fermions on the 3-brane at origin, the extra dimensions can be large and the gauge hierarchy problem can be solved for there does not exist the proton decay problem at all.

5. Discussion and conclusion

The extra gauge symmetry must be broken around or above the TeV scale. This can be done by Higgs mechanism, for example, the $SU(4)$ can be broken down to $SU(3)_C$ and $SU(2)_R$ can be broken by introducing the Higgs fields in their fundamental representations, and the extra $U(1)$ can be broken by introducing the Standard Model singlets which are charged under the extra $U(1)$. The right-handed neutrinos might also become massive during the extra gauge symmetry breaking. And with the ansatz that there exist discrete symmetries in the neighborhoods of the branes, one can discuss the general $E_6$ breaking on the space–time $M^4 \times M^1 \times M^1$, where the extra dimensions can be large and the KK states can be set arbitrarily heavy [3]. On the outlook, the gauge coupling unification, supersymmetry breaking, the $\mu$ problem, how to forbid the proton decay operators by $R$ symmetry, and how to explain the fermion mass hierarchy and mixing angles in our models deserve further study.

In short, we have studied the $N=2$ supersymmetric $E_6$ models on the 6-dimensional space–time where the supersymmetry and gauge symmetry can be broken by the discrete symmetry. On the space–time $M^4 \times S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2') \times S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2'')$, for the zero modes, we obtain the 4-dimensional $N=1$ supersymmetric models with gauge groups $SU(3) \times SU(2) \times SU(2) \times U(1)^2$, $SU(4) \times SU(2) \times SU(2) \times U(1)$, and $SU(3) \times SU(2) \times U(1)^3$ with one extra pair of Higgs doublets from the vector multiplet. In addition, considering that the extra space manifold is the annulus $A^2$ and disc $D^2$, we list all the constraints on constructing the 4-dimensional $N=1$ supersymmetric $SU(3) \times SU(2) \times U(1)^3$ models for the zero modes, and give the simplest model with $Z_9$ symmetry.

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Intersoliton forces in the Wess–Zumino model

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Abstract

The spectrum of supersymmetric domain wall solitons of the Wess–Zumino model is known to be discontinuous across a curve (of marginal stability) in the moduli space of quartic superpotentials. Here we show how this phenomenon can be understood from the behaviour of the long-range inter-soliton force, which we compute by a method due to Manton.

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1. Introduction

It is often stated that solitons of supersymmetric field theories that preserve the same fraction of supersymmetry will exert no force on each other. When true, this implies the existence of static supersymmetric multi-soliton solutions that can be interpreted as individual solitons in static marginal equilibrium due to a cancellation of attractive and repulsive forces. However, static supersymmetric multi-soliton solutions may not exist. This is typically the case for domain walls; i.e., solitons of (1 + 1)-dimensional field theories. Although it is possible to find (1 + 1)-dimensional models that admit multi-soliton solutions of the type described above [1], these seem to be the exception rather than the rule. In general, multi-soliton solutions of (1 + 1)-dimensional field theories are time-dependent, and this implies the existence of a force between the constituent solitons.

Consider the case of a field theory for a real scalar field \(\phi\) with a potential having three isolated degenerate minima, at \(\phi_a < \phi_b < \phi_c\). There exist static soliton solutions interpolating between the adjacent minima, and these are supersymmetric solutions of the supersymmetric version of this model, but there is no supersymmetric soliton interpolating between the non-adjacent minima. This is because all supersymmetric solutions correspond to flows of a first-order equation and there is no flow connecting \(\phi_a\) with \(\phi_c\). There must be some solutions that interpolate between the non-adjacent minima, but they are necessarily time-dependent. These solutions represent an \(ab\)-soliton followed by a \(bc\)-soliton moving under the influence of the force between them, at least for large separation. The leading-order term in an asymptotic expansion of the long-range force must be repulsive because an attractive force would imply the existence of a bound state of an \(ab\)-soliton with a \(bc\)-soliton and hence the existence of an \(ac\)-soliton but, as we have just explained, there is no such soliton. We shall confirm the repulsive nature of the long-range force, as a special case of a more general result, by adapting a method...
introduced by Manton [2] to compute the long-range attractive force between a soliton and its anti-soliton.

The situation for multi-component scalar field theories is much more complicated. Here we concentrate on domain walls of the bosonic sector of the Wess–Zumino model for a single complex scalar superfield. On reduction to 1 + 1 dimensions this becomes a model for a single complex scalar field \( z(t, s) \) with Lagrangian density

\[
\mathcal{L} = \frac{1}{2} [ |\dot{z}|^2 - |z|^2 - |\partial W(z)|^2 ] ,
\]

where the ‘superpotential’ \( W(z) \) is a holomorphic function, and

\[
\dot{z} \equiv \frac{\partial z}{\partial t}, \quad z' \equiv \frac{\partial z}{\partial s}, \quad \partial W = \frac{\partial W}{\partial z} .
\]

Critical points of \( W \) are degenerate global minima of the potential and solitons are minimal energy configurations that interpolate between them. In the supersymmetric context, the critical points of \( W \) are supersymmetric vacua and the solitons interpolating between them preserve 1/2 of the supersymmetry [3–5].

Polynomials provide a simple class of superpotentials. A polynomial superpotential of order \( n \) has \( n - 1 \) critical points, and hence \( n - 1 \) vacua, so in order for such a model to admit multi-soliton configurations we need \( n \geq 4 \). Here we shall concentrate on the simplest case of a quartic superpotential, which may (without loss of generality) be put in the form

\[
W(z) = z^4 - \frac{4}{3} \mu z^3 - 2z^2 + 4\mu z ,
\]

where \( \mu \) is a complex constant that parametrizes the space of physically-distinct quartic superpotentials [4]. Provided that \( \mu \neq \pm 1 \), there are then three (degenerate) supersymmetric vacua, at

\[
(1): \quad z = -1 , \quad (2): \quad z = 1 ,
\]

\[
(3): \quad z = \mu ,
\]

and hence, potentially, three soliton solutions interpolating between them. However, whether all three types of soliton actually exist depends on the value of \( \mu \). For example, if \( \mu = 0 \) then soliton solutions \( z(t) \) are real (because all three vacua lie on the real axis in the \( z \)-plane) and can connect only adjacent vacua; there are therefore only two types of soliton. On the other hand, the choice

\[
\mu = i \sqrt{3} \quad (5)
\]

yields a \( Z_3 \)-symmetric model for which the existence of all three solitons is guaranteed by symmetry given any one of them.\(^1\)

The \( \mu \)-dependence of the soliton spectrum of the WZ model with superpotential (3) was analysed in [4], where it was shown that the ‘2-soliton’ to ‘3-soliton’ frontiers in the \( \mu \)-plane are the two branches of a curve \( \Delta(\mu_1, \mu_2) = 0 \), where we have set \( \mu = \mu_1 + i\mu_2 \) and

\[
\Delta = -6\mu_1^2 - 6\mu_2^2 - \mu_1^4 + 2\mu_1^2\mu_2^2 + 3\mu_1^4 .
\]

This curve is shown in Fig. 1; when it is crossed from inside a ‘3-soliton’ region, one of the three solitons disappears from the spectrum.\(^2\) The curve \( \Delta = 0 \) is therefore a simple (and, apparently, the earliest) example of what is now called a curve of marginal stability. The reason for the discontinuity in the soliton spectrum across this curve was explained in [4]: consider what happens as we start from the \( Z_3 \)-symmetric case (5) and proceed down the imaginary \( \mu \) axis to the origin. As \( \mu_2 \) decreases, the soliton trajectory in the \( z \)-plane that starts at vacuum 1 (\( z = -1 \)) and ends at vacuum 2 (\( z = 1 \)) passes increasingly close to vacuum 3 (\( z = \mu \)). In other words, the 12-soliton (con-

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1 This case yields an integrable model in 1 + 1 dimensions [3].
2 This phenomenon was rediscovered by other methods in [6], where we call a WZ model is called a ‘Landau–Ginzburg’ model.
nnecting vacua 1 and 2) looks increasingly like a loose bound state of the 13-soliton and 32-soliton. This suggests the following picture: near the curve of marginal stability, one of the three solitons can be viewed as a bound state of the other two in which the constituents are held at a distance that goes to infinity on the curve of marginal stability, thus causing the bound state soliton to disappear from the spectrum.

The main aim of this Letter is to confirm this picture for the WZ model described above by a determination of the asymptotic intersoliton force using Manton’s method [2]. For real $\mu$ all solitons are real and hence solutions of a truncated theory involving only a single real scalar field. As explained above, the two solitons of this theory must repel each other, at least asymptotically. As we move the $z = \mu$ vacuum away from the real axis towards the curve of marginal stability we find that this repulsive asymptotic force goes to zero, changing sign as we cross the curve. So the leading-order force is attractive on the ‘3-soliton’ side of the curve, as we might expect. We shall argue, albeit less directly, that the next-to-leading order force is always repulsive. This implies the existence of a bound state near the curve of marginal stability (on the ‘3-soliton’ side) with a separation of the constituent solitons that diverges on the curve. This bound state therefore disappears from the spectrum as the curve is crossed, in agreement with the results of [4].

A similar result was obtained recently, by different methods, for a different scalar field theory [7]. We should also note that similar results have been obtained, again by different methods, for a different scalar field theory [7].

We shall begin with a brief review of the solitons of the WZ model with quartic superpotential. Although they are not known explicitly we will show that the soliton trajectories in field space can be found exactly; this allows some qualitative results of [4] to be made quantitative. Next, we show how Manton’s computation can be generalized to yield the asymptotic long-range force between solitons of multi-component scalar field theories. We then apply this result to the solitons of the WZ model with quartic superpotential. In particular, we show that the leading-order force vanishes on the curve of marginal stability. We conclude with a discussion of the next-to-leading order and the implications for soliton bound states.

2. WZ solitons for quartic superpotential

Let $z_i \ (i = 1, 2, 3)$ be the three critical points of the quartic superpotential (3). Given the existence of a soliton connecting vacua $i$ and $j$, its topological charge is

$$T_{ij} = 2\left[W(z_j) - W(z_i)\right].$$

These three charges form the three sides of a triangle in the complex $W$-plane. The triangle inequality

$$|T_{ij}| + |T_{jk}| > |T_{ik}|$$

ensures that any soliton in the spectrum of supersymmetric states must remain in it as $\mu$ is varied, unless the triangle degenerates. If this happens, the inequality is saturated and the minimum energy configuration with charge $T_{ik}$ need not be a one-soliton configuration. In fact, since solitons correspond to straight lines in the $W$-plane only two of the three possible solitons can exist whenever the three points $W(z_i)$ in the $W$-plane are colinear; this occurs when $\mu$ is real and $\mu$ lies on the curve $\Delta = 0$, with $\Delta$ given by (6). When $\mu$ is real, the long-range intersoliton force is repulsive (for reasons explained above) and, by continuity, it remains repulsive for $\mu$ in some neighbourhood of the real axis. Thus, in this region of moduli space only two of the three possible soliton states exist and there is therefore no discontinuity in the soliton spectrum across the real $\mu$ axis. This agrees with the analysis of [4] but it was also shown there, by a study of the qualitative behaviour of soliton trajectories in the $z$-plane, that there is a discontinuity across the curve $\Delta = 0$. We shall now confirm this by finding the exact soliton trajectories.

Supersymmetric solitons solve the first order ‘BPS’ equation

$$\vec{z}' = e^{-i\alpha} \partial W(z),$$

where $\alpha = \arg(T)$. Writing $z(x) = u(x) + iv(x)$, this yields the pair of coupled differential equations

$$u' = f(u, v), \quad v' = g(u, v)$$

where, for the quartic superpotential (3),

$$f(u, v) = 4\cos \alpha \left[u^3 - 3uv^2 - u - \mu_1 (u^2 - v^2 - 1) + 2uv \mu_2\right]$$

3 A recent demonstration of this may be found in [11].
Fig. 2. Plot showing the soliton $S_{12}$ disappearing from the spectrum as the $z = \mu$ vacuum crosses the curve of marginal stability in the $z$-plane. All plots have $\mu_1 = 0$ and $\mu_2$ is respectively 1, 0.7 and 0.65. The critical value for which $\Delta = 0$ if $\mu_1 = 0$ is $\mu_2 = \pm \sqrt{2/3} - 3 \approx \pm 0.681$. 

\[-4 \sin \alpha \left[ v^3 - 3u^2v + v + 2uv\mu_1 + (u^2 - v^2 - 1)\mu_2 \right],\]

\[g(u, v) = 4 \cos \alpha \left[ v^3 - 3u^2v + v + 2uv\mu_1 + (u^2 - v^2 - 1)\mu_2 \right] + 4 \sin \alpha \left[ u^3 - 3uv^2 - u - (v^2 - u^2 - 1)\mu_1 + 2uv\mu_2 \right].\]

These equations determine $u(x)$ and $v(x)$. When $\mu$ is not real we cannot find explicit solutions, but we can find the soliton trajectories in the $z$-plane. To this end, we observe that (10) implies that $\frac{dv}{du} - g(\mu) = 0$. However,

\[\frac{df}{dv} - g(\mu) = dh,\]  

where

\[h(u, v) = 4 \cos \alpha \left[ (u^3 - uv^2 - u)v - \mu_1 v(u^2 - \frac{1}{4}v^2 - 1) + \mu_2 u(v^2 - \frac{1}{4}u^2 + 1) \right] - 4 \sin \alpha \left[ 2v^2 - 2u^2 + v^4 + u^4 - 6u^2v^2 + 4\mu_1 u(v^2 - \frac{1}{4}u^2 + 1) + 4\mu_2 v(u^2 - \frac{1}{4}v^2 - 1) \right].\]  

Soliton trajectories are therefore curves of constant $h$ in the $z$-plane that pass through two of the critical points. All other curves of constant $h$ are ‘BPS-flows’ in that they solve the first-order equations (10), but not with the boundary conditions needed for finite energy.

Consider the 12-soliton interpolating between the vacua at $z = -1$ and $z = 1$. The curve of constant $h$ that passes through these points has

\[h_{12} = \frac{\mu_2}{|\mu|}, \quad \tan \alpha_{12} = \frac{\mu_2}{\mu_1}.\]  

The latter relation implies that $\arg T = \text{actan}(\mu_2/\mu_1)$, which is consistent with the fact that $T \propto \mu$ for the 12-soliton. The 12-soliton trajectory is therefore

\[0 = \mu_2 \left[ (1 - u^2)^2 + v^4 - 6u^2v^2 \right] - 4\mu_1 uv(u^2 - v^2 - 1) + 4|\mu|^2 v(u^2 - \frac{1}{4}v^2 - 1).\]  

In the limit as $\mu \to \infty$ this trajectory coincides with the real axis, but for finite $\mu$ it is energetically favourable for the 12-soliton to partially roll down the potential towards this third vacuum. A 12-soliton trajectory that passes very close to $z = \mu$ must either end or begin there, so we then have an infinitely separated 13 and 32 soliton but no 12 soliton. Demanding that (15) pass through $z = \mu$ yields precisely the same condition as found in [4] by requiring colinear topological charges; that is, either $\mu$ is real or it lies on the curve $\Delta = 0$. (See Fig 2.)
The other two soliton trajectories can be found similarly. In each case the soliton trajectory coincides with the union of the other two when $\mu_2 \Delta = 0$, although it disappears from the spectrum only on crossing the curve $\Delta = 0$. It should be noted that the relevant segments of the curve $\Delta = 0$ differ in all three cases. The above case, in which the 12-soliton is the one that disappears from the spectrum as we go from a '3-soliton' to a '2-soliton' region, corresponds to the segments with $\mu_1^2 < 1$, whereas the other two cases correspond to segments with $\mu_1^2 > 1$. We skip the details but note for future use that

$$
\tan \alpha_{13} = \frac{\mu_2}{\mu_1 + \Delta_+}, \quad \tan \alpha_{23} = \frac{\mu_2}{\mu_1 + \Delta_-}
$$

where

$$
\Delta_{\pm} = \pm \frac{\Delta}{4[3\mu_1 + \mu_1 \mu_2^2 - \mu_1^3 \pm 2]}
$$

It follows that $\alpha_{13} = \alpha_{23}$ when $\Delta = 0$, as expected.

3. The leading-order long-range force

We now plan to obtain a formula for the asymptotic force between two solitons of a $(1 + 1)$-dimensional field theory for a multi-component real scalar field $\phi(t, s)$ with Lagrangian density

$$
\mathcal{L} = \frac{1}{2} |\phi|^2 - \frac{1}{2} |\phi'|^2 - V.
$$

This of course includes (1) as a special case of a two-component real scalar field Lagrangian density. We assume that $V$ has multiple degenerate vacua at $\phi = \phi_i$ ($i = 1, 2, \ldots$), and that some vacua are connected by static soliton solutions. Let $\phi_{ij}(s)$ be a soliton connecting the $i$th vacuum to the $j$th vacuum. Near the $i$th vacuum we have

$$
V \approx \frac{1}{2} m_i^2 |\phi - \phi_i|^2.
$$

The asymptotic form of the $ij$-soliton solution near the $j$th vacuum (as $s \to -\infty$) is therefore

$$
z_{ij}(s) = z_j - m_j^{-1} t_{ij} e^{-m_j s},
$$

where $t_{ij}$ is tangent to the $ij$-soliton trajectory in field space as it approaches the $j$th vacuum. Similarly, the asymptotic form of the $ij$-soliton near the $i$th vacuum (as $s \to -\infty$) is

$$
z_{ij}(s) = z_i - m_i^{-1} t_{ij} e^{-m_i s}.
$$

Now consider a static configuration of an $ik$-soliton at the origin, $s = 0$, separated by a distance $L$ from a $kj$-soliton at $s = L$. A configuration that fits this description is

$$
\phi(s) = \phi_{ik}(s) + \phi_{kj}(s - L) - \phi_k.
$$

We should not expect this to be an exact solution of the field equations but it will be an approximate solution for large $L$ provided that there is a $b$ in the range $0 \ll b \ll L$ such that both individual static soliton solutions $\phi_{ik}(s)$ and $\phi_{kj}(s)$ are nearly equal to their common vacuum value $\phi_k$ near $s = b$. We shall write (22) as

$$
\phi(s) = \phi_{ik}(s) + \psi(s),
$$

where

$$
\psi(s) = \phi_{kj}(s - L) - \phi_k
$$

and assume that $\psi(s)$ is a small perturbation to the static solution $\phi_{ik}(s)$ in the region $s \leq b$. Since, for $s \approx b$,

$$
\phi_{ik}(s) = \phi_k - m_k^{-1} t_{ik} e^{-m_k s},
$$

$$
\psi(s) \sim m_k^{-1} t_{ik} e^{m_k s - L},
$$

this means that we must have $2b \ll L$, so we are now assuming that

$$
0 \ll 2b \ll L.
$$

The force exerted on the $ik$-soliton by the $kj$-soliton can now be found as follows [2]; the momentum of the $ik$-soliton is approximately given by

$$
P = -\int_{-\infty}^{b} \dot{\phi} \cdot \phi' \, ds.
$$

The force on it is therefore

$$
F \equiv \dot{P} = \left[ -\frac{1}{2} |\phi|^2 - \frac{1}{2} |\phi'|^2 + V \right]_{-\infty}^{b},
$$

\footnote{The sign, which agrees with [2], is chosen such that a negative force corresponds to an attractive intersoliton potential.}
which follows on use of the field equation
\[ \ddot{\phi} = \phi'' - \frac{\partial V}{\partial \phi}. \]  
(29)

Using the ansatz (23) and properties of \( \phi_{ik} \), notably
\[ |\phi_{ik}'|^2 = 2V(\phi_{ik}), \]  
(30)
we find that
\[ F = -\left[\phi_{ik}' \cdot \phi'' - \phi_{ik}' \cdot \phi\right]_{-\infty}^{b} + \mathcal{O}(\phi^2). \]  
(31)
This may be evaluated using the asymptotic forms (25) of \( \phi_{ik} \) and \( \phi \). The result is
\[ F = -2t_{ik} e^{-m_i L}. \]  
(32)
This generalizes the result of Manton to multi-component scalar field theories. For a single-component theory the tangent vectors are necessarily either parallel or antiparallel, so the force is either attractive or repulsive, never zero. For example, for a soliton-antisoliton pair we have \( i = j \) and hence the attractive force [2].
\[ F = -2t_{ik} e^{-m_i L}. \]  
(33)

4. Application to the WZ model

The above results are applicable to the WZ model because this has degenerate vacua at critical points \( z = z_i \) of the superpotential \( W \), with
\[ m_i^2 = |W'(z_i)|^2. \]  
(34)
We shall consider again the case of a quartic superpotential with critical points at \( z = -1, 1, \mu \) (with \( \mu = \mu_1 + i \mu_2 \)), and choose
\[ \mu_1^2 < 1, \]  
(35)
so that the 12-soliton is the one expected to appear as a bound state (of a 13-soliton and 32-soliton) near the curve of marginal stability. Consider a field configuration in which a 13-soliton is at \( s = 0 \) and a 32-soliton at \( s = L \), for large \( L \). In the region between these solitons, and far from both, we may linearize the first-order equations (10) about the \( z = \mu \) vacuum to get
\[ \begin{pmatrix} u' \\ v' \end{pmatrix} = M \begin{pmatrix} u - \mu_1 \\ v - \mu_2 \end{pmatrix}, \]  
(36)
where the matrix \( M \) is
\[ M = 4 \begin{pmatrix} \mu_1^2 - \mu_2^2 - 1 & 2\mu_1\mu_2 \\ 2\mu_1\mu_2 & \mu_1^2 - \mu_2^2 - 1 \end{pmatrix} \times \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}. \]  
(37)
This matrix has eigenvalues \( \pm m_3 \), where
\[ m_3 = \sqrt{1 + |\mu|^4}. \]  
(38)
The corresponding eigenvectors are
\[ e_{\pm}(\alpha) = \frac{1}{\sqrt{2(1 + \cos(\theta + \alpha))}} \begin{pmatrix} \sin(\alpha + \theta), -\cos(\alpha + \theta) \pm 1 \end{pmatrix}, \]  
(39)
where
\[ \tan \theta = \frac{2\mu_1\mu_2}{1 + \mu_1^2 - \mu_2^2}. \]  
(40)
These eigenvectors are tangents to the separatrix flows at \( z = \mu \). Note that they are orthogonal.

The generic solution of (36) has both \( e^{-m_3 s} \) and \( e^{m_3 s} \) terms, but the asymptotic 13-soliton solution \( z_{13} \) has only the \( e^{-m_3 s} \) exponential term. Thus, we should take
\[ t_{13} \propto e_{-}(\alpha_{13}). \]  
(41)
Similarly, the asymptotic 32-soliton solution, near the 3-vacuum, has only the \( e^{m_3 s} \) factor, so we should take
\[ t_{32} \propto e_{+}(\alpha_{32}). \]  
(42)
The sign of the constant of proportionality should be the same in both cases, so this yields the force
\[ F \propto -\sin[\frac{1}{2}(\alpha_{13} - \alpha_{32})] \sin[\frac{1}{2}(\alpha_{13} + \alpha_{32}) + \theta] + \sin^2[\frac{1}{2}(\alpha_{13} - \alpha_{32})] \]  
(43)
for some positive constant of proportionality.

Consider first the case for which \( \mu_2 = 0 \), so that both soliton solutions correspond to BPS-flows along the real axis in the \( z \)-plane from \( z = -1 \) to \( z = 1 \) (since \( \mu_1^2 < 1 \) by hypothesis). From (16) we then see that
\[ \alpha_{13} = 0, \pi. \]  
(44)
Similarly for \( \alpha_{32} \) but \( \alpha_{32} = \alpha_{13} + \pi \). To see why, note that a BPS flow just above the real axis will approach the vacuum at \( z = \mu \) anti-parallel to the real axis (\( e_+ \)) and then leave it parallel to the imaginary axis (\( -e_- \)).
To arrange for this flow to reach $z = 1$ (or close to it) we must rotate $-e_+$ back to $e_-$. This is achieved by a shift of the angle by $\pi$ since
\[ e_+ (\alpha + \pi) = -e_+(\alpha). \] (45)

Thus, in this case,
\[ \alpha_{32} = \alpha_{13} + \pi, \] (46)
and this leads to $F \propto (1 - \sin \theta)$. Since $\sin \theta = 0$ for real $\mu$ we thus deduce that $F \propto 1$ with positive constant of proportionality; that is, $F > 0$, so the force is repulsive, as expected.

We now turn to those cases for which $\mu$ is near the curve of marginal stability. In this case we see from (16) that
\[ (\alpha_{13} - \alpha_{23}) = \frac{-\mu_2 \Delta}{4 \mu_1^2 (1 - \mu_1^2)} + O(\Delta^2). \] (47)

It follows, after some calculation, that
\[
F \propto \mu_2^2 (1 + |\mu|^2) (1 + \mu_2^2 - \mu_1^2) \Delta \\
\times \left[ (1 - \mu_1^2)^3 \left( \mu_1^2 (1 - |\mu|^2)^2 \\
+ \mu_2^2 (1 + |\mu|^2)^2 (1 + \mu_2^2 - \mu_1^2)^2 \right) \right]^{-1} \\
+ O(\Delta^2). \] (48)

Since $\mu_1^2 < 1$, all factors other than $\Delta$ are positive. We thus have
\[ F(L) \sim c^2 [\Delta + O(\Delta^2)] e^{-m_3 L}, \] (49)
for some non-zero constant $c$. The leading-order asymptotic force is therefore repulsive when $\Delta > 0$, which corresponds to a point inside the ‘2-soliton’ region. It is attractive when $\Delta < 0$, which corresponds to a point inside a ‘3-soliton’ region. On the curve of marginal stability the leading-order asymptotic force is zero.

5. Soliton bound states

So far, we have analysed the behaviour of solitons in the WZ model with quartic superpotential near the curve of marginal stability in the moduli space of such superpotentials. On one side of this curve there are only two types of soliton and the long-range asymptotic force between these two solitons is repulsive. We have shown, however, that this repulsive asymptotic force vanishes on the curve of marginal stability, and becomes attractive after it is crossed.

An interesting question that this result raises is whether the long-range intersoliton force vanishes on the curve of marginal stability only to leading order in an asymptotic expansion (in powers of $e^{-m_3 L}$) or to all orders (assuming the existence of this expansion). If the force were to vanish exactly on the curve of marginal stability then we would expect to be able to find a one-parameter family of 12-soliton solutions corresponding to a 13 and 32 soliton at arbitrary separations. However, there are no such solutions because the 12-soliton trajectory necessarily coincides, on (the appropriate segment of) the curve of marginal stability, with the union of the 13 and 32-soliton trajectories. This fact suggests that it is only the leading-order asymptotic force that vanishes on the curve of marginal stability, and that the next-to-leading order term is non-zero. Given that it is non-zero it must be repulsive because an attractive force would imply a bound state ‘third soliton’ on the ‘2-soliton’ side of the curve of marginal stability. Near $\Delta = 0$ we thus expect an asymptotic expansion for the intersoliton force of the form
\[ F(L) \sim c^2 \Delta e^{-m_3 L} + \gamma^2 e^{-2m_3 L} + \ldots \] (50)
for some non-zero constant $\gamma$. When $\Delta < 0$ this force vanishes for
\[ e^{-m_3 L} \approx (c^2 / \gamma^2) |\Delta|. \] (51)
Since $\Delta$ is small the neglect of the higher-order terms in (50) is justified, and there is thus a minimum of the inter-soliton potential at $L \sim (1 / m_3) \log(1 / |\Delta|)$. This diverges on the curve of marginal stability, as we know must happen from the analysis of the soliton trajectories, and this explains the discontinuity of the spectrum on crossing this curve. As mentioned earlier, this result agrees qualitatively with a similar result obtained for a different scalar field model by different methods [7], as well as with results for other types of supersymmetric soliton.
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References

Quantum mechanics on noncommutative plane and sphere from constrained systems

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Abstract

It is shown that quantum mechanics on noncommutative (NC) spaces can be obtained by canonical quantization of some underlying constrained systems. Noncommutative geometry arises after taking into account the second class constraints presented in the models. It leads, in particular, to a possibility of quantization in terms of the initial NC variables. For a two-dimensional plane we present two Lagrangian actions, one of which admits addition of an arbitrary potential. Quantization leads to quantum mechanics with ordinary product replaced by the Moyal product. For a three-dimensional case we present Lagrangian formulations for a particle on NC sphere as well as for a particle on commutative sphere with a magnetic monopole at the center, the latter is shown to be equivalent to the model of usual rotor. There are several natural possibilities to choose physical variables, which lead either to commutative or to NC brackets for space variables. In the NC representation all information on the space variable dynamics is encoded in the NC geometry. Potential of special form can be added, which leads to an example of quantum mechanics on the NC sphere. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction and summary

Recently quantum mechanics on noncommutative spaces (NQM) have received a considerable discussion [1–7]. For two-dimensional plane it can be characterized by the following brackets \((\epsilon_{ab} = -\epsilon_{ba}, \quad a, b = 1, 2, \epsilon_{12} = 1)\)

\[
\{x_a, x_b\} = \theta \epsilon_{ab}, \quad \{x_a, p_b\} = \delta_{ab}, \quad \{p_a, p_b\} = 0,
\]

(1)

and by the Hamiltonian \(H = \frac{p^2}{2m} + V(x)\) with some potential \(V(x)\). To make this situation tractable, the prescription is to consider new variables \(\tilde{x}_a = x_a + \frac{\theta}{2} \epsilon_{ab} p_b, \quad \tilde{p}_a = p_a,\)

(2)

which obey the canonical brackets and thus can be quantized in the standard way. It leads to the
Schrödinger equation

\[ E\Psi(\tilde{x}) = \frac{1}{2m}\tilde{\theta}^2\Psi(\tilde{x}) + V\left(\tilde{x}_a - \frac{\theta}{2}\epsilon_{ab}\tilde{p}_b\right)\Psi(\tilde{x}), \tag{3} \]

where the last term can be rewritten [3,8,9] through the Moyal product

\[ V\left(\tilde{x}_a - \frac{\theta}{2}\epsilon_{ab}\tilde{p}_b\right)\Psi(\tilde{x}) = V(\tilde{x}) \star \Psi(\tilde{x}). \tag{4} \]

Thus one obtains quantum mechanics in terms of the commutative variables \( \tilde{x}, \tilde{p} \), but with the ordinary product replaced by the star product.

Let us recall that in some cases appearance of the noncommutative geometry [10] has a natural interpretation as resulting from the canonical quantization of some underlying constrained system. In particular, this interpretation is possible for the lowest level Landau problem [9,11] and for the open string in a B-field background [12–14]. Regarding the NC quantum mechanics on the plane, one possibility was proposed in [1], starting from higher derivative mechanical action. It leads to the NC particle with extra physical degrees of freedom. In this relation it is natural to ask whether a similar interpretation is possible for NC quantum mechanics of the scalar particle (1)–(4), as well as for the corresponding generalization on three-dimensional sphere. Here we demonstrate that it is actually the case. Our starting point will be some mechanical system (without higher derivatives) formulated in an appropriately extended configuration space. Nonphysical character of the corresponding extra degrees of freedom is supplied by second class constraints. The noncommutative geometry (1) arises after introduction the Dirac bracket corresponding to these constraints, while the prescription (2) becomes, in fact, the standard necessary step for the canonical quantization of a system with second class constraints [15,16].

The work is organised as follows. In Section 2 we discuss models which lead to quantum mechanics on the NC plane. The Lagrangian action, which is appropriate for at most quadratic potential, looks as follows

\[ S = \int d\tau \left[ -\frac{m}{2}\dot{v}^2 + 2(\dot{v} \dot{\theta})^{-1}v \right. \]
\[ \left. - \frac{2}{m\tilde{\theta}^2}v^2 - U(x) \right], \tag{5} \]

where \( x_\mu(\tau), v_\nu(\tau) \) are the configuration space variables and \( \theta_{ab} = \bar{\theta}_{ab} \). The variables \( v_\nu \) are subject to the second class constraints and can be omitted from the consideration after the Dirac bracket is introduced. The physical sector consist of \( x_a \) and the conjugated momentum \( p_a \). The Dirac bracket for \( x_a \) turns out to be nontrivial, with the noncommutativity parameter being \( \bar{\theta} = \theta \det^{-1}(1 - \frac{2m\tilde{\theta}}{\bar{\theta}}\partial U) \). The parameter (and rank of matrix of the constraint brackets) depend on the potential. It explains appearance of two phases [4–6] of the resulting NQM: for a critical value of the parameter, the model involves first class constraints instead of the second class ones. For the case \( \partial_a\partial_bU = \text{const} \) one can easily find the canonical variables (see Eq. (17)). Quantization leads to NQM (3) with the potential \( V = -\frac{m\tilde{\theta}^2}{8}\partial_a\partial_bU\partial_a\partial_bU + U \). For an arbitrary potential \( U(x) \), the noncommutativity parameter \( \bar{\theta} \) depends on \( x_a \) and one is faced with the problem of diagonalization of the brackets, Eq. (15). Surprisingly enough, the problem can be resolved if one starts from the action, which is obtained from (5) omitting the first term

\[ S = \int d\tau \left[ 2(\dot{v} - \dot{\bar{\theta}})^{-1}v - \frac{2}{m\tilde{\theta}^2}v^2 - V(x) \right]. \tag{6} \]

It can be considered as the action of ordinary particle (with position \( x_a \)) written in the first order form, with the “Chern–Simons term” for \( v \) added: \( \dot{\bar{\theta}}^{-1}v \). The action is similar to the one discussed by Lukierski et al. [1], but does not involve of higher derivatives. As a consequence, there are no “internal” oscillator modes in the physical sector. Below we show that this action leads to NQM of the scalar particle (1)–(4) with the same potential \( V \).

In Section 3 we demonstrate that the same procedure works for noncommutative sphere in three dimensions [19–21]. We propose the following action

\[ S = \int d\tau \left[ -\epsilon_{ijk}\dot{v}_i\dot{v}_jx_k - v^2 \right. \]
\[ \left. + \phi(x^2 - 1) - V(v^2) \right], \tag{7} \]

where \( \epsilon_{ijk} = \epsilon_{[ijk]} \), \( \epsilon_{123} = 1 \). The variables \( v_i \) play the same role as in the previous case (5). The variables \( x_i \) are restricted to lie on the sphere \( x^2 = 1 \). The kinematic constraint is included into the action by using of the Lagrangian multiplier \( \phi \). Dynamics is governed by second order differential equations, which is supplied
by the presence of the term $v^2$. We restrict ourselves to $SO(3)$-invariant potential $V(v^2)$. The combination $x_i v_i$ is not included into the potential since it would lead to deformation of the constraint system algebra as compared with the free case $V = 0$ (see below). In the Hamiltonian formalism the essential constraints of the model are

$$G_i = p_i + \epsilon_{ijk} v_j x_k = 0, \quad \pi_i = 0,$$

$$x_i^2 - 1 = 0, \quad x_i v_i = 0,$$

where $(x, \pi)$ and $(v, p)$ form canonical pairs (in these notations commutative relations appear in the standard form, see Eq. (41)). The constraints form the second class system. The corresponding Dirac bracket is constructed and brackets for the phase space variables are presented in $SO(3)$ covariant form. Using the constraints $G_1 = 0$ one can represent one of the variables $(x, v, p)$ through the remaining ones, which leads either to commutative or to NC brackets for space variables. The representations are discussed in Section 4. $(x, p)$-representation is characterized by NC space geometry and trivial dynamics for the corresponding space variables. In $(v, p)$-representation the geometry can be made commutative by transition to the canonical variables, the dynamics of which is governed then by nonlinear equations. In Section 5 we present and discuss slight modification of the action (7) which can be made commutative by transition to the canonical variables, the dynamics of which is governed then by nonlinear equations. In the end of the section some possible generalizations of the action (7) are discussed.

2. Particle on the noncommutative plane

Starting from the action (5), one finds in the Hamiltonian formalism the primary constraints

$$G_a = p_a + 2\theta_{ab}^{-1} v_b = 0,$$

and the Hamiltonian

$$H = -\frac{1}{2m} \pi^2 + \frac{2}{m} \pi \theta^{-1} v + U(x) + \lambda (p + \theta^{-1} v).$$

Here $p, \pi$ are conjugated momenta for $x, v$ and $\lambda$ is the Lagrangian multiplier for the constraint. Further analysis gives the secondary constraints

$$\dot{G}_a = 0 \quad \Rightarrow \quad T_a = \pi_a - 2\theta^{-1} \theta_{ab} v_b + \frac{m}{2} \theta_{ab} \partial_b U = 0,$$

as well as equations for determining the Lagrangian multipliers

$$F_k = -\frac{2}{m} \left( \pi - \theta^{-1} v \right),$$

$$F_{ab} = \delta_{ab} - \frac{m \theta^2}{4} \partial_a \partial_b U.$$  

Next step depends on the rank of the matrix $F$. If $\text{det} F = 0$, the model involves first class constraints (see also Eq. (13)), which explains appearance of two phases [4–6] of the resulting NQM. Let us consider the nondegenerated case $\text{det} F \neq 0$. Then the constraints form the second class system

$$[G_a, G_b] = 0, \quad [T_a, T_b] = -4\theta_{ab}^{-1},$$

$$[G_a, T_b] = 2 F_{ac} \theta_{cb}^{-1}.$$  

Introducing the Dirac bracket


$$- [A, G] \frac{1}{2} F^{-1} \theta [T, B]$$

$$- [A, T] \frac{1}{2} \theta F^{-1} [G, B],$$

the variables $v, \pi$ can be omitted from consideration, while for the remaining physical variables $x, p$ one obtains from Eq. (14) the following brackets:

$$[x_a, x_b] = \Delta^{-1} \delta_{ab}, \quad [x_a, p_b] = F_{ab}^{-1},$$

$$[p_a, p_b] = 0.$$  

The noncommutativity parameter depends on the potential through the quantity

$$\Delta = \text{det} \left( 1 - \frac{m \theta^2}{4} \partial_a \partial_b U \right).$$

Let us restrict ourselves to the case $\partial_a \partial_b U = \text{const}$. To quantize the system one needs to find the canonical variables [16], which in this case turn out to be

$$\tilde{x}_a = F_{ab} x_b + \frac{1}{2} \theta_{ab} p_b, \quad \tilde{p}_a = p_a.$$  

They obey the standard brackets $[\tilde{x}_a, \tilde{x}_b] = 0, \quad [\tilde{x}_a, \tilde{p}_b] = \delta_{ab}, \quad [\tilde{p}_a, \tilde{p}_b] = 0$. The Hamiltonian in terms of the canonical variables is (the term $F$ can
be equally included into the kinetic part of the Hamiltonian \[26\]
\[H_{\text{ph}} = \frac{1}{2m} \dot{\mathbf{p}}^2 - \frac{m \theta^2}{8} \partial_a U \partial_a U \bigg|_{\mathbf{x}(\tilde{x}, \tilde{\mathbf{p}})} + U \left[ F^{-1} \left( \tilde{x} - \frac{1}{2} \theta \tilde{\mathbf{p}} \right) \right], \tag{18}\]
where the term with derivatives of the potential comes from Eq. (17). The resulting system can be quantized now in the standard way. Note that the underlying potential \(U\) and the final one turn out to be different for this model. For example, starting from the harmonic oscillator \(U = \frac{1}{2} |x|^2\), one obtains the NQM which corresponds to oscillator with renormalized rigidity \(\tilde{k} = (1 - m \theta^2 k/4)^{-1} k\), namely
\[V = \left[ -\frac{m \theta^2}{8} \partial_a U \partial_a U + U \right] \bigg|_{\mathbf{x}(\tilde{x}, \tilde{\mathbf{p}})} = \frac{\tilde{k}}{2} \left( \frac{\tilde{x}}{2} - \frac{1}{2} \theta \tilde{\mathbf{p}} \right)^2. \tag{19}\]
Note also that in absence of the potential \((U = 0)\) the model (5) describes the free NC particle which is characterised by the equations of motion \(\dot{x}_a = \frac{1}{m} p_a, \dot{p}_a = 0\) and by the relations (1).

Let us return to the case of an arbitrary potential. As it was mentioned, the complicated brackets (15) arise due to the fact that the secondary constraints (17) involve derivative of the potential. While existence of the canonical variables is guaranteed by the known theorems \[16\], it is problematic to find a solution in the manifest form. One possibility to avoid the problem is to construct action which will create the primary constraints only. Since \(U(x)\) does not contain the time derivative, it cannot give contribution into the primary constraints. An appropriate action is\[2\]
\[S = \int d\tau \left[ 2(\dot{\mathbf{v}} - \dot{\tilde{x}}) \theta^{-1} \mathbf{v} - \frac{2}{m \theta^2} \mathbf{v}^2 - V(x) \right], \tag{20}\]
where \(x_a, v_a\) are the configuration space variables. Configuration space dynamics is governed by the second order equations which is supplied by the term \(\mathbf{v}^2\). Following the Dirac procedure one obtains the primary second class constraints
\[G_a = p_a + 2 \theta^{-1} \mathbf{v} b_a = 0, \tag{21}\]
\[T_a = \pi_a - 2 \theta^{-1} \mathbf{v} b_a = 0, \tag{22}\]
and the Hamiltonian
\[H = \frac{2}{m \theta^2} \mathbf{v}^2 + V(x) + \lambda_1 G + \lambda_2 T. \tag{22}\]

Remaining analysis is similar to the previous case. Introducing the Dirac bracket (14) (where \(F = 1\) now), the variables \(v, \pi\) can be omitted, while for \(x, p\) one has the brackets (1). Defining the canonical variables \(\tilde{x}_a = x_a + \frac{1}{2} \theta a_b p_b, \tilde{p}_a = p_a\), one obtains the physical Hamiltonian \(H = \frac{2}{m \theta^2} \mathbf{v}^2 + V(\tilde{x} - \frac{1}{2} \theta \tilde{\mathbf{p}})\), thus reproducing the NQM (3), (4) for the case of an arbitrary potential.

We have demonstrated that quantum mechanics on NC plane can be considered as resulting from direct canonical quantization of the underlying constrained systems (5), (6). It implies, that instead of the star product (3), (4), one can equally use now other possibilities to quantize the system. In particular, the conversion scheme [17] or the embedding formalism [18] can be applied. For example, it is not difficult to rewrite the formulation (20)–(22) as a first class constrained system. Namely, let us keep \(G\)-constraint only and define the deformed Hamiltonian as
\[\tilde{H} = \frac{2}{m \theta^2} \mathbf{v}^2 + V\left[ x - \frac{1}{2} \theta (\pi - 2 \theta^{-1} \mathbf{v}) \right] + \lambda G. \tag{23}\]
Since \([G, \tilde{H}] = 0\), it is equivalent formulation of the problem (22), the latter is reproduced in the gauge \(T = 0\). Now one can quantize all the variables canonically, while the first class constraint \(G = 0\) can be imposed as restriction on the wave function. It implies quantization in terms of the initial NC variables. Another possibility is to consider the gauges different from \(T = 0\). For example, one can take \(\pi = 0\), which can lead to simplification of the eigenvalue problem (3).

3. Particle on the noncommutative sphere

Here we show that dynamics on the NC sphere can be described in a similar fashion, starting from the action (7). From manifest form of the action it follows
that velocities do not enter into expressions for definition of conjugated momentum in the Hamiltonian formulation. On the first stage of the Dirac procedure one finds the primary constraints

\[ G_i \equiv p_i + \epsilon_{ijk} v_j x_k = 0, \quad T_i \equiv \pi_i = 0, \]

\[ p_\phi = 0, \]  \quad \text{(24)}

where \( p_i \) are conjugated momentum for \( v_i \) while \( \pi_i \) corresponds to \( x_i \). The Hamiltonian is

\[ H = v^2 - \phi(x_i^2 - 1) + V(v^2) + \lambda_i G_i \]

\[ + \bar{\lambda}_i T_i + \lambda p_\phi, \]  \quad \text{(25)}

where \( \lambda \) are the Lagrangian multipliers for the corresponding constraints. The constraints obey the following Poisson bracket algebra

\[ \{ G_i, G_j \} = 2\epsilon_{ijk} x_k, \quad \{ G_i, T_j \} = -\epsilon_{ijk} v_k, \]

\[ \{ T_i, T_j \} = 0. \]  \quad \text{(26)}

Matrix composed from the brackets admits two null-vectors \( w_1 = (0, v_i), w_2 = (v_i, -2x_i) \), so the system \((G, T)\) involve two first class constraints at this stage: \( v_i T_i \) and \( v_i G_i - 2x_i T_j \). From (24) one has the consequences \( v_i p_i = 0, x_i p_i = 0 \). At the second stage of the Dirac procedure there appear the equations

\[ \dot{p}_\phi = 0 \quad \Rightarrow \quad x_i^2 - 1 = 0, \]

\[ \dot{G}_i = 0 \quad \Rightarrow \quad -2(1 + V') v_i \]

\[ + 2\epsilon_{ijk} \bar{\lambda}_j x_k - \epsilon_{ijk} \bar{\lambda}_j v_k = 0, \]

\[ \dot{T}_i = 0 \quad \Rightarrow \quad 2\phi x_i - \epsilon_{ijk} \bar{\lambda}_j v_k = 0, \]  \quad \text{(27)}

where \( V' = \partial V/\partial v^2 \). From these equations one extracts three secondary constraints

\[ S = x_i^2 - 1 = 0, \quad \bar{S} = x_i v_i = 0, \]

\[ \Phi = \phi + \frac{1}{2} v_i^2 (1 + V') = 0, \]  \quad \text{(28)}

while the remaining equations involve the Lagrangian multipliers. They will be resolved in the manifestly SO(3)-covariant form below. On the next step there arise equations for the Lagrangian multipliers only

\[ \dot{\lambda}_i = \lambda + \{ \Phi, G_j \} \bar{\lambda}_j \]

\[ \dot{\bar{\lambda}}_i = -x_i \lambda_i = 0, \]

\[ \dot{\bar{S}} = 0 \quad \Rightarrow \quad v_i \bar{\lambda}_i + x_i \lambda_j = 0, \]  \quad \text{(29)}

which finishes the Dirac procedure for revealing the constraints. To determine the Lagrangian multipliers one has now Eqs. (27), (29). Their consequences are \( x_i \lambda_i = x_i \bar{\lambda}_i = v_i \lambda_i = v_i \bar{\lambda}_i = 0 \). Using these equations, one resolves Eqs. (27), (29) as

\[ \lambda_i = (1 + V') p_i, \quad \bar{\lambda}_i = \lambda = 0. \]  \quad \text{(30)}

The Hamiltonian equations of motion for the model can be obtained with the help of Eqs. (25), (30). They will be discussed in the next section. Since all the multipliers have been determined, the constraints (24), (28) form the second class system and thus can be taken into account by transition to the Dirac bracket. After introduction of the Dirac bracket corresponding to the pair \( p_\phi = 0 \), \( \phi = 0 \), the variables \( \phi, p_\phi \) can be omitted from consideration. The Dirac brackets for the remaining variables coincide with the Poisson one. To find the Dirac bracket which corresponds to the remaining eight constraints one needs to invert \( 8 \times 8 \) matrix composed from Poisson brackets of these constraints. To simplify the problem, we prefer to do this in two steps: first, let us construct an intermediate Dirac bracket which corresponds to the constraints \( G_a = 0, T_a = 0, a = 1, 2 \), and then bracket which corresponds to the remaining constraints \( G_3, T_3, S, \bar{S} \). Consistency of this procedure is guaranteed by the known theorems [16]. On the first step one has the Poisson brackets

\[ \{ G_a, G_b \} = 2\epsilon_{ab} x_3, \quad \{ G_a, T_b \} = -\epsilon_{ab} v_3, \]

\[ \{ T_a, T_b \} = 0. \]  \quad \text{(31)}

Then the intermediate Dirac bracket is

\[ [A, B]_{D1} = [A, B] - [A, G_a] \frac{\epsilon_{ab}}{v_3} [T_b, B] \]

\[ - [A, T_a] \frac{2x_3}{v_3} \epsilon_{ab} [T_b, B]. \]  \quad \text{(32)}

Now one can use the equations \( G_a = 0, T_a = 0 \) in any expression. As a consequence, the remaining constraints can be taken in the form

\[ G_3 \equiv x_i p_i = 0, \quad T_3 \equiv \pi_3 = 0, \]

\[ S \equiv x_i^2 - 1 = 0, \quad \bar{S} \equiv x_3 (v_3 + J_3) = 0, \]  \quad \text{(33)}

and obey the D1-algebra

\[ [G_3, S]_{D1} = -\frac{4x_3}{J_3}, \quad [G_3, \bar{S}]_{D1} = -2 \left( 1 + \frac{p_a^2}{J_3^2} \right). \]
\[ [T_3, S]_{D1} = 0, \quad [T_3, \overline{S}]_{D1} = -\frac{J_3^2 - p_3^2}{x_3^2 J_3}, \]
\[ [S, \overline{S}]_{D1} = \frac{4x_3^3 p_3}{J_3^2}, \quad \] (34)

where \( J_i \) are the rotation generators: \( J_i \equiv \epsilon_{ijk} x_j p_k \).

The corresponding matrix can be easily inverted, and the final expression for the Dirac bracket is

\[ [A, B]_D = [A, B] - [A, G_3] \frac{x_3^2 p_3}{J_3^2 - p_3^2} [T_3, B] \]
\[ - [A, G_3] \frac{J_3}{4x_3} [S, B] \]
\[ + [A, T_3] \frac{x_3^2 p_3}{J_3^2 - p_3^2} [G_3, B] \]
\[ + [A, T_3] \frac{x_3 (J_3^2 + p_3^2)}{2(J_3^2 - p_3^2)} [S, B] \]
\[ - [A, T_3] \frac{x_3^2 J_3}{J_3^2 - p_3^2} [\overline{S}, B] \]
\[ + [A, S] \frac{J_3}{4x_3} [G_3, B] \]
\[ - [A, S] \frac{x_3 (J_3^2 + p_3^2)}{2(J_3^2 - p_3^2)} [T_3, B] \]
\[ + [A, \overline{S}] \frac{x_3^2 J_3}{J_3^2 - p_3^2} [T_3, B], \] (35)

where all the brackets on the r.h.s. are D1-brackets.

Note that the complete constraint system (24), (28) is \( SO(3) \)-covariant. Consequently, one expects that the final expressions for the brackets can be rewritten in \( SO(3) \)-covariant form also. It is actually the case. For example, from Eq. (35) one obtains for the variables \( x_i \)

\[ \{x_1, x_2\}_D = -\frac{x_3}{J_3^2} \left[ 2 + \frac{x_3^2 (2p_i^2 + J_3^2)}{J_3^2 - p_3^2} \right], \]
\[ \{x_1, x_3\}_D = -\frac{1}{J_3 p_i^2} \left[ \frac{2x_3 p_i}{J_3} \epsilon_{ac} p_c + 2p_a \right. \]
\[ \left. + \frac{J_3^2 + 2p_a^2}{J_3} \right] \epsilon_{ac} x_c. \] (36)

Using the equalities

\[ J^2 \equiv J_i^2 = p_i^2 = x_i^2, \quad J_3^2 - p_3^2 = -x_3^2 p_3^2, \]
\[ x_3 p_3 p_a - J_3 \epsilon_{ab} p_b + p_a^2 x_a = 0, \] (37)

which are true on the constraint surface (24), (28), Eqs. (36) can be presented in \( SO(3) \)-covariant form \( \{x_i, x_j\}_D = \frac{1}{J_3^2} \epsilon_{ijk} x_k \). Other brackets can be computed from Eq. (35) in a similar fashion. After tedious calculations one obtains the following result

\[ \{x_i, x_j\} = \frac{1}{J_3^2} \epsilon_{ijk} x_k, \quad \{x_i, p_j\} = \frac{1}{J_3^2} J_i x_j, \]
\[ \{p_i, p_j\} = -\frac{1}{J_3^2} \epsilon_{ijk} x_k, \] (38)
\[ \{x_i, x_j\} = \frac{1}{J_3^2} \epsilon_{ijk} x_k, \quad \{x_i, x_j\} = \frac{1}{J_3^2} x_i x_j, \]
\[ \{p_i, p_j\} = \frac{1}{J_3^2} (\delta_{ij} + x_i x_j). \quad \] (39)

Since \( \{x_i, J^2\} = 0 \), the operator \( J^2 \) can be included into redefinition of \( x_i \): \( \tilde{x}_i \equiv J^2 x_i \), then \( \tilde{x}_i \) obeys \( SU(2) \) algebra \( \{\tilde{x}_i, \tilde{x}_j\} = \epsilon_{ijk} \tilde{x}_k \), and is constrained to lie on the fuzzy sphere \( \lambda^2 = \langle J^2 \rangle^2 \). Quantum realization and irreducible representations of such a kind of an algebraic structure were considered, in particular, in [20,21]. One notes that the algebra obtained (38) has much simpler structure as compared with the one proposed in [20] from algebraic considerations.

Since the second class constraints were taken into account, one can now use them in any expression. In particular, from Eqs. (24), (28) it follows that as the physical sector variables one can choose either \( (x_i, p_i) \) or \( (v_i, p_i) \), or \( (x_i, v_i) \). Relation between these representations is given by the first equation from (24), which can be written in one of the following forms\(^3\)

\[ p_i = -\epsilon_{ijk} v_j x_k, \quad v_i = -\epsilon_{ijk} x_j p_k, \]
\[ x_i = \frac{1}{v_j^2} \epsilon_{ijk} v_j p_k. \quad \] (40)

Let us point out that for any given choice, the remaining nonphysical variable looks formally as the rotation generator in the corresponding representation. Eqs. (40) relate different representations of the particle dynamics on NC sphere which are discussed in the next section.

\(^3\) The representations for NC plane can be obtained in a similar fashion starting from Eq. (21), and are not interesting due to linear character of the constraints.
4. Three representations for the particle dynamics on noncommutative sphere

To discuss classical dynamics of the particle on NC sphere it will be sufficient to consider the free case \( V = 0 \). In what follows, we will preserve \( SO(3) \) covariance which implies that two of the constraints are not resolved in the manifest form. Note also that the variables \( \phi, p_\phi \) are trivially constrained \( \phi = 0, p_\phi = 0 \) and thus are omitted from consideration.

Noncommutative \((x_i, p_i)\)-representation. Taking \( x, p \) as the basic variables, their algebra is

\[
[x_i, x_j] = \frac{1}{J^2} \epsilon_{ijk} x_k, \quad [x_i, p_j] = \frac{1}{J^2} J_i x_j, \\
[p_i, p_j] = -\frac{1}{2} \epsilon_{ijk} x_k.
\]

Equations of motion follow from (25), (30)

\[\\hat{x}_i = 0, \quad \hat{p}_i = \epsilon_{ijk} x_j p_k,
\]

and are accompanied by two constraints

\[
x_i^2 - 1 = 0, \quad x_i p_i = 0.
\]

The physical Hamiltonian has the form \( H_{ph} = p^2 \).

One notes that \( \{x_i, J^2\} = 0 \), so \( J^2 \) can be absorbed into redefinition of \( x_i \) as \( \tilde{x}_i = J^2 x_i \). The algebra acquires then the form

\[
[\tilde{x}_i, \tilde{x}_j] = \epsilon_{ijk} \tilde{x}_k, \quad [\tilde{x}_i, p_j] = \epsilon_{ijk} p_k, \\
[p_i, p_j] = -\frac{1}{2 J^2} \epsilon_{ijk} \tilde{x}_k.
\]

Commutative \((v_i, p_i)\)-representation. In this case the bracket algebra is

\[
[v_i, v_j] = -\frac{1}{2 p^2} v_i [p_j, p_i], \\
[v_i, p_j] = \frac{1}{2} \left( \delta_{ij} - \frac{v_i v_j + p_i p_j}{v^2} \right), \\
[p_i, p_j] = -\frac{1}{2 p^2} v_i [p_j, p_i].
\]

Dynamics turns out to be nontrivial for both variables

\[
\dot{v}_i = p_i, \quad \dot{p}_i = -v_i, \quad v_i^2 = p_i^2, \\
v_i p_i = 0,
\]

which implies \( \dot{v} + v = 0 \) for the configuration space variable. Note that the representation turns out to be symmetric under the change \( v \rightarrow -v, p \rightarrow v \).

Let us compare these two representations of the particle dynamics on NC sphere. Since the NC geometry has been obtained by using the Dirac bracket, there exists canonical transformation to new variables, in terms of which the bracket acquires the canonical form [16]. The corresponding theorem states that constraints of a theory become a part of the new variables after this transformation. Being applied to the case under consideration, it means that the new variables will have the following structure:

\[
(x_i, \pi_i, v_i, p_i)
\]

\[
\Rightarrow \left( \tilde{x}_i = x_i - \frac{\epsilon_{ijk} v_j p_k}{v^2}, \pi_i, \tilde{p}_3, \tilde{v}_3, \tilde{p}_a, \tilde{v}_a \right),
\]

where \( \tilde{v}_a, \tilde{p}_a \) are the physical variables with the canonical brackets, in particular: \( \{ \tilde{v}_a, \tilde{p}_a \} = 0 \). From the expression (24), (47) one notes that the theorem naturally selects \((v, p)\)-representation for transition to the canonical brackets, which is the reason for the name: “commutative representation”. From Eq. (46) it follows that the canonical coordinates have nontrivial equations of motion (see also the next section). In contrast, in the NC \((x, p)\)-representation the configuration space dynamics (42) turns out to be trivial. Thus, the NC description implies that all information on the dynamics is encoded in NC geometry. Similar situation was observed for \( SO(n) \) nonlinear sigma-model in [22] and for the Green–Schwarz superstring in the covariant gauge in [23].

\((x_i, v_i)\)-representation coincides with \((x_i, p_i)\)-representation.

5. Particle on commutative sphere with a magnetic monopole at the center and the rotor

In this section we show that slight modification of the action (7) gives description for a particle with a monopole at the center of the sphere [24]. It will be demonstrated also that this model is equivalent to the model of usual rotor.
Let us consider the action (7) with the variables $x$ and $v$ interchanged in the first term
$$S = \int d\tau \left[ -\epsilon_{ijk} \dot{x}_i x_j v_k - v^2 + \phi(x_i^2 - 1) - V(v^2) \right].$$
(48)

Canonical momentum for $x_i$ is denoted through $p_i$ while $\pi_i$ corresponds to the variable $v_i$ (the notations are opposite to the ones adopted for the model (7)). In these notations analysis of the model turns out to be similar to the previous case, so we present the final results only. The essential constraints of the theory are
$$G_i \equiv p_i + \epsilon_{ijk} x_j v_k = 0, \quad T_i \equiv \pi_i = 0,$$
$$S = x_i^2 - 1 = 0, \quad S \equiv x_i v_i = 0,$$
and can be taken into account by transition to the Dirac bracket. After that, dynamics of the model can be presented in one of the following three forms.

$(x_i, v_j)$-representation. In terms of these variables the bracket algebra is
$$\{x_i, x_j\} = 0, \quad [x_i, v_j] = \epsilon_{ijk} x_k,$$
$$[v_i, v_j] = \epsilon_{ijk} v_k,$$
(50)
while their dynamics is governed by the equations (free case)
$$\dot{x}_i = -2\epsilon_{ijk} x_j v_k, \quad \dot{v}_i = 0, \quad x_i^2 - 1 = 0,$$
$$x_i v_i = 0.$$
(51)

For the physical Hamiltonian one has the expression (remember that $v$ is noncommutative variable)
$$H_{ph} = v^2 + V(v^2).$$
(52)

The algebra obtained (50) corresponds to the particle on commutative sphere with a monopole at the center (note the relations (37)) [24].

$(x_i, p_j)$-representation. In this case one has the brackets
$$\{x_i, x_j\} = 0, \quad [x_i, p_j] = \delta_{ij} - x_i x_j,$$
$$[p_i, p_j] = -(x_i p_j - x_j p_i).$$
(53)

Equations of motion turn out to be nontrivial for both variables
$$\dot{x}_i = 2 p_i, \quad \dot{p}_i = -2 p_i^2 x_i, \quad x_i^2 - 1 = 0,$$
$$x_i p_i = 0.$$
(54)

which implies $\ddot{x}_i + 4 p_i^2 x_i = 0$ for the configuration space variables. Eqs. (53), (54) correspond to model of the rotor and can be equally obtained from the action
$$S = \int d\tau \left[ \frac{1}{2} \dot{x}_i^2 + \phi(x_i^2 - 1) \right].$$
(55)

Thus we have demonstrated canonical equivalence of the models (50), (51) and (53), (54). They correspond to different choices of physical variables in the underlying action (48). Equivalently, they are related by the change of variables $v_i = \epsilon_{ijk} x_j p_k$.

Resolving the remaining constraints from Eq. (54), it is not difficult to find the canonical variables of the model $x_a = \tilde{x}_a$, $p_a = \tilde{p}_a - \frac{(\dot{x}_a \dot{x}_b)}{\overline{x}_a \overline{x}_b} \tilde{x}_a$, $a = 1, 2$, which obey $\{\tilde{x}_a, \tilde{x}_b\} = \{\tilde{p}_a, \tilde{p}_b\} = 0$, $\{\tilde{x}_a, \tilde{p}_b\} = \delta_{ab}$. Dynamics is governed by the nonlinear equations $\dot{\tilde{x}}_a = \tilde{p}_a + \frac{(\dot{x}_a \dot{x}_b)}{\overline{x}_a \overline{x}_b} \tilde{x}_a$, $\tilde{p}_a = 0$. Let us point out that the relation established between the particle with a monopole and the rotor allows one to construct NC quantum mechanics corresponding to the geometry given in Eq. (50), with the nontrivial potential (52), following the same procedure as in Section 2. Since the bracket kernel of (53) is degenerated, see Eq. (54), the star product constructed using all six variables turns out to be nonassociative [25]. This subject will be discussed elsewhere.

$(v_i, p_j)$-representation. For these variables the algebra is $(J_i \equiv \epsilon_{ijk} v_j p_k)$
$$\{v_i, v_j\} = \epsilon_{ijk} v_k,$$
$$\{v_i, p_j\} = -\frac{1}{p_2} (v_i J_j - v_j J_i),$$
$$\{p_i, p_j\} = -\epsilon_{ijk} v_k,$$
(56)

while the equations of motion are similar to $(x, v)$-representation
$$\dot{v}_i = 0, \quad \dot{p}_i = 2\epsilon_{ijk} v_j p_k, \quad v_i^2 = \tilde{p}_i^2,$$
$$p_i v_i = 0.$$
(57)

Comparing this representation with $(x, p)$-representation one observes the same property as for NC sphere: transition from commutative description (53) to NC description (56) implies trivial dynamics for space variables in the latter representation.

Thus we have presented the Lagrangian formulations for a particle on the noncommutative sphere (7) as well as for a particle on the commutative sphere.
with a monopole at the center \((48)\), the latter is shown to be canonically equivalent to the model of rotor. In both cases the desired algebraic structure \((41), (50)\) arises as the Dirac bracket corresponding to the second class constraints presented in the model. After introduction of the Dirac bracket, the constraints can be used to represent part of variables through the remaining ones. There exist several \((SO(3)\) covariant) possibilities to choose the basic variables, which leads to different representations for the two models. In both cases there is the “commutative representation” which is appropriate for determining the canonical variables starting from the known constraint system. Using relation between NC and commutative representations one is able to construct quantum mechanics which corresponds to the NC representation.

In conclusion, let us comment on possible generalizations of the model \((7)\). One possibility is to consider immersion of the model into a locally invariant system. Let us omit the term \(\phi(x^2 - 1)\) in the action \((1)\). Then the formulation involves one first class constraint which corresponds to the local symmetry \(\delta x_i = \gamma v_i\). Thus, one is able now to consider different gauges of the model \((x^2 - 1 = 0\) and \(v^2 - 1 = 0\) are equally admissible now). We suggest that it can give unified description of the three models considered in this work. Other possibility may be NC quantum mechanics on three-dimensional plane. To construct it, one needs to modify the action \((7)\) in such a way that only primary constraints of the type \((24)\) are generated and form the second class system. These problems will be considered elsewhere.

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The induced Chern–Simons term at finite temperature

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Abstract

It is argued that the derivative expansion is a suitable method to deal with finite temperature field theory, if it is restricted to spatial derivatives only. Using this method, a simple and direct calculation is presented for the radiatively induced Chern–Simons-like piece of the effective action of (2 + 1)-dimensional fermions at finite temperature coupled to external gauge fields. The gauge fields are not assumed to be subjected to special constraints, and in particular, they are not required to be stationary nor Abelian. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Finite temperature field theory [1] is notoriously more difficult to deal with than its zero temperature counterpart. Lorentz symmetry is reduced and time and space play different roles. This is apparent in the imaginary time formalism, in which the time is Wick-rotated and compactified to a circle. As a consequence, tools such as the perturbative expansion or the derivative expansion, that were quite useful at zero temperature may become unreliable. These remarks apply, in particular, to (2 + 1)-dimensional QED. In this theory, due to the compactification, topologically non-trivial (large) gauge transformations are supported at finite temperature [2]. In principle, one would expect that the coefficient of the radiatively induced Chern–Simons (CS) term in the effective action [3] would be correctly quantized [4], consistently with gauge invariance of the partition function at any temperature [5]. This turns out not to be the case when that coefficient is computed using perturbation theory [2,6]. Existing (Abelian and non-Abelian) exact results for particular configurations [5,7–9] show that gauge invariance is an exact symmetry but perturbation theory fails to see this; perturbation theory is designed to describe the effective action in the neighborhood of a vanishing gauge field and large gauge transformations necessarily move the gauge field configuration away from the perturbative region. The derivative expansion is also problematic as already shown in a (0 + 1)-dimensional setting [10] (using the real time formalism). The problem is clear in the imaginary time formalism since the energy becomes a discrete variable. This suggests that the trouble exists only if one insists in taking a derivative expansion in both time and space. Technically,
to make a derivative expansion of the effective action functional $W$, means to replace the given configuration $A_\mu(x)$ by a family of configurations $A_\mu(\lambda x)$, where $\lambda$ is a bookkeeping parameter, and then expand $W(\lambda)$ in powers of $\lambda$. At finite temperature, this procedure is inconsistent, since the external (bosonic) fields are required to be periodic in the (Euclidean) time and the dilatation in the temporal direction breaks this constraint. On the other hand, no problem should appear if the derivative expansion is restricted to the spatial directions (or at least, no new pathologies as compared to the zero temperature case). Moreover, gauge invariance can be accounted for if the bookkeeping parameter is introduced in the form $A_\mu(x_0, \lambda x)$, where $\lambda_\mu = (x_0, \lambda x)$.

The exact results in [5,8,9] were obtained for particular configurations of the gauge field using specially adapted methods. In this Letter we show that general configurations are also amenable to explicit calculation using the method of expanding the effective action in the number of spatial covariant derivatives. We present a direct calculation of the induced CS term at finite temperature which yields a simple explicit expression for this quantity. This expression holds for general configurations and agrees with all previously known exact results. Of course, in addition to this, the effective action contains further contributions which are completely regular (i.e., gauge and parity invariant and ultraviolet finite) which are also calculable within the same general scheme.

The fermionic effective action is defined as $W = -\log \text{Det } D$, where $D = \gamma_\mu D_\mu + m$ is the Dirac operator, $m$ is the fermion mass and $D_\mu = \partial_\mu + A_\mu$ is the covariant derivative. The Dirac operator acts on a single-particle Hilbert space which contains space–time, Dirac and internal degrees of freedom (i.e., those associated to the gauge group). The finite temperature $T = 1/\beta$ is introduced by compactifying the Euclidean time. As usual the fermionic wave functions are antiperiodic and the gauge fields are periodic in the time variable.

As is well known, the effective action is afflicted by several related pathologies, namely, ultraviolet divergences, many-valuation and anomalies in some of the classical symmetries of the Dirac operator. Our purpose is to isolate precisely those pieces of the effective action which can have an anomalous or many-valued contribution. This excludes ultraviolet finite pieces, which are always regular. The expansion in the number of spatial covariant derivatives [11,12] is appropriate for this kind of calculations, since this expansion preserves gauge invariance order by order and, in addition, terms beyond second order are ultraviolet finite.

A further simplification can be achieved by selecting only those terms which have abnormal parity, i.e., those containing a Levi-Civita pseudo-tensor, since the normal parity component of the effective action can be renormalized preserving all classical symmetries and is one-valued. In passing we note that “parity” (defined as space reflection but including $m \rightarrow -m$) is a symmetry of the classical action. Therefore, we would expect the normal (abnormal) parity component of the effective action to be an even (odd) function of the mass. This symmetry may be anomalously broken as a consequence of gauge invariance, as in the massless case [3].

2. The current

Following Schwinger [13], we will work with the current, which is better behaved than the effective action. The current is defined through the relation $\delta W = \int d^3 x \text{ tr}(J_\mu \delta A_\mu)$ (where $\delta A_\mu$ is a local variation, $J_\mu(x)$ is a matrix in internal space and tr refers to this space), and thus

$$J_\mu(x) = -\frac{1}{\gamma_\mu \text{tr}_D + m} |x\rangle.$$  \hspace{1cm} (1)  

(Trp refers to Dirac space.) In view of our previous remarks, our approach will be to compute the relevant terms of the spatial current $J_\mu$ and then reconstruct the effective action from it. This requires to expand the current in powers of $D_\mu$ retaining only terms with $\epsilon_{ij}$ and with precisely one $D_\mu$. A suitable way to deal with the matrix element at coincident points, which

1 This shows that each term of the expansion is independent of the method used in the calculation. This fails to hold for other expansions, such as the commutator expansion.

2 Our conventions follow those in [11,12] with $\eta = +1$.  

3 This remark applies to the massive case only. In the massless case [3], or if $|\mu| > |m|$ (where $\mu$ is the chemical potential) [20], the normal parity term combines with the abnormal parity one to yield a one-valued partition functional $\text{Det } D$.  

combines well with gradient expansion, is to use the
method of symbols [14], adapted to finite temperature
[11,12] and improved by Platenov and Banin [15,16].

This gives

\[ J_i(x) = -\frac{1}{p} \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} \mathrm{tr}_D(x)\gamma_i \frac{1}{\gamma_\nu \Delta_\nu + m} |0\rangle. \]  

(2)

In this formula [0] is the constant wave function, i.e.,
\( (x|0 = 1 \) which is periodic rather than antiperiodic),
p_0 = 2\pi (n + 1/2)/\beta runs over the fermionic Matsubara
frequencies, and finally, \( \Delta_\mu \) is related to the origi
nal covariant derivative through a double similarity transformation,

\[ \tilde{\Delta}_\nu = \epsilon^{j\nu\rho} \partial_j e^{-i\nu \rho} \partial_\nu e^{-i\rho} \Delta_i \]

\[ \left( \partial_j = \frac{\partial}{\partial p_j} \right). \]  

(3)

The inner similarity transformation yields \( \Delta_\mu \rightarrow \Delta_\mu + ip_\mu \) and corresponds to the original method of symbols. In that method, the integration over \( p \)
cancels all contributions except those where \( \Delta_i \) appears
inside a commutator (more precisely, in the form \{\( \Delta_i \), \}). In the improved method, this cancellation is
achieved prior to momentum integration by means of the
outer similarity transformation in (3). Explicitly:

\[ \tilde{\Delta}_0 = ip_0 + D_0 - i E_i \partial_i^p + \frac{1}{2} \{ \Delta_i, E_j \} \partial_i^p \partial_j^p + O(D_i^3), \]

\[ \tilde{\Delta}_1 = i p_1 + \frac{i}{2} F_{ij} \partial_j^p + O(D_i^3), \]  

(4)

where \( F_{\mu\nu} = \{ \Delta_\mu, D_\nu \} \) and \( E_i = F_{\nu i} \) is the electric
field. The improvement does not extend to the compacti
cated time coordinate, and indeed at finite temperature \( D_0 \) appears in two different ways which pre
serve gauge invariance, (i) inside commutators, \{\( D_0 \), \},
and (ii) through the (untracted) Polyakov loop \( \Omega(x) = T \exp(-\int_{t_0}^{t+\beta} A_0(t, x) dt) \) [12].

For our present purposes it will be sufficient to retain
in (4) only those terms with at most one spatial covariant derivative. Upon substitution of these expressions in (2), and taking the Dirac trace keeping only the abnormal parity terms, produces

\[ J_i^-(x) = 2i m \epsilon_{ij} \frac{1}{\beta} \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} (x) \frac{1}{\Delta} E_i \frac{1}{\Delta} |0\rangle \]

+ \( O(D_i^3), \)  

(5)

where \( \Delta = m^2 + p^2 + (p_0 - i D_0)^2 \). It is noteworthy
that the result is directly ultraviolet finite and also
consistent, i.e., a true variation.4

The current (5) picks up contributions from all
terms with two spatial indices in the effective action,
however, most of those terms are perfectly regular, in
the sense that they are ultraviolet finite, one-valued,
strictly gauge invariant and anomaly free, therefore,
we can simplify the calculation by isolating only the
anomalous terms, namely

\[ J_i^{an} = im \epsilon_{ij} \frac{1}{\beta} \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} (x) \frac{1}{\Delta^2} E_j |0\rangle. \]  

(6)

This “anomalous” current is still consistent. The difference
between the two currents, \( J_i^+ - J_i^{an} \), involves
\{\( D_0, E_i \)\} and derives from (fully calculable) regular
terms in the effective action which are quadratic in \( E_j \)[17]. On the contrary, the anomalous current does not
contain such a commutator and so it cannot derive from one of those terms. After carrying out the mo
mentum integration and the sum over frequencies, the
anomalous current takes the simple explicit form

\[ J_i^{an} = \epsilon_{ij} \langle x | \{ \phi(D_0), E_j \} |0 \rangle, \]  

(7)

where we have introduced the function

\[ \phi(x) = \frac{i}{8\pi} \frac{\sinh(\beta m)}{\cosh(\beta m) + \cosh(\beta a)}. \]  

(8)

This current preserves parity (as defined above), and
can be written in a manifestly gauge invariant form by
using the identity \( e^{-D_0} = \Omega(x) \) [12]:

\[ J_i^{an} \]

\[ = \frac{i}{4\pi} \epsilon_{ij} \left\{ \frac{\sinh(\beta m)}{2 \cosh(\beta m) + \Omega + \Omega^{-1}}, E_j \right\}. \]  

(9)

Note that this current gives the static limit of the
gauge field self-energy instead of the long-wave limit
[18]. The latter limit follows from restoring the regular
terms stripped from the anomalous piece. Our criterion
for selecting the anomalous piece is that the remainder
is strictly gauge invariant, however, regarding the
self-energy after rotation to real time, the anomalous
piece is analytical at zero momentum and energy
and the non-analytical behavior is contained in the
remainder [17].

4 Consistency requires \( \int d^3x \mathrm{tr}(\delta_1 A_1, \delta_2 A_2) \) to be symmetric
under the exchange of the labels 1 and 2.
At this step we can already verify that the zero temperature limit comes out correctly. Indeed, for large $\beta$
\[
J_i^{00} = \frac{i}{4\pi} \epsilon(m) \epsilon_{ij} E_j \quad (T = 0)
\]
(10)
(where $\epsilon(m)$ stands for the sign of $m$). This current derives from
\[
W_{an} = -\frac{1}{2} \epsilon(m) W_{CS} \quad (T = 0),
\]
where $W_{CS}$ is the Chern–Simons action
\[
W_{CS} = \frac{i}{4\pi} \int d^3 x \epsilon_{ij} \text{tr}(A_0 F_{ij} - A_i \partial_0 A_j).
\]
Eq. (11) is the standard induced Chern–Simons term at zero temperature [2,6,19].

3. The effective action

Using the techniques developed in [11,12] it is possible to reconstruct the anomalous piece of the effective action from its current while preserving manifest gauge invariance at every step of the calculation. This method will be presented elsewhere [17]. A more direct approach, which we will follow here, is to proceed by fixing the gauge.\(^5\) We choose the gauge so that $A_0(x)$ is stationary
\[
\partial_0 A_0(x) = 0.
\]
Such a gauge always exists (globally) [11]. In this gauge, (9) becomes
\[
J_i^{00}(x) = \epsilon_{ij} \{ \psi(A_0(x)), E_j(x) \}.
\]
(14)
The final step is to recover the effective action associated to this current. To this end we split the current into two parts $J_i^{(1)} + J_i^{(2)}$, corresponding to the decomposition of the electric field into $E_i = [D_0, A_i] - \partial_0 A_0$. Noting that in our gauge $A_0$ and $D_0$ commute, it is readily verified that the first piece derives from
\[
W^{(1)} = \int d^3 x \epsilon_{ij} \text{tr}(\psi(A_0) A_i [D_0, A_j]).
\]

Thus the remainder must satisfy
\[
\delta W^{(2)} = -\int d^3 x \epsilon_{ij} \text{tr}(\delta A_i [\psi(A_0), \partial_0 A_j]).
\]
(16)
To proceed, we further restrict the gauge (by means of a suitable subsequent stationary gauge transformation) so that $A_0(x)$ is everywhere a diagonal matrix, i.e.,
\[
\{ A_0(x), \partial_0 A_0(x) \} = 0.
\]
For subsequent reference we note that in this gauge
\[
[D_0, [D_0, A_i]] = [D_0, E_i].
\]
(18)
With this choice of gauge, (16) has the simple solution
\[
W^{(2)} = -\int d^3 x \epsilon_{ij} \text{tr}(\Phi(A_0) \delta_i A_j)
\]
(19)
provided that $\Phi'(a) = 2\psi(a)$ or, in our case,
\[
\Phi(a) = \frac{i}{2\pi \beta} \text{tanh}^{-1}(\text{tanh}(\beta m/2) \text{tanh}(\beta a/2)).
\]
(20)
Using the convenient notation of differential forms (for the spatial indices only), the final result obtained by adding up (15) and (19), takes the form
\[
W_{an} = \int d x_0 \text{tr}(\Phi(A_0)(A^2 - B) + \psi(A_0) A[D_0, A])
\]
(21)
(where $A = A_i dx_i$ and $B = dA + A^2 = \frac{1}{2} F_{ij} dx_i dx_j$ is the magnetic field). This action reproduces the spatial components of the current. Since any further contributions to the effective action cannot contain $A_i$, they would be of the form $\int d x_0 \text{tr}(f(A_0) dA_0 dA_0)$ and this vanishes identically in our gauge.

A subtle, but important, point is that, depending on the space manifold and the gauge group, a given configuration $A_0(x)$ may not be globally diagonalizable (i.e., with continuity at all points). Nevertheless, the “diagonal” gauge (17) always exists locally, that is, for each of the patches covering the space manifold. Without loss of generality we can assume that the support of the local variation in (16) takes place inside one of the patches, thus integration by parts in $x$ is allowed and the correct current is obtained from (19). Because the current (14) is patch independent, this suggests that the integrand in (21) is also patch independent. As shown in the next section, this is actually correct, confirming that the functional $W_{an}$ is well

\(^5\) The gauge conditions (13) and (17) depend only on $A_0$ and so they do not interfere with an arbitrary local variation of $A_i(x)$, as required to obtain the current and reconstruct the effective action from it.
defined. For simplicity, we will momentarily assume $A_0(x)$ to be diagonal in a global gauge.

Eqs. (10) and (11) show that the calculation is consistent with the known zero temperature limit, and this can also be verified directly from $W_{an}$. Next we can study the behavior of this functional under gauge transformations. We have to distinguish between the “gauge fixing” transformation needed to bring the original configuration to the gauge where $A_0$ is stationary and diagonal, on the one hand, and the allowed gauge transformations which preserve the gauge conditions (13) and (17), on the other. Of course, gauge invariance of the partition function under the gauge fixing transformation cannot be studied using our functional. Such gauge invariance follows from general arguments, e.g., by using the $\xi$-function version of the effective action [5]. The gauge conditions (13) and (17) are preserved by two kinds of gauge transformations, namely, stationary and discrete transformations [11]. The first class is that of transformations which are stationary and diagonal, and it is easily verified that $W_{an}$ is invariant under such transformations. The second class refers to those transformations of the form $U = \exp(x_0A)$, where $A$ is a constant diagonal matrix with entries $k_j = 2\pi in_j/\beta$ (with $n_j$ integer). Under discrete transformations $A_0$ transforms covariantly (i.e., homogeneously) whereas $A_0 \rightarrow A_0 + A$. The current (14) transforms covariantly (due to periodicity of $\psi(a)$) thus $W_{an}$ can only change by constant additive terms, that is, terms that do not give a contribution to the transformed current. This can be verified directly from the functional as follows. First, at a formal level the function $\Phi(a)$ is periodic and $W_{an}$ is invariant. More correctly, on its Riemann surface $\Phi(A_0)$ is invariant. More correctly, on its Riemann surface.

4. Topological issues

Let us now analyze the general case in which the diagonal gauge is not assumed to exist globally. In this case it will be convenient to work in the global gauge where $A_0$ is stationary but not necessarily diagonal, and rewrite $W_{an}$ in that gauge. This yields

$$W_{an} = \int dx_0 \text{tr}(\Phi(A_0) \left( A^2 - B \right) + \phi(A_0) A[D_0, A]).$$

(22)

In this formula $A$ denotes the vector field $A$ of the diagonal gauge rotated covariantly back to the stationary gauge. That is, if $U$ denotes the stationary gauge transformation which brings $A_0$ to a (local) diagonal gauge,

$$A = A + dU U^{-1}.$$  

(23)
A crucial property of (22) is that it remains unchanged under the replacement
\[ A \rightarrow A' = A + C, \]  
provided that the field \( C(x) \) satisfies
\[ \partial_0 C = [A_0, C] = 0. \]  
This is readily verified noting that \( C \) is a 1-form and \( A_0, D_0 \) and \( C \) are all commuting quantities.

By construction \( A \) transforms covariantly (under transformations of the global stationary gauge and fixed local diagonal gauge), but is not globally defined since the gauge transformation \( U \) only exists locally. Let \( M^{(k)} \) denote a set of local charts covering the space manifold. In each chart we can take a diagonalizing gauge transformation \( U^{(k)} \) and this defines a corresponding “covariant vector field” \( A^{(k)} \). What have to be verified is that the 2-form in (22) takes the same value in any of the charts.

To show this let us note that the relation (18), which was written in the diagonal gauge, in the stationary gauge becomes
\[ [D_0, [D_0, A^{(k)}]] = [D_0, E]. \]  
Therefore the field \( C^{(k, \ell)} = A^{(k)} - A^{(\ell)} \) (which is stationary due to (23)) satisfies
\[ [A_0, [A_0, C^{(k, \ell)}]] = 0. \]  
This already implies that \( C^{(k, \ell)} \) commutes with \( A_0 \) and satisfies (25). As a consequence any two \( A^{(k)} \) at the same point are related as in (24) and \( W_{an} \) is path independent.

It is noteworthy that the field \( A_0'(x) \), denoting \( A_0 \) in a diagonal gauge, is (or can be taken to be) equal in all patches. Indeed, the local eigenvalues of \( A_0(x) \), \( a_\ell(x) \), can be labeled at each point so that they are continuous functions on the space manifold. Thus, without loss of generality we can choose the local diagonal gauges in such a way that the eigenvalues are ordered in all patches in the same way and so the \( A_0'(x) \), which are diagonal matrices containing the eigenvalues, will be the same matrix in all patches. However, although \( A_0'(x) \) is global, it needs not correspond to a global gauge; in general, the eigenvectors corresponding to \( a_\ell(x) \) cannot be chosen in a continuous way on the space manifold. Assuming the non-degenerated case for simplicity, the choice of eigenvectors in each patch will differ by a phase (and perhaps normalization, depending on the gauge group). These phases are contained in the diagonal transition matrices \( U^{(\ell)}{^{-1}} U^{(k)} \), which contain the non-trivial topology of \( A_0(x) \) (under gauge transformations).

When we bring a completely general gauge field configuration to an \( A_0 \)-stationary gauge the result is not unique. The different possible stationary gauges so obtained are related either by a stationary gauge transformation or by a discrete transformation [11]. The discrete transformations are of the form \( U = \exp(\chi_0 \Lambda) \), where \( \Lambda(x) \) is stationary, commutes with \( A_0(x) \) and has eigenvalues \( \lambda_j = 2\pi i n_j / \beta \) (with \( n_j \) integer). The discussion of how \( W_{an} \) changes under these transformations is essentially the same as that given before for the case of a global diagonal gauge. It can be noted, however, that the discrete transformations need not be topologically small if the diagonal gauge is not global (an explicit SU(2) example is shown in [2], also discussed in [11].)

5. Conclusions

In summary, we have shown that the method of expanding in the number of spatial covariant derivatives is suitable to make computations at finite temperature, and in particular it is able to retain all subtleties usually tied to the effective action, such as topological pieces, many-valuation and anomalies. At the practical level, the method have been shown to be quite efficient, yielding a simple explicit form for the induced CS term at finite temperature, without assuming special constraints on the gauge field configuration.

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Path integral measure in Regge calculus from the functional Fourier transform

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Abstract

The problem of fixing measure in the path integral for the Regge-discretised gravity is considered from the viewpoint of it being “best approximation” to the already known formal continuum general relativity (GR) measure. A rigorous formulation may consist in treating the measure as functional on the space of the metric functionals. We require coincidence of the measures for the discrete and continuous versions of the theory on some sufficiently large (dense) set of metric functionals which exist and admit exact definitions and calculation in the both versions. This set consists of generalisation of the usual finite-dimensional plane waves to the functional space so that the discrete measure follows by means of the functional Fourier transform. The possibility for such set to exist is due to the Regge manifold being a particular case of general Riemannian one (Regge calculus is a minisuperspace theory). Only a certain continuum measure among the local ones (the scale invariant Misner measure) is found to be reducible in this way to the well defined Regge discretisation, and we find the two versions for the latter depending on what metric tensor, covariant or contravariant one, is taken as fundamental field variable. The closed expressions for the measure are obtained in the two simple cases of Regge manifold. These turn out to be quite reasonable one of them indicating to possibility of passing in backward direction when appropriately defined continuum limit of the Regge measure would reproduce the original continuum GR measure.

Regge calculus still remains the most natural discrete regularisation of general relativity (GR) promising from the viewpoint of constructing well-defined quantum gravity theory [1]. The result of quantisation being expressed in the form of the path integral, the key question is that of the choice of the integration measure. In particular, the earliest quantum formulation of 3D Regge calculus [2] is based on specific property of 6j-symbols whose product for large values of arguments reduces to a kind of the path integral with the Regge calculus action, the arguments of 6j-symbols being interpreted as linklengths. There are a number of models generalising these results to the physical 4D case [1], of which the Barrett–Crane one [3] attracts much attention for it analogously reproduces path integral with the Regge calculus action [4]. The path integral measure in numerical simulations is usually chosen as the simplest among the invariant ones [5]. Normalising the measure w.r.t. the DeWitt supermetric would allow to fix the measure uniquely [6]. However, in the 4D case this construction turns out to suffer from unrenormalisable UV divergences provided by singular nature of Regge manifold; dis-
cretisation of the Faddeev–Popov ghost field improves the situation, but the measure turns out to be singular at the point of superspace of metrics corresponding to the flat spacetime [7]. The reason for this is rather simple and connected with the change of gauge content of the theory in the flat spacetime when a certain variations of linklengths become gauge ones since they do not change geometry. The same unpleasant feature displays also in the canonical quantisation of the (3 + 1)D (continuous time) Regge calculus [8]. The singularity of the measure at the flat spacetime makes extracting physical consequences from the theory a difficult task because of the absence of the perturbative expansion around the flat spacetime. Therefore, it may be useful to study the problem of quantum Regge calculus within another framework. Thus far Regge calculus has been treated as independent theory without any reference to the continuum GR. Now consider it as general Riemannian one, the functional on Riemannian general Riemannian one, the functional on Riemannian metrics can be viewed at the same time as that on continuum GR. In other words, instead of the principle “first discretise, then quantise” we are trying to employ in some sense the reverse sequence of operations, “first quantise, then discretise”. So we need to define a notion of “the best approximation” to the known formal expression for the continuum measure
\[ d\mu_C = \prod_x g^{-5/2} d^{10}g_{ik}, \quad g \equiv \det \|g_{ik}\|. \] (1)

This simplest local invariant measure has been proposed by Misner [9]. The DeWitt approach [10] based on normalising measure w.r.t. a group-invariant metric \[ \|\delta g_{ik}\|^2 = \int (g^{ik} g^{lm} + C g^{il} g^{km}) \delta g_{il} \delta g_{km} \sqrt{g} d^4x \] for the space of the \( g_{ik} \) rejects the Misner measure (in fact, this leads in the 4D case to the pure product of differentials \( \prod_x d^{10}g_{ik} \)). However, there is one else point of view on normalising measure w.r.t. the such supermetric, if one considers the invariant volume element \( d\Omega = \sqrt{g} d^4x \) instead of \( d^4x \) as fundamental object not depending on metric. Then one gets just Misner measure, Eq. (1). Example is the Polyakov’s treatment of the 2D gravity for the needs of string theory: thus normalised w.r.t. the supermetric the Polyakov’s measure is scale-invariant [11]. Proceeding from the time evolution in the canonical quantisation framework, however, Leutwyler has suggested the measure \[ \prod_x g^{-3/2} g^{00} d^{10}g_{ik} \] [12], not so simple. Although derived from consideration of a nonunitary \( S \)-matrix, this result has been reproduced in [13] and proved to satisfy the requirement to cancel all the (UV) divergences of the type \( \delta^{(4)}(0) \) which arise in the theory due to its nonlinearity. The specific role of the time coordinate \( x^0 \) is connected with the particular (Hamiltonian) way of regularisation of the UV divergent integrals which is not unique: changing the regularisation procedure can lead to, e.g., coordinate \( x^1 \) being singled out.

Now, what choice of the \( d\mu_C \) is more appropriate from the viewpoint of the subsequent passing to the Regge lattice? There is no UV divergences nor the single coordinate in the completely discrete Euclidean Regge calculus, so the criteria leading to the Leutwyler measure seem to be not actual in this respect. Instead, the (scale) invariance argument is usually considered to be reasonable in the Regge lattice aspect. But second and more important argument in favour of Eq. (1) is that, namely, this version admits the natural reduction to the Regge lattice by the method of the present Letter as discussed after Eq. (19).

The notion of “the best approximation” to \( d\mu_C \) can be given a strict sense by treating the measure as a functional \( \int \cdot d\mu \) on the space of the functionals of metric. Since Regge metric is a particular case of general Riemannian one, the functional on Riemannian metrics can be viewed at the same time as that on Regge ones. Thus, the two measures, continuum (1) and discrete Regge one of interest \( d\mu_R \) can be defined on the same set of functional metrics. Looking for such set dense in the space of metric functionals in the appropriate topology and requiring that both measures would coincide on this set we can define \( d\mu_R \). The exponents of linear functionals of metric (“functional plane waves”) just present such set, probably the only one which gives possibility to have expressions definable and calculable on the functional level.

The above approach is natural also from the axiomatic point of view where a functional subject to a certain set of axioms, the Osterwalder–Schrader ones, can be considered as functional Fourier transform of a measure of some quantum field theory [14], the so-called characteristic functional usually referred to in the physical literature as generating functional. The analog of the characteristic functional considered in our case takes the form
\[ \hat{\mu}_C(f) = \int e^{i\tilde{g}(f)} d\mu_C(g), \] (2)
where there are the two possibilities for the linear metric functional $g(f)$,

$$
\int f_{ik} g^{ik} \sqrt{g} \, d^4 x \equiv g_1(f) \quad (3)
$$

or

$$
\int f^{ik} g_{ik} \sqrt{g} \, d^4 x \equiv g_2(f) \quad (4)
$$

depending on what metric tensor, $g^{ik}$ or $g_{ik}$, is chosen as true field variable; the $f_{ik}(x)$ or $f^{ik}(x)$ is probe function (since quantum fields are generally treated as distributions, the probe functions are usually supposed to be infinitesimally differentiable with compact supports).

Strictly speaking, the measure involved in the definition of the characteristic functional should include also $\exp(-S)$, the $S$ being the (Euclidean) gravity action. Occurrence of this factor would make the explicit calculations not easier than defining and calculating the gravity path integral itself. Therefore, we are trying to define Regge analog $d\mu_R$ of the $d\mu_C$ separately from $\exp(-S)$. A point of view on omitting this factor within strict framework of characteristic functional may consist in saying that the strong coupling limit ($S \to 0$) is considered.

Thus, our approach to definition of the Regge measure $d\mu_R$ amounts to setting

$$
\tilde{\mu}_R(f_R) = \tilde{\mu}_C(f_R) \quad (5)
$$
on a discretised version $f_R$ of the probe functions. The only natural choice for the tensor $f_R$ on Regge manifold is to take it being piecewise-constant in the piecewise-affine frame, that is, constant on each the 4-simplex whenever $g_{ik}$ is constant on it. Then one tries to define $\tilde{\mu}_C(f_R)$ (where $f_R$ is not smooth but is a limit of smooth functions).

Strictly speaking, the measure $d\mu_C$ does not exist as mathematical object, a regularisation is implicit. There should be some care with this regularisation. For example, the measure $d\mu_C$ looks formally positive, and regularisation should keep this property. Convenience of the characteristic functional is, in particular, just that the positivity property looks rather simple if written in terms of this functional,

$$
\sum_{\alpha, \beta=1}^N c_\alpha c_\beta \tilde{\mu}_C(f_\alpha - f_\beta) \geq 0 \quad (6)
$$

for any sequence of the probe functions $f_\alpha$ and complex numbers $c_\alpha$, $\alpha = 1, 2, \ldots, N$. If then $d\mu_R$ is defined via (5), it is positivity immediately follows from (6). So we imply that positivity of the measure, if required, is ensured in the continuum GR; then it is guaranteed for our construction of the Regge measure too.

Now turn to our characteristic functional (2) which proves to be the product over points,

$$
\tilde{\mu}_C(f) = \prod_s I(x), \quad (7)
$$
of the factors (for $g(f) = g_1(f)$)

$$
I = I_1 = \int e^{if_{ik} g^{ik} \sqrt{g} \, d^4 x} g^{-\frac{5}{2} + \epsilon} d^{10} g_{ik}. \quad (8)
$$

Here a nonzero $\epsilon$ is introduced because, as mentioned above, the measure $d\mu_C$ does not exist without regularisation, therefore not specifying the latter this measure can be understood whenever this is possible in the sense of analytical continuation from the sufficiently large positive $\epsilon$ where (8) can be defined to the point $\epsilon = 0$ of interest; the $d^4 x$ is an infinitesimal “bare” (i.e., corresponding to the Euclidean metric $g_{ik} = \delta_{ik}$) 4-volume associated to a point. To calculate this (and $I = I_2$ for $g(f) = g_2(f)$) note that

$$
g^{-\frac{5}{2} + \epsilon} d^{10} g_{ik} = (\det(g^{ik} \sqrt{g}))^{-\frac{5}{2} + \epsilon} d^{10} (g^{ik} \sqrt{g}) = (\det(g_{ik} \sqrt{g}))^{-\frac{5}{2} + \epsilon} d^{10} (g_{ik} \sqrt{g}). \quad (9)
$$

Perform the following change of variables,

$$
g^{ik} \sqrt{g} = \sum_A e^i_A \lambda_A e^k_A, \quad (10)
$$

where $e^i_A = 0$ at $A > i$, $e^i_i = 1$ at $A = i$ (this is the Gaussian decomposition of the symmetrical matrix into the product of a diagonal and a triangle one with unity diagonal elements). Integral turns out to be Gaussian over $d^6 e^i_A$ and factorisable over $d^4 \lambda_A$. Physical region of positivity of $g^{ik} \sqrt{g}$ is picked out by inequalities $\lambda_A > 0 \forall A$. Correspondingly, over $\lambda_A$ the cosine (sine) Fourier transfer in that region is
two covariant tensor in a 4-simplex. But whereas unambiguously parameterise a symmetrical rank \( \sigma \)

\[ f(\sigma) \]

link \((ab)\) does not depend on the choice of \( \sigma \) constant in a simplex \( \sigma \)

\[ \text{Important is only proportionality to accidentally, the same follows for sine transform).} \]

\[ I(x) = 2 \sum_{(ab) \subset \sigma} f(\sigma) \frac{dV}{d\tau_{(ab)}} \]

expression for \( I_2 \) corresponding to the choice \( g(f) = g_2(f) \) follows by replacing \( \epsilon \) by \( \epsilon/3 \) and substituting \( f \) by \( f^{ik} \).

Now consider reduction of \( \exp(i g(f)) \) and \( \tilde{\mu}(f) \) to the Regge lattice; the functional relating these two reduced objects will be just the discrete measure of interest. The piecewise-affine frame is fixed by attributing the coordinates \( x^i, i = 1, 2, 3, 4 \) to each vertex \( a \). The length squared \( s(ab) \) of the link \((ab)\) connecting the vertices \( a \) and \( b \),

\[ s(ab) = l_{ab} \eta_{ab} g(\sigma), \quad l_{ab} \equiv x_{a}^{i} - x_{b}^{i}, \quad (ab) \subset \sigma \]

is a particular example of the so-called edge components \( f(\sigma) \) of a symmetrical second rank tensor \( f^{ik} \) constant in a 4-simplex \( \sigma \) [15],

\[ f(\sigma) = l_{ab}^{(ab)} f^{ik}(\sigma). \]

Here \( g_1(\sigma) \) and \( f^{ik}(\sigma) \) are the values of the tensors in a simplex \( \sigma \) containing the link \((ab)\). The edge components unambiguously parameterise a symmetrical rank two covariant tensor in a 4-simplex. But whereas \( s(\sigma) \) does not depend on the choice of \( \sigma \) containing the link \((ab)\), the \( f(\sigma) \) may do so. However, the number of variables \( f(\sigma) \) does not depend on the choice of \( \sigma \) containing the link \((ab)\), the \( f(\sigma) \) may do so. However, the number of variables \( s(\sigma) \) parameterising the metric, i.e., the number of links. Therefore, the condition is required that the variables \( f(\sigma) \) should not depend on \( \sigma \supset (ab) \). The possibility to have \( f(\sigma) \) constrained by this condition becomes evident if one imagines that \( f(\sigma) \) are the new squared linklengths of our Regge manifold instead of \( s(ab) \), the scheme of linking and coordinates of vertices being the same (some of these linklengths can be made imaginary, if necessary, in the sense of analytical continuation); the metric tensor in the piecewise-affine frame in the 4-simplices of thus constructed Regge lattice will be just \( f^{ik}(\sigma) \). Then according to the Ref. [15] the functional (3) on Regge lattice functions takes the form

\[ g_1(f_R) = 2 \sum_{(ab) \subset \sigma} f(\sigma) \frac{dV}{d\tau_{(ab)}} \]

\[ = 2 \sum_{(ab) \subset \sigma} f(\sigma) \frac{dV}{d\tau_{(ab)}} \]

Here \( V(\sigma) \) is the volume of \( \sigma \), \( V = \sum_{\sigma} V_\sigma \) is the volume of the manifold (in the compact case).

Analogously, let \( f^{(ab)} \) be independent variables living on the links. Let us define the contravariant symmetrical rank two tensor \( f^{ik} \) constant inside each the 4-simplex \( \sigma \),

\[ f^{ik}(\sigma) = \sum_{(ab) \subset \sigma} f^{(ab)} \eta_{ab}^{ik}. \]

Using this ansatz we get

\[ g_2(f_R) = \sum_{(ab) \subset \sigma} f^{(ab)} s(ab) V_{(ab)}. \]

Here we have introduced notation for a volume associated to a link,

\[ V_{(ab)} = \sum_{\sigma \supset (ab)} V_\sigma. \]

Next reduce the expression (7), (8) to the Regge lattice when \( f(x) \) is piecewise-constant, \( f(x) = f(\sigma) \) whenever \( x \in \sigma \). Then \( I(x) \) is piecewise-constant too, and for \( g(f) = g_1(f) \) we have

\[ \prod_{x} I(x) = \prod_{\sigma \ni x} I_1(\sigma) \]

\[ = \prod_{\sigma} I_1(\sigma)^{N_\sigma} \sim \prod_{\sigma} (\det f(\sigma))^{-\epsilon N_\sigma} \]

(and analogously for \( g(f) = g_2(f) \) with the replacement \( \epsilon \to \epsilon/3 \) where only dependence on \( f \) is shown. Here \( N_\sigma \) is a number of points contained in a simplex \( \sigma \); of course, the continuum measure is defined in the limit \( N_\sigma \to \infty \) starting from the originally finite \( N_\sigma \). If integration over metric is made, information on
the simplex size is lost, and the only choice symmetrical w.r.t. the different simplices and points is to consider \( N \) being equal to the same value \( N' \) for all the \( \sigma \)'s. Then, if we keep \( N \) finite before taking the limit \( \epsilon \to 0 \), we can redefine \( N \epsilon \to \epsilon \) (or \( N \epsilon / 3 \to \epsilon \) for \( g(f) = g_2(f) \)), so that

\[
\hat{\mu}_C(f_R) \sim \prod_\sigma (\det(f(\sigma)))^{-\epsilon}. \tag{19}
\]

This corresponds to the naive idea that the product over points should turn, up to a normalisation factor, into the same product but over simplices. Also we observe that it is, namely, the measure (1) for which this correspondence takes place; were it not scale invariant, we would have some finite exponent in the Fourier transform instead of \( \epsilon \) and would lose possibility to rescale it as \( N \epsilon \to \epsilon \) above to avoid infinite exponent in the limit \( N \to \infty \). Take into account parameterisation of the tensors \( f_{ik} \), \( f_{ik} \) in terms of the edge components (13), (15). Then

\[
\text{det}\| f_{ik}(\sigma) \| = (\text{det}\| l_{ab}^{i} \|)^{-2} \Delta_1(f; \sigma) \tag{20}
\]

where \( \Delta_1(f; \sigma) \) is the so-called bordered determinant [16] composed of the variables \( f_{(ab)} \) living on the links \( (ab) \) belonging to \( \sigma \). Note that \( (\Delta_1(s; \sigma))^2 = V_\sigma \), the volume of \( \sigma \). The \( \| l_{ab}^{i} \| \) means the matrix of any four link vectors \( l_{ab}^{i} \) of the simplex not laying in the same 3-plane. In the case \( f = f^{ik} \) we can find even more simple expression,

\[
\text{det}\| f^{ik}(\sigma) \| = (\text{det}\| l_{ab}^{i} \|)^{2} \Delta_2(f; \sigma), \tag{21}
\]

where the summation runs over all the unordered combinations of the four links of the simplex, \( (a_i b_i) \), \( i = 1, 2, 3, 4 \) not laying in the same 3-plane.

Finally, we write out up to the normalisation factor the relation which fixes the Regge measure if we choose for the fundamental metric field the \( g^{ik} \).

\[
\int \exp\left( 2i \sum_{(ab)} f_{(ab)} \frac{\partial V}{\partial s_{(ab)}} \right) d \mu_R^{(1)}(s) = \prod_\sigma (\Delta_1(f; \sigma))^{-\epsilon}, \tag{22}
\]

or \( g_{ik} \).

\[
\int \exp\left( i \sum_{(ab)} f_{(ab)} s_{(ab)} V_{(ab)} \right) d \mu_R^{(2)}(s) = \prod_\sigma (\Delta_2(f; \sigma))^{-\epsilon}. \tag{23}
\]

This is the main result of the Letter in the implicit form.

Consider the simplest case of (closed) Regge manifold consisting of the two identical 4-simplices \( \sigma_1 \), \( \sigma_2 \) with mutually identified vertices. Then \( f_{(k)}(\sigma_1) = f_{ik}(\sigma_2) \) or \( f^{ik}(\sigma_1) = f^{ik}(\sigma_2) \) if parameterised by \( f_{(ab)} \) or \( f_{(ab)}^{(i)} \) according to (13) or (15). Now using \( f_{(ab)} \) and \( f_{ik} \) (or \( f^{ik} \)) as Fourier conjugate variables is equally convenient, because the number of links \( (ab) \) coincides with the number 10 of the components of \( f_{ik} \) (or \( f^{ik} \)) taken in one of the simplices. So we do not need to parameterise tensors by the edge components and can write immediately

\[
\int \exp\left( \frac{2i}{4!} f_{ik} g^{ik} \sqrt{g} \text{det}\| l_{ab}^{i} \| \right) d \mu_R^{(1)}(g) = (\text{det}\| f_{ik} \|)^{-2\epsilon} \tag{24}
\]

or

\[
\int \exp\left( \frac{2i}{4!} f^{ik} g_{ik} \sqrt{g} \text{det}\| l_{ab}^{i} \| \right) d \mu_R^{(2)}(g) = (\text{det}\| f^{ik} \|)^{-2\epsilon} \tag{25}
\]

instead of (22), (23) (again, up to a normalisation factor). Here we have taken into account that the product in the RHS consists of the two identical factors, so the exponent is simply rescaled, \( \epsilon \to 2\epsilon \).

The inverse Fourier transform is then straightforward and gives

\[
d \mu_R^{(1)} = d \mu_R^{(2)} = (\text{det}\| g_{ik} \|)^{-5/2} d^{10} g_{ik} \tag{26}
\]

up to normalisation, or, in terms of linklengths,

\[
d \mu_R = V^{-5} d^{10} s, \tag{27}
\]

the \( V \) being the volume of the simplex, \( V = (\Delta_1(s))^2 \). Thus, the relations for the measure Eqs. (22), (23) obtained lead to the reasonable result for the particular case of the simplest Regge lattice. In the general case the \( g_{ik} \) in the different simplices are different but
related via common edges; the answer is convolution of the expressions like Eq. (27) of the two types, Eqs. (30), (31) below. The above example deals with the strongly curved spacetime; next consider the simplest Regge minisuperspace model of the flat spacetime. Take the flat 4-parallelepiped with all it is diagonals emitted from one of it is vertices and compactified toroidally by imposing periodic boundary conditions (on the linklengths). This is the simplest, consisting of 24 4-simplices elementary cell of the periodic Regge lattice [17] specified here by the conditions of compactness and flatness. The flatness means that the linklengths of the body and hyperbody diagonals can be expressed in terms of the linklengths of the 4 parallelepiped edges and 6 face diagonals. Equivalently, the metric terms of the linklengths of the 4-simplices can be taken the same in all the 24 4-simplices. Since the number of components $g_{ik}$ coincides with the number 10 of independent linklengths, as in the example above, we again may work not passing to the variables $s$. Further, if we study the measure on the Regge minisuperspace constrained by additional conditions on the linklengths or metric, we need the same number of the conditions also on the Fourier conjugate variables $f$. In our case metric $g_{ik}$ being the same in all the 24 equivalent 4-simplices, the Fourier transform of the measure of interest depends on $f^{ik}(\sigma)$ through the sum $f^{ik}(\sigma_1) + f^{ik}(\sigma_2) + \cdots + f^{ik}(\sigma_{24})$. The most symmetrical way of setting the conditions on $f^{ik}(\sigma)$ is to equate these for all the 4-simplices, $f^{ik}(\sigma) \equiv f^{ik}$, and analogously for $f_{ik}$. Finally, in the RHS we have the product of the 24 identical factors, so the exponent $\epsilon$ is rescaled to 24$\epsilon$.

\[
\int \exp(i f_{ik} g^{ik} \sqrt{g} \det]|f_{ab}||) \, d\mu_{(ab)}^{(3)}(g) = (\det |f_{ik}|)^{-24\epsilon},
\]

\[
\int \exp(i f^{ik} g^{ik} \sqrt{g} \det]|f_{ab}||) \, d\mu_{(ab)}^{(4)}(g) = (\det |f^{ik}|)^{-24\epsilon}.
\]

The answer is notationally the same as in the above example, Eq. (26).

Thus, in the two examples, those of strongly curved and flat Regge manifolds we have obtained the same expressions for the measure written in terms of metric. This means that the measure cannot crucially depend on the curvature. On the other hand, the example of the flat spacetime might be relevant to the continuum limit of the Regge calculus. Indeed, if one triangulates a fixed smooth manifold with the help of Regge manifolds and tends the maximal linklength $a$ of these manifolds to zero making triangulation finer and finer, then the angle defects of these Regge manifolds tend to zero as $R a^2$, $R$ being typical curvature of the smooth manifold. The result we have obtained is just the expression for the continuum measure, although for a specific case when the product over points runs over only one point.

In general case we can write out explicit expression for the measure as convolution of elementary measures like (27) for all the 4-simplices,

\[
d\mu_{(ab)}^{(1)} = \prod_{(ab)} \left[ d\left( \frac{\partial V}{\partial s_{(ab)}} \right) \right]
\times \int \left\{ \prod_{\sigma} \Delta_2(h_{(ab)}^\sigma ; \sigma)^{-5/2+\epsilon}
\times \prod_{(ab)} \left[ \delta \left( \sum_{\sigma \supset (ab)} h_{(ab)}^\sigma \right) - \frac{\partial V}{s_{(ab)}} \right]
\times \prod_{\sigma \supset (ab)} d\mu_{(ab)}^{(1)} \right\} \right\}
\]

or

\[
d\mu_{(ab)}^{(2)} = \prod_{(ab)} \left[ d(s_{(ab)} V_{(ab)}) \right]
\times \int \left\{ \prod_{\sigma} \Delta_1(s_{(ab)}; \sigma)^{-5/2+\epsilon}
\times \prod_{(ab)} \left[ \delta \left( \sum_{\sigma \supset (ab)} s_{(ab)} - s_{(ab)} V_{(ab)} \right)
\times \prod_{\sigma \supset (ab)} d\mu_{(ab)}^{(2)} \right\} \right\},
\]

where $s_{(ab)}$ and $h_{(ab)}^\sigma$ are dummy variables living on the pairs 4-simplex—edge. It is taken into account that $\Delta_1^{-\epsilon}$ and $\Delta_2^{-5/2+\epsilon}$ or $\Delta_1^{-5/2+\epsilon}$ and $\Delta_2^{-\epsilon}$ are mutually connected by Fourier transform, as it follows from the relation of $\Delta_1$ and $\Delta_2$ to determinants of the co- and contravariant metric. The Eqs. (30), (31) constitute the main result of the Letter in the explicit form.
Despite that the two versions of the Regge measure coincide in the above simple cases, probably the crucial difference between them would display in the 2D model. There an analog of the Eq. (30) could not be derived directly by Fourier transform of the continuum measure because the functional plane waves as functionals of $g^{ik}\sqrt{g}$ do not depend on the conformal degree of freedom of the metric and thus do not form a dense set. But even being derived via analytic continuation from the dimensionality $n \neq 2$, the $d\mu_{(1)}^R$ given by Eq. (30) (where now $V$ is the total square) is degenerate for it does not depend on the differential of the global conformal degree of freedom, while the $d\mu_{(2)}^R$ does so. This can serve as some argument in favour of the version $d\mu_{(2)}^R$, although the absence of the 2D puzzle cannot be the criterium for the 4D case.

The expressions (30), (31) remind those for Feynman diagrams in the usual quantum field theory. However, the role of propagators is played by the fourth order polynomials raised to the negative half-integer power and with nontrivial position of zeroes. This makes analytic evaluation of nontrivial such graph quite difficult. But prior to that the problem of regularising the original continuous measure should be considered. In the simple examples of the present Letter the explicit form of this regularisation turns out to be unimportant for the formal expression for the resulting Regge measure in the limit $\epsilon \to 0$. In the general case it is unclear whether expressions (30), (31) remain finite at $\epsilon \to 0$ without any additional regularisation of $d\mu_C$ or not; if not, this would mean that the final formal expressions for $d\mu_R$ would generally depend on this regularisation.

To conclude, it is possible to define Regge measure in a strict way treating it as functional from the requirement for it to coincide with known continuum measure functional on their common dense set of definition. It turns out that, first, the well defined counterpart on Regge lattice exists only for a certain (Misner) measure among the local ones and, second, the number of different measures obtained in such the way (the extent of ambiguity of the procedure) is practically two.

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References

Erratum to: “Sudakov effects in BBNS approach”


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1. On p. 56, Eq. (15) should read as

\[(x - y) e^{-i(x-y)\cdot P} = \frac{1}{m_B^2} e^{-i(x-y)\cdot P}.
\]

2. In Eq. (18), the \(f_\pi \) and \(K^{-1} \left(-i \sqrt{u v m_B^2} \right) \) should be substituted by \(f_2 \) and \(\frac{b_2}{-2i \sqrt{u v m_B^2}} K^{-1} \left(-i \sqrt{u v m_B^2} \right) \), respectively.

3. On p. 57, the second paragraph is modified as follows:

Define \( f \) as the value of the hard spectator scattering contribution divided by the lowest order result

\[ f = \frac{S_{b+h}}{f_2 f_3} \]

where \( f_2 \) and \( f_3 \) represent the contribution of twist-2 and twist-3 terms. The numerical result (for \((V - A) \otimes (V - A)\) operator insertion) is: Twist-2: in BBNS approach, \( f_2 = 0.043 \); in the modified PQCD approach, \( f_2 = 0.057 + i 0.0037 \); Twist-3: in BBNS approach, \( f_3 \) cannot be calculated, it is expressed in terms of phenomenological parameters \( \rho_H \) and \( \phi_H \) [1].

\[
\begin{align*}
 f_3 &= \frac{\pi \alpha_c C_F f_B f_{2\pi}^2}{m_B^2 f_0^2} \frac{1}{N_c^2} \int_0^1 d\xi \phi_B(\xi) \int_0^1 dx \phi_A(x) \frac{2\mu_c}{m_B} (1 + \rho_H e^{i\phi_H}) \ln \frac{m_B}{M_b},
\end{align*}
\]

where \( \rho_H \leq 1 \), \( M_b = 0.5 \) GeV; in the modified PQCD approach, \( f_3 = 0.0166 - i 0.0144 \). Compare the two results, we can get \( \rho_H = 0.97 \), which is within the constraint of \( \rho_H \leq 1 \), and the strong phase is very large, it is \( \phi_H = -83.7^\circ \).

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In Eqs. (43) and (46) the product should be expanded up to $k$. It reflects the physical meaning of the correlation term between the $(i) - (i+1)$ and $(k) - (k+1)$ borders:

$$2(Z_i - Z_{i+1})H_{i+1}H_{i+2} \cdots H_{k-1}H_k(Z_k - Z_{k+1}),$$

which has to involve all $(i+1) - (k)$ internal medium gaps.
Erratum to:

“Low lying $S = -1$ excited baryons and chiral symmetry”


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In this Erratum we correct the values quoted in Phys. Lett. B 527 (2002) 99–105 for the coupling constants of the $S = -1$ s-wave resonances. In that work the modulus of the couplings should be changed to the modulus squared. Therefore, one should replace Eqs. (13), (14), (16) and (18) by:

\[
\begin{align*}
|g_{KN}|^2 &= 0.51, \quad |g_{\pi \Sigma}|^2 = 0.052, \quad |g_{\pi \Lambda}|^2 = 1.0, \quad |g_{K \Xi}|^2 = 11, \\
|g_{KN}|^2 &= 0.61, \quad |g_{\pi \Sigma}|^2 = 0.073, \quad |g_{\pi \Lambda}|^2 = 1.1, \quad |g_{K \Xi}|^2 = 12, \\
|g_{KN}|^2 &= 7.4, \quad |g_{\pi \Sigma}|^2 = 2.3, \quad |g_{\pi \Lambda}|^2 = 2.0, \quad |g_{K \Xi}|^2 = 0.12, \\
|g_{KN}|^2 &= 2.6, \quad |g_{\pi \Sigma}|^2 = 7.2, \quad |g_{\pi \Lambda}|^2 = 4.2, \quad |g_{\eta \Sigma}|^2 = 3.5, \quad |g_{K \Xi}|^2 = 12.
\end{align*}
\]

We acknowledge Dr. D. Jido for communicating this erratum to us.

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